VOCATIONAL MATHEMATICS

BY

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PREFACE

The author has had, during the last ten years, considerable experience in organizing and conducting intermediate and secondary technical schools. During this time he has noticed the inability of the regular teachers in mathematics to give the pupils the training in commercial and rule of thumb methods of solving mathematical problems that are so necessary in everyday life. A pupil graduates from the course in mathematics without being able to "commercialize" or apply his mathematical knowledge in such a way as to meet the needs of trade and industry.

It is to overcome this difficulty that the author has prepared this book on vocational mathematics. He does not believe in doing away with the regular course of mathematics but in supplementing it with a practical course. This course may take the place of the first year algebra and the first year geometry in vocational classes in which it is not desirable to give the traditional course in algebra and geometry.

This book may be used by the regular teacher in mathematics and by the shop teacher. It can be used in the shop in teaching mathematics and in providing drill problems upon the shop work. A course based upon the contents of the book should be provided before pupils finish their training, so that they may become skillful in applying the principles of mathematics to the daily needs of manufacturing life.

In revising the manuscript the author has had the assistance of his teachers in the Lawrence Industrial School, the Lowell Industrial School, the Fall River Technical High School, and

of many other teachers, practical men, and manufacturing firms. Valuable material has also been obtained from standard handbooks, such as Kent's.

To mention the names of all persons to whom the author is indebted is impossible. Acknowledgment should be made to the following persons and firms who have kindly consented to furnish cuts and information, and have offered valuable suggestions and problems: Mr. George W. Evans, Principal Charlestown (Mass.) High School; Dr. David Snedden; Mr. Charles R. Allen; Mr. Fred. W. Turner; Mr. Peter Gartland, Principal South Boston High School; Mr. John Casey, Worcester (Mass.) Trade School; Mr. John L. Sullivan, Principal Chicopee (Mass.) Industrial School; Mr. William Hunter, Fitchburg Industrial Course; Mr. J. Gould Spofford, Principal Quincy Industrial School; Mr. Edward R. Markham, Cambridge Technical High School; Principal Joseph J. Eaton, Saunders Trades School, Yonkers, N. Y.; Miss Bessie Kingman, Brockton (Mass.) High School; Mr. E. W. Boshart, Director of Industrial Arts, Mt. Vernon, N. Y.; Mr. G. A. Boate, Technical High School, Newton, Mass.; Mr. G. R. Smith, Bradford, England; Mr. H. R. Carter, Belfast, Ireland; New York Central Lines, Apprenticeship Department; Mr. H. E. Thomas, Tuskegee Institute, Ala.; Brown & Sharp Co.; Simond's Guide for Carpenters; R. M. Starbuck & Sons; Garvin Machine Co.; The Crane Co.; The William Powell Co.; Crosby Steam Gage and Valve Co.; Mr. Peter Lobben; Mr. H. P. Faxon; Bardons and Oliver; Stanley Rule and Level Co.; Pittsburgh Valve Foundry and Construction Co.; Becker Milling Machine; Braeburn Steel Co.; Fore River Ship Building Co.; *Engineering Workshop Machines and Processes;* American Injector Co.; B. F. Sturtevant Co.; E. W. Bliss Co.; Dillon Boiler Works; Whitcomb-Blaisdell Co.; Hoefer Drill Co.; Bradford Lathe Co.; and Detroit Screw Works.

The author will be very thankful for any suggestions relating to the work.

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VOCATIONAL MATHEMATICS

PART I — REVIEW OF ARITHMETIC

CHAPTER I

Notation and Numeration

A **unit** is one thing; as, one book, one pencil, one inch.

A **number** is made up of units and *tells how many units* are taken.

An **integer** is a whole number.

A single figure expresses a certain number of units and is said to be in the units column. For example, 5 or 8 is a single figure in the units column; 53 is a number of two figures and has the figure 3 in the units column and the figure 5 in the tens column, for the second figure represents a certain number of tens. Each column has its own name, as shown below.

hundred-billions	ten-billions	billions	hundred-millions	ten-millions	millions	hundred-thousands	ten-thousands	thousands	hundreds	tens	units
1	3	8,	6	9	5,	4	0	7,	1	2	5

Reading Numbers. — For convenience in reading and writing numbers they are separated into groups of three figures each by commas, *beginning at the right:*

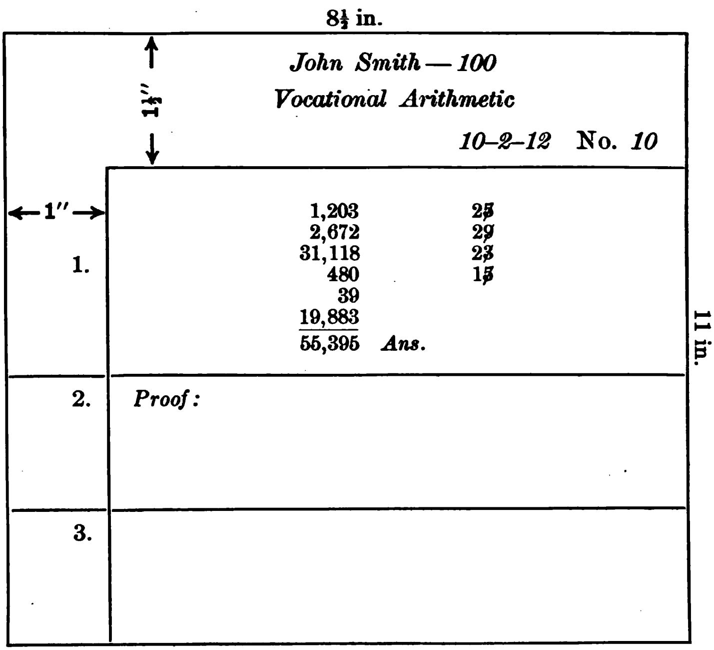
138,695,407,125.

The first group is 125 units.
The second group is 407 thousands.
The third group is 695 millions.
The fourth group is 138 billions.

The preceding number is read one hundred thirty-eight billion, six hundred ninety-five million, four hundred seven thousand, one hundred twenty-five; or 138 billion, 695 million, 407 thousand, 125.

Standard Mathematics Sheet. — To avoid errors in solving problems the work should be done in such a way as to show each step, and should make it easy to check the answer when found. A sheet of paper of standard size, 8½ in. by 11 in., should be used. Rule this sheet as in the following diagram, set down each example with its proper number in the margin, and clearly show the different steps required for the solution. To show that the answer obtained is correct, the proof should follow the example itself.

STANDARD MATHEMATICS SHEET



The pupil should write or print his name and class, the date when the problem is finished, and the number of the problem on the Standard

Mathematics Sheet. If the question contains several divisions or problems, they should be tabulated — (*a*), (*b*), etc. — at the left of the problems inside the margin line. There should be a line drawn between problems to separate them.

Addition

Addition is the process of finding the sum of two or more numbers. The result obtained by this process is called the *sum* or *amount.*

The **sign of addition** is an upright cross, +, called *plus.* The sign is placed between the two numbers to be added.

Thus, 9 inches + 7 inches (read nine inches plus seven inches).

The **sign of equality** is two short horizontal parallel lines, =, and means equals or equal to.

Thus, the statement that 8 feet + 6 feet = 14 feet, means that six feet added to eight feet (or 8 feet plus 6 feet) equals fourteen feet.

To find the sum or amount of two or more numbers.

EXAMPLE. — What is the total weight of a machine made up of parts weighing 1203, 2672, 31,118, 480, 39, and 19,883 lb., respectively?

[The abbreviation for pounds is lb.]

	1,203	$2\cancel{5}$
	2,672	$2\cancel{9}$
	31,118	$2\cancel{3}$
	480	$1\cancel{5}$
	39	
	19,883	
Sum	55,395 lb.	

The sum of the units column is 3 + 9 + 0 + 8 + 2 + 3 = 25 units or 20 and 5 more; 20 is tens, so leave the 5 under the units column and add the 2 tens in the tens column. The sum of the tens column is 2 + 8 + 3 + 8 + 1 + 7 + 0 = 29 tens. 29 tens equal 2 hundreds and 9 tens. Place the 9 tens under the tens column and add the 2 hundreds to the hundreds column. 2 + 8 + 4 + 1 + 6 + 2 = 23 hundreds; 23 hundreds are equal to 2 thousands and 3 hundreds. Place the 3 hundreds under the hundreds column and add the 2 thousands to the next column. 2 + 9 + 1 + 2 + 1 = 15 thousands or 1 ten-thousand and 5 thousands. Add the 1 ten-thousand to the ten-thousands column

and the sum is $1 + 1 + 3 = 5$. Write the 5 in the ten-thousands column. Hence, the sum or weight is 55,395 lb.

PROOF. — Repeat the process, beginning at the top of the right-hand column.

Exactness is very important in arithmetic. There is only one correct answer. Therefore it is necessary to be accurate in performing the numerical calculations. A check of some kind should be made on the work. The simplest check is to estimate the answer before solving the problem. A comparison can be made between them. If there is a great discrepancy then the work is probably wrong. It is also necessary to be exact in reading the problem.

EXAMPLES

1. Write the following numbers as figures and add them: Seventy-five thousand three hundred eight; seven million two hundred five thousand eight hundred forty-nine.

2. In a certain year the total output of copper from the mines was worth $58,638,277.86. Express this amount in words.

3. Solve the following:

$$386 + 5289 + 53666 + 3001 + 291 + 38 = ?$$

4. On a shelf there were three kegs of bolts. The first keg weighed 203 lb., the second 171 lb., and the third 93 lb. How many pounds of bolts were there on the shelf?

5. On the platform in an electrical shop there were a motor-generator and two motors. The motor-generator weighed 275 lb., one of the motors 385 lb., and the other motor 492 lb. What weight did the platform support?

6. Solve the following:

$$6027 + 836 + 4901 + 3{,}800{,}031 + 28{,}639 + 389{,}661 = ?$$

7. Four coal sheds in a shop contained respectively 1498 lb., 4628 lb., 6125 lb., and 12,133 lb. What was the whole amount of coal in the shop?

8. An engineer ordered coal and found that the first load weighed 5685 lb., the second 5916 lb., the third 5495 lb., and the fourth 5280 lb. What was the total weight?

9. Wire for electric lights was run around four sides of three rooms. If the first room was 13 ft. long and 9 ft. wide; the second 18 ft. long and 18 ft. wide; and the third 12 ft. long and 7 ft. wide, what was the total length of wire required? Remember that electric lights require two wires.

10. Find the sum:

46 lb + 135 lb. + 72 lb. + 39 lb. + 427 lb. + 64 lb. + 139 lb.

Subtraction

Subtraction is the process of finding the difference between two numbers, or of finding what number must be added to a given number to equal a given sum. The *minuend* is the number from which we subtract; the *subtrahend* is the number subtracted; and the *difference* or *remainder* is the result of the subtraction.

The **sign of subtraction** is a short horizontal line, —, called *minus*, and is placed before the number to be subtracted.

Thus, $12 - 8 = 4$ is read twelve minus (or less) eight equals four.

To find the difference of two numbers.

Example.—A reel of wire contained 8074′. If 4869′ were used in wiring a house, how many feet remained on the reel?

[Feet and inches are represented by ′ and ″ respectively.]

Minuend	8074 ft.
Subtrahend	4869
Remainder	3205 ft.

Write the smaller number under the greater, with units of the same order in the same vertical line. 9 cannot be taken from 4, so change 1 ten to units. The 1 ten that was changed from the 7 tens makes 10 units, which added to the 4 units makes 14 units. Take 9 from the 14 units and 5 units remain. Write the 5 under the unit column. Since 1 ten was changed from 7 tens, there are 6 tens left, and 6 from 6 leaves 0. Write 0 under the tens column. Next, 8 hundred cannot be taken from 0 hundred, so 1 thousand

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EXAMPLES

1. A strip of plate measures 85″ in length. How many pieces 6″ long can be cut from it? Would there be a remainder?

2. How many pieces 2″ long can be cut from a brass plate 62′ long, if no allowance is made for waste in cutting?

3. If the cost of constructing 362 miles of railway was $4,561,200, what was the cost per mile?

4. If a job which took 379 hours was divided equally among 25 men, how many even hours would each man work, and how much overtime would one of the number have to put in to complete the job?

5. The "over-all" dimension on a drawing was 18′ 9″. The distance was to be spaced off into 14-inch lengths, beginning at one end. How many such lengths could be spaced? How many inches would be left at the other end?

6. If a locomotive consumed 18 gallons of fuel oil per mile of freight service, how far could it run with 2036 gallons of oil?

7. If 48 screws weigh one pound, how many cases each containing 36 screws could be filled from a stock of 29 lb. of screws?

8. A plate measures 68″ in length. How many pieces 15″ long can be cut from it? How much will be left over?

9. A machinist desired to know the number of lots of 42 lb. each, contained in 3276 lb. of screws. Solve.

10. If a freight train made a distance of 112 miles in 8 hours, what was the average speed per hour?

11. If the circumference of the driving wheel of a locomotive was 22 feet, how many turns did it make in going 88 miles?

12. Divide 38,910 by 3896.

REVIEW EXAMPLES

1. A load of castings came to a machine shop. It was not desirable to weigh the castings on the wagon, so they were weighed in 8 lots as follows: 196 lb., 389 lb., 876 lb., 899 lb., 212 lb., and 847 lb. What was the total weight?

2. Five steel bars are placed end to end. If each bar is 29 ft. long, what is the total length?

3. A steam fitter found that the weight per foot of $3\frac{1}{2}''$ wrought iron pipe is 9 lb.; what is the weight of a piece of $3\frac{1}{2}''$ wrought iron pipe 12 feet long?

4. An accident happened in a mill. A number of men were sent out to make the repairs. The following number of hours was reported:

8 men 10 hours each	3 men 5 hours each
4 men 65 hours each	4 men 21 hours each
7 men 14 hours each	6 men 11 hours each

What was the total number of hours worked?

5. The consumption of water for a city during the month of December was 116,891,213 gallons and for January 115,819,729 gallons. How much was the decrease in consumption?

6. An order to a machine shop called for 598 machines each weighing 1219 pounds. What was the total weight?

7. If an I beam weighs 24 lb. per foot of length, find the weight of one measuring 16′ 9″ long.

8. Multiply 641 and 225.

9. Divide 24,566 by 319.

10. An order was sent for ties for a railroad 847 miles long. If each mile required 3017 ties, how many ties would be needed?

11. How many gallons per minute are discharged by two pipes if one discharges 25 gallons per minute and the other 6 gallons less?

Factors

The **factors** of a number are the integers which when multiplied together produce that number.

Thus, 21 is the product of 3 and 7 ; hence, 3 and 7 are the factors of 21.

Separating a number into its factors is called *factoring.*

A number that has no factors but itself and 1 is a *prime* number.

The prime numbers up to 25 are 2, 3, 5, 7, 11, 13, 17, 19 and 23.

A prime number used as a factor is a *prime factor.*

Thus, 3 and 5 are prime factors of 15.

Every prime number except 2 and 5 ends with 1, 3, 7, or 9.

To find the prime factors of a number.

EXAMPLE. — Find the prime factors of 84.

$$\begin{array}{r} 2\underline{)84} \\ 2\underline{)42} \\ 3\underline{)21} \\ 7\underline{)7} \\ 1 \end{array}$$

The prime number 2 divides 84 evenly, leaving the quotient 42, which 2 divides evenly. The next quotient is 21 which 3 divides, giving a quotient 7. 7 divided by 7 gives the last quotient 1 which is indivisible. The several divisors are the prime factors. So 2, 2, 3, and 7 are the prime factors of 84.

PROOF. — The product of the prime factors gives the number.

EXAMPLES

Find the prime factors:

1. 63	4. 636	7. 1155
2. 60	5. 1572	8. 7007
3. 250	6. 2800	9. 13104

Cancellation

To reject a factor from a number divides the number by that factor; to reject the same factors from both dividend and divisor does not affect the quotient. This process is called *cancellation.*

This method can be used to advantage in many everyday calculations.

EXAMPLE. — Divide $12 \times 18 \times 30$ by $6 \times 9 \times 4$.

$$\begin{array}{l} \textit{Dividend} \\ \textit{Divisor} \end{array} \frac{\overset{2}{\cancel{12}} \times \overset{\overset{1}{\cancel{2}}}{\cancel{18}} \times \overset{15}{\cancel{30}}}{\underset{1}{\cancel{6}} \times \underset{1}{\cancel{9}} \times \underset{\underset{1}{\cancel{2}}}{\cancel{4}}} = 30 \ \textit{Quotient.}$$

By this method it is not necessary to multiply before dividing. Indicate the division by writing the divisor under the dividend with a line between. Since 6 is a factor of 6 and 12, and 9 of 9 and 18, respectively, they may be cancelled from both divisor and dividend. Since 2 in the dividend is a factor of 4 in the divisor it may be cancelled from both, leaving 2 in the divisor. Then the 2 being a factor of 30 in the dividend, is cancelled from both, leaving 15. The product of the uncancelled factors is 30. Therefore, the quotient is 30.

PROOF. — If the product of the divisor and the quotient equal the dividend, the answer is correct.

EXAMPLES

Indicate and find quotients by cancellation:

1. Divide $36 \times 27 \times 49 \times 38 \times 50$ by $70 \times 18 \times 15$.

2. What is the quotient of $36 \times 48 \times 16$ divided by $27 \times 24 \times 8$?

3. How many pounds of washers at 50 cents a pound must be given in exchange for 15 pounds of bolts at 40 cents a pound?

4. There are 16 ounces in a pound; 30 pounds of steel will produce how many horseshoes, if each weighs 6 ounces?

5. How many pieces of steel rod, each weighing 10 pounds, at 20 cents a pound, must be given in exchange for 10 bars of $\frac{3}{16}''$ iron rod, each weighing 5 pounds, at 4 cents per pound?

6. Divide the product of 10, 75, 9, and 96 by the product of 5, 12, 15, and 9.

7. If 24 men, working 9 hours a day, can do a piece of work in 12 days, how many days will it take 18 men, working 8 hours a day, to do the work?

Greatest Common Divisor

The **greatest common divisor** of two or more numbers is the greatest number that will exactly divide each of the numbers.

To find the greatest common divisor of two or more numbers.

EXAMPLE. — Find the greatest common divisor of 90 and 150.

$$90 = 2 \times 3 \times 5 \times 3$$
$$150 = 2 \times 3 \times 5 \times 5$$
Ans. $30 = 2 \times 3 \times 5$

$$\begin{array}{r|rr} 2 & 90 & 150 \\ \hline 5 & 45 & 75 \\ \hline 3 & 9 & 15 \\ \hline & 3 & 5 \end{array}$$

$2 \times 3 \times 5 = 30$ *Ans.*

First Method

The prime factors common to both 90 and 150 are 2, 3, and 5. Since the greatest common divisor of two or more numbers is the product of their common factors, 30 is the greatest common divisor of 90 and 150.

```
                          90)150(1
                             90
                             60)90(1
                                60
Greatest Common Divisor         30)90(3
                                   90
```

Second Method

To find the greatest common divisor when the numbers cannot be readily factored, divide the larger by the smaller, then the last divisor by the last remainder until there is no remainder. The last divisor will be the greatest common divisor. If the greatest common divisor is to be found of more than two numbers, find the greatest common divisor of two of them, then of this divisor and the third number, and so on. The last divisor will be the greatest common divisor of all of them.

EXAMPLES

Find the greatest common divisor:

1. 270, 810.
2. 264, 312.
3. 504, 560.
4. 288, 432, 1152.
5. 72, 153, 315, 2187.

Least Common Multiple

The product of two or more numbers is called a **multiple** of each of them; 4, 6, 8, 12 are multiples of 2. The multiple

of two or more numbers is called the **common multiple** of the numbers; 60 is a common multiple of 4, 5, 6.

The **least common multiple** of two or more numbers is *the smallest* common multiple of the number; 30 is the least common multiple of 3, 5, 6.

To find the least common multiple of two or more numbers.

EXAMPLE. — Find the least common multiple of 21, 28, and 30.

First Method

$$21 = 3 \times 7$$
$$28 = 2 \times 2 \times \cancel{7}$$
$$30 = \cancel{2} \times \cancel{3} \times 5$$

Take all the factors of the first number, all of the second not already represented in the first, etc. Thus,

$$3 \times 7 \times 2 \times 2 \times 5 = 420 \text{ } L.\ C.\ M.$$

Second Method

$$\begin{array}{r|rrr} 2 & 21 & 28 & 30 \\ \hline 3 & 21 & 14 & 15 \\ \hline 7 & 7 & 14 & 5 \\ \hline & 1 & 2 & 5 \end{array}$$

$$2 \times 3 \times 7 \times 1 \times 2 \times 5 = 420 \text{ } L.\ C.\ M.$$

Divide any two or more numbers by a prime factor contained in them, like 2 in 28 and 30. Write 21 which is not divided by the 2 for the next quotient together with the 14 and 15. 3 is a prime factor of 21 and 15 which gives a quotient of 7 and 5 with 14 written in the quotient undivided. 7 is a prime factor of 7 and 14 which gives a remainder of 1, 2; and 5 undivided is written down as before. The product 420 of all these divisors and the last quotients is the least common multiple of 21, 28, and 30.

EXAMPLES

Find the least common multiple:

1. 18, 27, 30. 2. 15, 60, 140, 210. 3. 24, 42, 54, 360.

4. 25, 20, 35, 40. 5. 24, 48, 96, 192.

6. What is the shortest length of rope that can be cut into pieces 32′, 36′, and 44′ long?

Fractions

A fraction is one or more equal parts of a unit. If an apple be divided into two equal parts, each part is one-half of the apple, and is expressed by placing the number 1 above the number 2 with a short line between: $\frac{1}{2}$. A fraction always indicates division. In $\frac{1}{2}$, 1 is the dividend and 2 the divisor; 1 is called the *numerator* and 2 is called the *denominator*.

A common fraction is one which is expressed by a numerator written above a line and a denominator below. The numerator and denominator are called the *terms of the fraction*.

A proper fraction is a fraction whose value is less than 1; its numerator is less than its denominator, as $\frac{3}{4}$, $\frac{5}{8}$, $\frac{6}{7}$, $\frac{11}{12}$. An **improper fraction** is a fraction whose value is 1 or more than 1; its numerator is equal to or greater than its denominator, as $\frac{4}{4}$, $\frac{19}{16}$. A number made up of an integer and a fraction is a **mixed number**. Read with the word *and* between the whole number and the fraction: $4\frac{9}{16}$, $3\frac{7}{8}$, etc.

The *value of a fraction* is the *quotient* of the numerator divided by the denominator.

EXERCISE

Read the following:

1. $\frac{5}{8}$	3. $12\frac{1}{2}$	5. $5\frac{1}{2}$	7. $9\frac{1}{32}$	9. $\frac{7}{4}$
2. $\frac{17}{16}$	4. $8\frac{1}{2}$	6. $6\frac{7}{8}$	8. $12\frac{3}{64}$	

Reduction of Fractions

Reduction of fractions is the process of changing their form without changing their value.

To reduce a fraction to higher terms.

Multiplying the denominator and the numerator of the given fraction by the same number does not change the value of the fraction.

EXAMPLE. — Reduce $\frac{5}{8}$ to thirty-seconds.

$$\frac{5}{8}\times\frac{4}{4}=\frac{20}{32} \quad Ans.$$

The denominator must be multiplied by 4 to obtain 32; so the numerator must be multiplied by the same number so that the value of the fraction may not be changed.

EXAMPLES

Change the following:

1. $\frac{8}{9}$ to 27ths.
2. $\frac{11}{12}$ to 60ths.
3. $\frac{5}{8}$ to 40ths.
4. $\frac{7}{8}$ to 56ths.
5. $\frac{9}{10}$ to 50ths.
6. $\frac{9}{25}$ to 75ths.
7. $\frac{19}{36}$ to 144ths.
8. $\frac{47}{56}$ to 168ths.
9. $\frac{17}{18}$ to 522ds.
10. $\frac{125}{375}$ to 9375ths.

A fraction is said to be in its *lowest terms* when the numerator and the denominator are prime to each other.

To reduce a fraction to its lowest terms.

Dividing the numerator and the denominator of a fraction by the same number does not change the value of the fraction. The process of dividing the numerator and denominator of a fraction by a number common to both may be continued until the terms are prime to each other.

EXAMPLE. — Reduce $\frac{12}{16}$ to fourths.

$$\frac{12}{16}=\frac{3}{4} \quad Ans.$$

The denominator must be divided by 4 to give the new denominator 4; then the numerator must be divided by the same number so as not to change the value of the fraction.

If the terms of a fraction are large numbers, find their greatest common divisor and divide both terms by that.

EXAMPLE. — Reduce $\frac{2166}{2888}$ to fourths.

(1)
```
          2166)2888(1
               2166
G. C. D.        722)2166(3
                    2166
```

(2) $\frac{2166}{2888}=\frac{3}{4}$ *Ans.*

EXAMPLES

Reduce to lowest terms:

1. $\frac{8}{16}$ 3. $\frac{1162}{1828}$ 5. $\frac{21}{78}$ 7. $\frac{300}{800}$ 9. $\frac{114}{285}$
2. $\frac{240}{480}$ 4. $\frac{14}{16}$ 6. $\frac{16}{128}$ 8. $\frac{180}{162}$ 10. $\frac{112}{1888}$

To reduce an integer to an improper fraction.

EXAMPLE. — Reduce 25 to fifths.

25 times $\frac{5}{5} = \frac{125}{5}$ *Ans.*

In 1 there are $\frac{5}{5}$. In 25 there must be 25 times $\frac{5}{5}$, or $\frac{125}{5}$.

To reduce a mixed number to an improper fraction.

EXAMPLE. — Reduce $16\frac{4}{7}$ to an improper fraction.

$$\begin{array}{rl} 16\frac{4}{7} & \\ \underline{7} & \text{sevenths} \\ 112 & \\ \underline{4} & \text{sevenths} \\ 116 & \text{sevenths}, = \frac{116}{7}. \end{array}$$

Since in 1 there are $\frac{7}{7}$, in 16 there must be 16 times $\frac{7}{7}$, or $\frac{112}{7}$.
$\frac{112}{7} + \frac{4}{7} = \frac{116}{7}$.

EXAMPLES

Reduce to improper fractions:

1. $3\frac{7}{8}$ 3. $17\frac{1}{4}$ 5. $13\frac{7}{8}$ 7. $359\frac{5}{16}$
2. $16\frac{1}{32}$ 4. $12\frac{1}{2}$ 6. $27\frac{3}{15}$ 8. $482\frac{19}{45}$
9. $25\frac{1}{30}$ 10. Reduce 250 to 16ths.
11. Change 156 to a fraction whose denominator shall be 12.
12. In \$730 how many fourths of a dollar?
13. Change $12\frac{5}{8}$ to 16ths. 14. Change $24\frac{5}{9}$ to 18ths.

To reduce an improper fraction to an integer or mixed number divide the numerator by the denominator.

EXAMPLE. — Reduce $\frac{385}{16}$ to an integer or mixed number.

$$\begin{array}{r} 24 \\ 16\overline{)385} \\ \underline{32} \\ 65 \\ \underline{64} \\ 1 \end{array}$$

$24\frac{1}{16}$ *Ans.*

Since $\frac{16}{16}$ equal 1, $\frac{385}{16}$ will equal as many times 1 as 16 is contained in 385, or $24\frac{1}{16}$ times.

EXAMPLES

Reduce to integers or mixed numbers:

1. $\frac{70}{85}$	4. $\frac{8824}{24}$	7. $\frac{807}{27}$	10. $\frac{8075}{25}$
2. $\frac{219}{16}$	5. $\frac{88768}{26}$	8. $\frac{912}{89}$	11. $\frac{89168}{64}$
3. $\frac{8982}{75}$	6. $\frac{860}{39}$	9. $\frac{25000}{100}$	12. $\frac{72}{22}$

When fractions have the same denominator their denominator is called a *common denominator.*

Thus, $\frac{10}{12}$, $\frac{6}{12}$, $\frac{2}{12}$ have a common denominator.

The *smallest common denominator* of two or more fractions is their least common denominator.

Thus, $\frac{10}{12}$, $\frac{6}{12}$, $\frac{2}{12}$ become $\frac{5}{6}$, $\frac{3}{6}$, $\frac{1}{6}$ when changed to their least common denominator.

The common denominator of two or more fractions is a *common multiple* of their denominators.

The least common denominator of two or more fractions is the *least common multiple* of their denominators.

EXAMPLE. — Reduce $\frac{3}{4}$ and $\frac{5}{6}$ to fractions having a common denominator.

$$\frac{3}{4} \times \frac{6}{6} = \frac{18}{24}$$
$$\frac{5}{6} \times \frac{4}{4} = \frac{20}{24}$$
$$\frac{3}{4} = \frac{18}{24} \text{ and } \frac{5}{6} = \frac{20}{24}$$

The common denominator must be a common multiple of the denominators 4 and 6, and since 24 is the product of the denominators, it is a common multiple of them. Therefore, 24 is a common denominator of $\frac{3}{4}$ and $\frac{5}{6}$.

To reduce fractions to fractions having the least common denominator.

EXAMPLE. — Reduce $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{12}$ to fractions having the least common denominator.

$$\begin{array}{r|rrr} 2 & 3 & 6 & 12 \\ \hline 3 & 3 & 3 & 6 \\ \hline & 1 & 1 & 2 \end{array}$$

$2 \times 3 \times 2 = 12$ *L. C. M.*

$\frac{2}{3} = \frac{8}{12}$; $\frac{5}{6} = \frac{10}{12}$; $\frac{7}{12} = \frac{7}{12}$. *Ans.*

The least common denominator must be the least common multiple of the denominators 3, 6, 12, which is 12.

Divide the least common multiple 12 by the denominator of each fraction, and multiply both terms by the quotient. If the

denominators should be prime to each other, their product would be their least common denominator.

EXAMPLES

Reduce to fractions having a common denominator:

1. $\frac{1}{3}, \frac{3}{5}$
2. $\frac{3}{4}, \frac{2}{3}$
3. $\frac{5}{8}, \frac{7}{9}$
4. $\frac{5}{7}, \frac{1}{14}, \frac{1}{2}$
5. $\frac{5}{7}, \frac{2}{3}, \frac{2}{5}$
6. $\frac{7}{8}, \frac{5}{6}, \frac{4}{9}$
7. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$
8. $\frac{1}{5}, \frac{3}{12}, \frac{5}{9}, \frac{7}{8}$

Reduce to fractions having least common denominator:

1. $\frac{6}{5}, \frac{7}{8}, \frac{7}{12}$
2. $\frac{3}{4}, \frac{7}{8}, \frac{5}{16}$
3. $\frac{3}{15}, \frac{4}{21}, \frac{2}{3}$
4. $\frac{4}{5}, \frac{5}{12}, \frac{3}{20}, \frac{5}{60}$
5. $\frac{5}{7}, \frac{3}{2}, \frac{5}{14}, 4$
6. $\frac{4}{5}, \frac{5}{6}, \frac{7}{8}, \frac{5}{9}$
7. Which fraction is larger, $\frac{6}{7}$ or $\frac{7}{8}$?

Addition of Fractions

Only fractions with a common denominator can be added. If the fractions have not the same denominator, reduce them to a common denominator, add their numerators, and place their sum over the common denominator. The result should be reduced to its lowest terms. If the result is an improper fraction, it should be reduced to an integer or mixed number.

EXAMPLE. — Add $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{9}{16}$.

1.
$$\begin{array}{r|rrr} 2 & 4 & 6 & 16 \\ \hline 2 & 2 & 3 & 8 \\ \hline & 1 & 3 & 4 \end{array} \quad 48\ L.\ C.\ M.$$

2. $\frac{3}{4} + \frac{5}{6} + \frac{9}{16} = \frac{36}{48} + \frac{40}{48} + \frac{27}{48} = \frac{103}{48}$. *Ans.*

The least common multiple of the denominators is 48. Dividing this by the denominator of each fraction and multiplying both terms by the quotient give $\frac{36}{48}$, $\frac{40}{48}$, $\frac{27}{48}$. The fractions are now like fractions, and are added by adding their numerators and placing the sum over the common denominator. Hence, the sum is $\frac{103}{48}$, or $2\frac{7}{48}$.

EXAMPLE. — Add $5\frac{3}{5}$, $7\frac{7}{10}$, and $6\frac{7}{15}$.

$$\begin{aligned} 5\tfrac{3}{5} &= 5\tfrac{18}{30} \\ 7\tfrac{7}{10} &= 7\tfrac{21}{30} \\ 6\tfrac{7}{15} &= 6\tfrac{14}{30} \\ \hline & 18\tfrac{53}{30} = 19\tfrac{23}{30}. \quad \textit{Ans.} \end{aligned}$$

First find the sum of the fractions, which is $\frac{53}{30}$, or $1\frac{23}{30}$. Add this to the sum of the integers, 18. $18 + 1\frac{23}{30} = 19\frac{23}{30}$.

EXAMPLES

1. Find the "over-all" dimension of a drawing if the separate parts measure $\frac{5}{16}''$, $\frac{3}{8}''$, $\frac{1}{2}''$, and $\frac{9}{16}''$, respectively.

2. Find the sum of $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{15}{16}$, and $\frac{31}{32}$.

3. Find the sum of $3\frac{3}{8}$, $4\frac{3}{4}$, and $2\frac{1}{16}$.

4. Four castings weigh respectively $8\frac{7}{8}$ lb., $5\frac{1}{2}$ lb., $11\frac{3}{4}$ lb., and $7\frac{5}{8}$ lb. What is their total weight?

5. The diameter of two holes is $3\frac{7}{8}''$ and the distance between the sides of the holes is $\frac{3}{4}''$. What is the distance from the outside of one hole to the outside of the other?

6. Two brass rods measure $8\frac{5}{16}''$ and $5\frac{1}{4}''$ What is their combined length?

7. A board was cut into two pieces, one $8\frac{3}{8}''$ and the other $5\frac{9}{16}''$ long. If $\frac{1}{16}''$ was allowed for waste in cutting, what was the length of the board?

8. Three pieces of rod contain $38\frac{1}{8}$, $12\frac{1}{2}$, and $53\frac{3}{8}$ feet respectively. What is their total length in feet?

9. Add: $10\frac{1}{5}$, $7\frac{2}{7}$, 11, $\frac{14}{35}$.

Subtraction of Fractions

Only fractions with a common denominator can be subtracted. If the fractions have not the same denominator, reduce them to a common denominator, and write the difference of their numerators over the common denominator. The result should be reduced to its lowest terms.

EXAMPLE. — Subtract $\frac{2}{3}$ from $\frac{5}{6}$.

$\frac{5}{6} - \frac{2}{3} = \frac{15}{18} - \frac{12}{18} = \frac{3}{18}$

$\frac{3}{18} = \frac{1}{6}$. *Ans.*

The least common denominator of $\frac{5}{6}$ and $\frac{2}{3}$ is 6. $\frac{5}{6} = \frac{5}{6}$, and $\frac{2}{3} = \frac{4}{6}$. Their difference is $\frac{1}{6}$.

EXAMPLE. — From $11\frac{1}{3}$ subtract $5\frac{5}{6}$.

$11\frac{1}{3} = 10\frac{8}{6}$

$4\frac{5}{6} = \underline{4\frac{5}{6}}$

$6\frac{3}{6} = 6\frac{1}{2}$. *Ans.*

When the fractions are changed to their least common denominator, they are $11\frac{2}{6} - 4\frac{5}{6}$. $\frac{5}{6}$ cannot be subtracted from $\frac{2}{6}$, hence 1 is taken from 11 units, changed to sixths, and added to the $\frac{2}{6}$ which makes $\frac{8}{6}$. $10\frac{8}{6} - 4\frac{5}{6} = 6\frac{3}{6} = 6\frac{1}{2}$.

EXAMPLES

1. The distance between two holes is $5\frac{3}{4}''$ measured from the centers. If the holes are $\frac{7}{16}''$ in diameter, what is the length of metal plate between them?

2. From a steel bar $26\frac{5}{8}''$ long were cut the following pieces: one $7\frac{1}{4}''$, one $6\frac{7}{8}''$, one $3\frac{3}{4}''$ long. If after cutting these pieces, the length of the bar was $8\frac{5}{8}''$, what was the amount of waste in cutting?

3. A piece of steel on a lathe is $1''$ in diameter. In the first cut $\frac{3}{32}''$ were taken off, in the second cut $\frac{2}{64}''$, in the third cut $\frac{1}{16}''$, in the fourth $\frac{1}{16}''$. What was the diameter of the finished piece?

4. A bolt is $15\frac{1}{4}''$ long. How much must be cut from it to make it $11\frac{3}{6}''$ long?

5. In wiring a house five men work 14 hours; one man works 1 hour and 20 minutes; a second man works 2 hours and 15 minutes; a third man works 5 hours; and a fourth man, $4\frac{3}{4}$ hours. How many hours did the fifth man work?

6. It is $3\frac{1}{4}''$ between the centers of two holes of the same size. The distance between the sides of the holes is $1\frac{1}{2}''$ What is the diameter of each hole?

7. There were $48\frac{1}{2}$ gallons in the tank. First $4\frac{1}{2}$ gallons were used, then $5\frac{1}{3}$ gallons, and last $2\frac{3}{4}$ gallons. How many gallons were left in the tank?

8. What is the difference between $\frac{9}{17}$ and $\frac{18}{34}$?

9. What is the difference between $32\frac{7}{8}$ and $3\frac{11}{64}$?

10. An electrician had a reel of 300 feet of copper wire. He used at various times $50\frac{1}{2}'$, $32\frac{1}{4}'$, $109\frac{2}{3}'$, and $27\frac{7}{12}'$. How much wire was left?

Multiplication of Fractions

To multiply fractions, multiply the numerators together for the new numerator and multiply the denominators together for the new denominator.

Cancel when possible. The word *of* between two fractions is equivalent to the sign of multiplication.

To multiply a mixed number by an integer, multiply the whole number and the fraction separately by the integer then add the products.

To multiply two mixed numbers, change each to an improper fraction and multiply.

EXAMPLE. — Multiply $\frac{4}{5}$ by $\frac{3}{4}$.

$\frac{4}{5}$ multiplied by $\frac{3}{4}$ is the same as $\frac{3}{4}$ *of* $\frac{4}{5}$. 3 and 5 are prime to each other so that answer is $\frac{3}{5}$. This method of solution is the same as multiplying the numerators together for a new numerator and the denominators for a new denominator. Cancellation shortens the process.

EXAMPLE. — Find the product of $124\frac{3}{4}$ and 5.

$$\begin{array}{r} 124\frac{3}{4} \\ 5 \\ \hline 3\frac{3}{4} \\ 620 \\ \hline 623\frac{3}{4} \end{array} \text{ Ans.}$$

$$5 \times \frac{3}{4} = \frac{15}{4} = 3\frac{3}{4}$$

If the fraction and integer are multiplied separately by 5, the result is 5 times $\frac{3}{4} = \frac{15}{4} = 3\frac{3}{4}$, and 5 times $124 = 620$. $620 + 3\frac{3}{4} = 623\frac{3}{4}$.

EXAMPLES

1. What is the length of a bar of iron that has been cut into 8 pieces, each $\frac{5}{16}''$ in length?

2. What is $\frac{1}{4}$ of $\frac{7}{8}$?

3. What is the cost of $5\frac{1}{2}$ lb. of iron castings at $4\frac{1}{4}$ cents a pound?

4. If a car is filled with machinery of a certain kind in $4\frac{1}{2}$ hours, how long will it take to fill 10 cars?

5. If $\frac{3}{8}$ of the shell of a stationary boiler is considered as the heating surface, how many square feet of heating surface are there in a boiler containing $98\frac{7}{16}$ sq. ft.?

6. Multiply $5\frac{7}{16}$ by $3\frac{1}{4}$.

7. What is the cost of $4\frac{1}{2}$ pounds of castings for making bench lathes at $3\frac{1}{2}$ cents per pound?

8. What is the cost of $5\frac{1}{2}$ lb. of hammered copper at $31\frac{1}{2}$ cents per pound?

9. If $\frac{3}{4}$ lb. of bolts are used to make a small machine, how many pounds of bolts must be made for 14 such machines?

10. If $10\frac{3}{4}$ bundles of shingles are used on one side of a square hip-roof, how many bundles will the whole roof require?

Division of Fractions

To divide one fraction by another, invert the divisor and proceed as in multiplication of fractions. Change integers and mixed numbers to improper fractions.

EXAMPLE. — Divide $\frac{4}{5} \times \frac{3}{8}$ by $\frac{5}{6} \times \frac{3}{5}$.

$\frac{4}{5} \times \frac{3}{8} \div (\frac{5}{6} \times \frac{3}{5}) =$

$$\frac{\cancel{4}}{5} \times \frac{3}{\underset{\cancel{2}}{\cancel{8}}} \times \frac{\overset{\cancel{2}}{\cancel{6}}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{3}} = \frac{3}{5} \quad \textit{Ans.}$$

The divisor $\frac{6}{5} \times \frac{5}{3}$ is inverted and the result obtained by the process of cancellation.

EXAMPLE. — Divide $3156\frac{3}{4}$ by 5.

$$\begin{array}{l} \quad 631\frac{7}{20} \ \ \textit{Ans.} \\ 5\overline{)3156\frac{3}{4}} \\ \ \underline{30} \\ \quad 15 \\ \quad \underline{15} \qquad 1\frac{3}{4} = \frac{7}{4} \\ \qquad 6 \\ \qquad \underline{5} \\ \qquad 1\frac{3}{4} \qquad \frac{7}{4} \div 5 = \frac{7}{4} \times \frac{1}{5} = \frac{7}{20} \end{array}$$

When the integer of a mixed number is large, it may be divided as follows: 5 in $3156\frac{3}{4}$, 631 times, with a remainder of $1\frac{3}{4}$. This remainder divided by 5 gives $\frac{7}{20}$, which is placed at the right of the quotient.

EXAMPLE. — Divide 3682 by $5\frac{1}{2}$.

$$\begin{array}{l} 5\frac{1}{2})\ 3682 \\ \underline{2} \qquad\ \underline{2} \\ 11\ \overline{)7364} \\ \qquad 669\frac{5}{11} \quad \textit{Ans.} \end{array}$$

When the dividend is a large number and the divisor a mixed number as in $3682 \div 5\frac{1}{2}$, multiply both dividend and divisor by 2, when the divisor becomes 11 halves and the dividend 7364 halves. Multiplying both dividend and divisor by the same number does not change the quotient. Dividing, the quotient is $669\frac{5}{11}$.

A fraction having a fraction for one or both of its terms is called a *complex fraction.*

To reduce a complex fraction to a simple fraction.

EXAMPLE. — Reduce $\dfrac{4\frac{2}{3}}{7\frac{5}{6}}$ to a simple fraction.

$$\frac{4\frac{2}{3}}{7\frac{5}{6}} = \frac{\frac{14}{3}}{\frac{47}{6}} = \frac{14}{3} \div \frac{47}{6} = \frac{14}{3} \times \frac{6}{47} = \frac{28}{47} \quad \textit{Ans.}$$

Change $4\frac{2}{3}$ and $7\frac{5}{6}$ to improper fractions, $\frac{14}{3}$ and $\frac{47}{6}$, respectively. Perform the division indicated with the aid of cancellation and the result will be $\frac{28}{47}$.

EXAMPLES

1. Divide $\frac{41}{64}$ by $\frac{1}{2}$.
2. Divide $\frac{7}{16}$ by $\frac{3}{4}$.
3. Divide $\frac{29}{32}$ by $\frac{1}{4}$.
4. Divide $\frac{7}{8}$ by $\frac{1}{4}$.
5. Divide $\frac{3}{4}$ by $\frac{5}{8}$.
6. $384\frac{3}{5} \div 5 = ?$
7. $296 \div 10\frac{1}{2} = ?$
8. $28{,}769 \div 7\frac{5}{8} = ?$
9. $\dfrac{7\frac{1}{9}}{\frac{16}{27}} = ?$
10. $\dfrac{\frac{1}{4} \text{ of } \frac{7}{8}}{\frac{2}{3} \times \frac{5}{6}} = ?$

Drill in the Use of Fractions

Addition

1. $\frac{1}{2}+\frac{1}{2}=?$
2. $\frac{1}{2}+\frac{1}{4}=?$
3. $\frac{1}{2}+\frac{1}{8}=?$
4. $\frac{1}{2}+\frac{1}{16}=?$
5. $\frac{1}{2}+\frac{1}{32}=?$
6. $\frac{1}{2}+\frac{1}{64}=?$
7. $\frac{1}{4}+\frac{1}{2}=?$
8. $\frac{1}{4}+\frac{1}{4}=?$
9. $\frac{1}{4}+\frac{1}{8}=?$
10. $\frac{1}{4}+\frac{1}{16}=?$
11. $\frac{1}{4}+\frac{1}{32}=?$
12. $\frac{1}{4}+\frac{1}{64}=?$
13. $\frac{5}{8}+\frac{1}{2}=?$
14. $\frac{5}{8}+\frac{1}{4}=?$
15. $\frac{5}{8}+\frac{1}{8}=?$
16. $\frac{5}{8}+\frac{1}{16}=?$
17. $\frac{5}{8}+\frac{1}{32}=?$
18. $\frac{5}{8}+\frac{1}{64}=?$
19. $\frac{1}{8}+\frac{1}{2}=?$
20. $\frac{1}{8}+\frac{1}{4}=?$
21. $\frac{1}{8}+\frac{1}{8}=?$
22. $\frac{1}{8}+\frac{1}{16}=?$
23. $\frac{1}{8}+\frac{1}{32}=?$
24. $\frac{1}{8}+\frac{1}{64}=?$
25. $\frac{1}{16}+\frac{1}{2}=?$
26. $\frac{1}{16}+\frac{1}{4}=?$
27. $\frac{1}{16}+\frac{1}{8}=?$
28. $\frac{1}{16}+\frac{1}{16}=?$
29. $\frac{1}{16}+\frac{1}{32}=?$
30. $\frac{1}{16}+\frac{1}{64}=?$
31. $\frac{3}{4}+\frac{1}{2}=?$
32. $\frac{3}{4}+\frac{1}{4}=?$
33. $\frac{3}{4}+\frac{1}{8}=?$
34. $\frac{3}{4}+\frac{1}{16}=?$
35. $\frac{3}{4}+\frac{1}{32}=?$
36. $\frac{3}{4}+\frac{1}{64}=?$
37. $\frac{1}{32}+\frac{1}{2}=?$
38. $\frac{1}{32}+\frac{1}{4}=?$
39. $\frac{1}{32}+\frac{1}{8}=?$
40. $\frac{1}{32}+\frac{1}{16}=?$
41. $\frac{1}{32}+\frac{1}{32}=?$
42. $\frac{1}{32}+\frac{1}{64}=?$
43. $\frac{1}{64}+\frac{1}{2}=?$
44. $\frac{1}{64}+\frac{1}{4}=?$
45. $\frac{1}{64}+\frac{1}{8}=?$
46. $\frac{1}{64}+\frac{1}{16}=?$
47. $\frac{1}{64}+\frac{1}{32}=?$
48. $\frac{1}{64}+\frac{1}{64}=?$
49. $\frac{7}{8}+\frac{1}{2}=?$
50. $\frac{7}{8}+\frac{1}{4}=?$
51. $\frac{7}{8}+\frac{1}{8}=?$
52. $\frac{7}{8}+\frac{1}{16}=?$
53. $\frac{7}{8}+\frac{1}{32}=?$
54. $\frac{7}{8}+\frac{1}{64}=?$

Subtraction

1. $\frac{1}{2}-\frac{1}{2}=?$
2. $\frac{1}{2}-\frac{1}{4}=?$
3. $\frac{1}{2}-\frac{1}{8}=?$
4. $\frac{1}{2}-\frac{1}{16}=?$
5. $\frac{1}{2}-\frac{1}{32}=?$
6. $\frac{1}{2}-\frac{1}{64}=?$
7. $\frac{1}{2}-\frac{1}{4}=?$
8. $\frac{1}{4}-\frac{1}{4}=?$
9. $\frac{1}{4}-\frac{1}{8}=?$
10. $\frac{1}{4}-\frac{1}{16}=?$
11. $\frac{1}{4}-\frac{1}{32}=?$
12. $\frac{1}{4}-\frac{1}{64}=?$
13. $\frac{5}{8}-\frac{1}{2}=?$
14. $\frac{5}{8}-\frac{1}{4}=?$
15. $\frac{5}{8}-\frac{1}{8}=?$
16. $\frac{5}{8}-\frac{1}{16}=?$
17. $\frac{5}{8}-\frac{1}{32}=?$
18. $\frac{5}{8}-\frac{1}{64}=?$
19. $\frac{1}{2}-\frac{1}{8}=?$
20. $\frac{1}{4}-\frac{1}{8}=?$
21. $\frac{1}{8}-\frac{1}{8}=?$

22. $\frac{1}{8} - \frac{1}{16} = ?$
23. $\frac{1}{8} - \frac{1}{32} = ?$
24. $\frac{1}{8} - \frac{1}{64} = ?$
25. $\frac{1}{2} - \frac{1}{16} = ?$
26. $\frac{1}{4} - \frac{1}{16} = ?$
27. $\frac{1}{8} - \frac{1}{16} = ?$
28. $\frac{1}{16} - \frac{1}{16} = ?$
29. $\frac{1}{16} - \frac{1}{32} = ?$
30. $\frac{1}{16} - \frac{1}{64} = ?$
31. $\frac{3}{4} - \frac{1}{2} = ?$
32. $\frac{3}{4} - \frac{1}{4} = ?$
33. $\frac{3}{4} - \frac{1}{8} = ?$
34. $\frac{3}{4} - \frac{1}{16} = ?$
35. $\frac{3}{4} - \frac{1}{32} = ?$
36. $\frac{3}{4} - \frac{1}{64} = ?$
37. $\frac{1}{2} - \frac{1}{32} = ?$
38. $\frac{1}{4} - \frac{1}{32} = ?$
39. $\frac{1}{8} - \frac{1}{32} = ?$
40. $\frac{1}{16} - \frac{1}{32} = ?$
41. $\frac{1}{32} - \frac{1}{32} = ?$
42. $\frac{1}{32} - \frac{1}{64} = ?$
43. $\frac{1}{2} - \frac{1}{64} = ?$
44. $\frac{1}{4} - \frac{1}{64} = ?$
45. $\frac{1}{8} - \frac{1}{64} = ?$
46. $\frac{1}{16} - \frac{1}{64} = ?$
47. $\frac{1}{32} - \frac{1}{64} = ?$
48. $\frac{1}{64} - \frac{1}{64} = ?$
49. $\frac{7}{8} - \frac{1}{2} = ?$
50. $\frac{7}{8} - \frac{1}{4} = ?$
51. $\frac{7}{8} - \frac{1}{8} = ?$
52. $\frac{7}{8} - \frac{1}{16} = ?$
53. $\frac{7}{8} - \frac{1}{32} = ?$
54. $\frac{7}{8} - \frac{1}{64} = ?$

Multiplication

1. $\frac{1}{2} \times \frac{1}{2} = ?$
2. $\frac{1}{2} \times \frac{1}{4} = ?$
3. $\frac{1}{2} \times \frac{1}{8} = ?$
4. $\frac{1}{2} \times \frac{1}{16} = ?$
5. $\frac{1}{2} \times \frac{1}{32} = ?$
6. $\frac{1}{2} \times \frac{1}{64} = ?$
7. $\frac{1}{4} \times \frac{1}{2} = ?$
8. $\frac{1}{4} \times \frac{1}{4} = ?$
9. $\frac{1}{4} \times \frac{1}{8} = ?$
10. $\frac{1}{4} \times \frac{1}{16} = ?$
11. $\frac{1}{4} \times \frac{1}{32} = ?$
12. $\frac{1}{4} \times \frac{1}{64} = ?$
13. $\frac{5}{8} \times \frac{1}{2} = ?$
14. $\frac{5}{8} \times \frac{1}{4} = ?$
15. $\frac{5}{8} \times \frac{1}{8} = ?$
16. $\frac{5}{8} \times \frac{1}{16} = ?$
17. $\frac{5}{8} \times \frac{1}{32} = ?$
18. $\frac{5}{8} \times \frac{1}{64} = ?$
19. $\frac{1}{8} \times \frac{1}{2} = ?$
20. $\frac{1}{8} \times \frac{1}{4} = ?$
21. $\frac{1}{8} \times \frac{1}{8} = ?$
22. $\frac{1}{8} \times \frac{1}{16} = ?$
23. $\frac{1}{8} \times \frac{1}{32} = ?$
24. $\frac{1}{8} \times \frac{1}{64} = ?$
25. $\frac{1}{16} \times \frac{1}{2} = ?$
26. $\frac{1}{16} \times \frac{1}{4} = ?$
27. $\frac{1}{16} \times \frac{1}{8} = ?$
28. $\frac{1}{16} \times \frac{1}{16} = ?$
29. $\frac{1}{16} \times \frac{1}{32} = ?$
30. $\frac{1}{16} \times \frac{1}{64} = ?$
31. $\frac{3}{4} \times \frac{1}{2} = ?$
32. $\frac{3}{4} \times \frac{1}{4} = ?$
33. $\frac{3}{4} \times \frac{1}{8} = ?$
34. $\frac{3}{4} \times \frac{1}{16} = ?$
35. $\frac{3}{4} \times \frac{1}{32} = ?$
36. $\frac{3}{4} \times \frac{1}{64} = ?$
37. $\frac{1}{32} \times \frac{1}{2} = ?$
38. $\frac{1}{32} \times \frac{1}{4} = ?$
39. $\frac{1}{32} \times \frac{1}{8} = ?$
40. $\frac{1}{32} \times \frac{1}{16} = ?$
41. $\frac{1}{32} \times \frac{1}{32} = ?$
42. $\frac{1}{32} \times \frac{1}{64} = ?$
43. $\frac{1}{64} \times \frac{1}{2} = ?$
44. $\frac{1}{64} \times \frac{1}{4} = ?$
45. $\frac{1}{64} \times \frac{1}{8} = ?$
46. $\frac{1}{64} \times \frac{1}{16} = ?$
47. $\frac{1}{64} \times \frac{1}{32} = ?$
48. $\frac{1}{64} \times \frac{1}{64} = ?$
49. $\frac{7}{8} \times \frac{1}{2} = ?$
50. $\frac{7}{8} \times \frac{1}{4} = ?$
51. $\frac{7}{8} \times \frac{1}{8} = ?$
52. $\frac{7}{8} \times \frac{1}{16} = ?$
53. $\frac{7}{8} \times \frac{1}{32} = ?$
54. $\frac{7}{8} \times \frac{1}{64} = ?$

Division

1. $\frac{1}{2} \div \frac{1}{2} = ?$
2. $\frac{1}{2} \div \frac{1}{4} = ?$
3. $\frac{1}{2} \div \frac{1}{8} = ?$
4. $\frac{1}{2} \div \frac{1}{16} = ?$
5. $\frac{1}{2} \div \frac{1}{32} = ?$
6. $\frac{1}{2} \div \frac{1}{64} = ?$
7. $\frac{1}{4} \div \frac{1}{2} = ?$
8. $\frac{1}{4} \div \frac{1}{4} = ?$
9. $\frac{1}{4} \div \frac{1}{8} = ?$
10. $\frac{1}{4} \div \frac{1}{16} = ?$
11. $\frac{1}{4} \div \frac{1}{32} = ?$
12. $\frac{1}{4} \div \frac{1}{64} = ?$
13. $\frac{5}{8} \div \frac{1}{2} = ?$
14. $\frac{5}{8} \div \frac{1}{4} = ?$
15. $\frac{5}{8} \div \frac{1}{8} = ?$
16. $\frac{5}{8} \div \frac{1}{16} = ?$
17. $\frac{5}{8} \div \frac{1}{32} = ?$
18. $\frac{5}{8} \div \frac{1}{64} = ?$
19. $\frac{1}{8} \div \frac{1}{2} = ?$
20. $\frac{1}{8} \div \frac{1}{4} = ?$
21. $\frac{1}{8} \div \frac{1}{8} = ?$
22. $\frac{1}{8} \div \frac{1}{16} = ?$
23. $\frac{1}{8} \div \frac{1}{32} = ?$
24. $\frac{1}{8} \div \frac{1}{64} = ?$
25. $\frac{1}{16} \div \frac{1}{2} = ?$
26. $\frac{1}{16} \div \frac{1}{4} = ?$
27. $\frac{1}{16} \div \frac{1}{8} = ?$
28. $\frac{1}{16} \div \frac{1}{16} = ?$
29. $\frac{1}{16} \div \frac{1}{32} = ?$
30. $\frac{1}{16} \div \frac{1}{64} = ?$
31. $\frac{3}{4} \div \frac{1}{2} = ?$
32. $\frac{3}{4} \div \frac{1}{4} = ?$
33. $\frac{3}{4} \div \frac{1}{8} = ?$
34. $\frac{3}{4} \div \frac{1}{16} = ?$
35. $\frac{3}{4} \div \frac{1}{32} = ?$
36. $\frac{3}{4} \div \frac{1}{64} = ?$
37. $\frac{1}{32} \div \frac{1}{2} = ?$
38. $\frac{1}{32} \div \frac{1}{4} = ?$
39. $\frac{1}{32} \div \frac{1}{8} = ?$
40. $\frac{1}{32} \div \frac{1}{16} = ?$
41. $\frac{1}{32} \div \frac{1}{32} = ?$
42. $\frac{1}{32} \div \frac{1}{64} = ?$
43. $\frac{1}{64} \div \frac{1}{2} = ?$
44. $\frac{1}{64} \div \frac{1}{4} = ?$
45. $\frac{1}{64} \div \frac{1}{8} = ?$
46. $\frac{1}{64} \div \frac{1}{16} = ?$
47. $\frac{1}{64} \div \frac{1}{32} = ?$
48. $\frac{1}{64} \div \frac{1}{64} = ?$
49. $\frac{7}{8} \div \frac{1}{2} = ?$
50. $\frac{7}{8} \div \frac{1}{4} = ?$
51. $\frac{7}{8} \div \frac{1}{8} = ?$
52. $\frac{7}{8} \div \frac{1}{16} = ?$
53. $\frac{7}{8} \div \frac{1}{32} = ?$
54. $\frac{7}{8} \div \frac{1}{64} = ?$

Decimal Fractions

A **power** is the product of equal factors, as $10 \times 10 = 100$. $10 \times 10 \times 10 = 1000$. 100 is the second power of 10. 1000 is the third power of 10.

A **decimal fraction** or **decimal** is a fraction whose denominator is 10 or a power of 10. A common fraction may have any number for its denominator, but a decimal fraction must always have for its denominator 10, or a power of 10. A decimal is written at the right of a period (.), called the decimal point. A figure at the right of a decimal point is called a decimal figure.

$\frac{5}{10} = .5$; $\frac{25}{100} = .25$; $\frac{7}{100} = .07$; $\frac{16}{1000} = .016$.

A **mixed decimal** is an integer and a decimal; as, 16.04.

To *read* a decimal, read the decimal as an integer, and give it the denomination of the right-hand figure. To *write* a decimal, write the numerator, prefixing ciphers when necessary to express the denominator, and place the point at the left. There must be as many decimal places in the decimal as there are ciphers in the denominator.

EXAMPLES

Read the following numbers:

1. .7	7. .4375	13. .0000054	19. 9.999999
2. .07	8. .03125	14. 35.18006	20. .10016
3. .007	9. .21875	15. .0005	21. .000155
4. .700	10. .90625	16. 100.000104	22. .26
5. .125	11. .203125	17. 9.1632002	23. .1
6. .0625	12. .234375	18. 30.3303303	24. .80062

Express decimally:

1. Four tenths.
2. Three hundred twenty-five thousandths.
3. Seventeen thousand two hundred eleven hundred-thousandths.
4. Seventeen hundredths.
5. Fifteen thousandths.
6. Five hundredths.
7. Six ten-thousandths.
8. Eighteen and two hundred sixteen hundred-thousandths.
9. One hundred twelve hundred-thousandths.
10. 10 millionths.
11. 824 ten-thousandths.
12. Twenty-nine hundredths.
13. 324 and one hundred twenty-six millionths.
14. 7846 hundred-millionths.
15. $\frac{15}{100}$, $\frac{289}{1000000}$, $\frac{1}{10000000}$, $\frac{1000}{1000}$, $15\frac{5}{1000}$, $500\frac{5}{10}$.

16. $\frac{568}{100000000}$, $\frac{1}{100}$, $\frac{2123}{10000}$, $\frac{3}{10}$, $\frac{28654}{1000000}$.

17. One and one tenth.

18. One and one hundred-thousandth.

19. One thousand four and twenty-nine hundredths.

Reduction of Decimals

Ciphers *annexed* to a decimal do not change the value of the decimal; these ciphers are called decimal ciphers. For each cipher *prefixed* to a decimal, the value is diminished tenfold. The denominator of a decimal — when expressed — is always 1 with as many ciphers as there are decimal places in the decimal.

To reduce a decimal to a common fraction.

Write the numerator of the decimal omitting the point for the numerator of the fraction. For the denominator write 1 *with as many ciphers annexed as there are decimal places in the decimal. Then reduce to lowest terms.*

EXAMPLE. — Reduce .25 and .125 to common fractions.

$$.25 = \frac{25}{100} = \frac{\overset{1}{\cancel{25}}}{\underset{4}{\cancel{100}}} = \frac{1}{4} \quad Ans.$$

Write 25 for the numerator and 1 for the denominator with two 0's, which makes $\frac{25}{100}$; $\frac{25}{100}$ reduced to lowest terms is $\frac{1}{4}$.

$$.125 = \frac{125}{1000} = \frac{\overset{1}{\cancel{125}}}{\underset{8}{\cancel{1000}}} = \frac{1}{8} \quad Ans.$$

.125 is reduced to a common fraction in the same way.

EXAMPLE. — Reduce $.37\frac{1}{2}$ to a common fraction.

$$\frac{37\frac{1}{2}}{100} = \frac{\frac{75}{2}}{100} = \frac{\overset{3}{\cancel{75}}}{2} \times \frac{1}{\underset{4}{\cancel{100}}} = \frac{3}{8} \quad Ans.$$

$37\frac{1}{2}$ has for its denominator 1 with 00, which equals $\frac{37\frac{1}{2}}{100}$.

This is a complex fraction which reduced to lowest terms is $\frac{3}{8}$.

EXAMPLES

Reduce to common fractions:

1. .09375	**6.** 2.25	**11.** $.16\frac{2}{3}$	**16.** $.87\frac{1}{2}$
2. .15625	**7.** 16.144	**12.** $.33\frac{1}{3}$	**17.** $.66\frac{2}{3}$
3. .015625	**8.** 25.0000100	**13.** $.06\frac{1}{4}$	**18.** $.36\frac{7}{8}$
4. .609375	**9.** 1084.0025	**14.** .140625	**19.** $.83\frac{1}{3}$
5. .578125	**10.** $.12\frac{1}{2}$	**15.** .984375	**20.** $.62\frac{1}{2}$

To reduce a common fraction to a decimal.

Annex decimal ciphers to the numerator and divide by the denominator. Point off from the right of the quotient as many places as there are ciphers annexed. If there are not figures enough in the quotient, prefix ciphers.

The division will not always be exact, *i.e.* $\frac{1}{7} = .142\frac{6}{7}$ or $.142^{+}$.

EXAMPLE. — Reduce $\frac{3}{4}$ to a decimal.

$$\begin{array}{r} .75 \\ 4\overline{)3.00} \\ \underline{28} \\ 20 \end{array}$$

$$\tfrac{3}{4} = .75$$

EXAMPLES

Reduce to decimals:

1. $\frac{1}{20}$	**6.** $\frac{7}{8}$	**11.** $\frac{1}{80}$	**16.** $\frac{7}{8}$	**21.** $\frac{1}{200}$
2. $\frac{1}{150}$	**7.** $\frac{11}{16}$	**12.** $\frac{1}{250}$	**17.** $16\frac{1}{4}$	**22.** $25.12\frac{1}{2}$
3. $\frac{1}{380}$	**8.** $\frac{15}{32}$	**13.** $\frac{1}{1250}$	**18.** $66\frac{2}{3}$	**23.** $33\frac{1}{3}$
4. $\frac{1}{8}$	**9.** $\frac{7}{32}$	**14.** $12\frac{1}{2}$	**19.** $\frac{15}{20}$	**24.** $\frac{7}{24}$
5. $\frac{3}{4}$	**10.** $\frac{1}{750}$	**15.** $\frac{6}{11}$	**20.** $\frac{5}{9}$	**25.** $\frac{1}{380}$

Addition of Decimals

To add decimals, write them so that their decimal points are in a column. Add as in integers, and place the point in the sum directly under the points above it.

EXAMPLE. — Find the sum of 3.87, 2.0983, 5.00831, .029, .831.

3.87	
2.0983	
5.00831	
.029	
.831	
11.83661	*Ans.*

Place these numbers, one under the other, with decimal points in a column, and add as in addition of integers. The sum of these numbers should have the decimal point in the same column as the numbers that were added.

EXAMPLES

Find the sum:

1. 5.83, 7.016, 15.0081, and 18.3184.
2. 12.031, 0.0894, 12.0084, and 13.984.
3. .0765, .002478, .004967, .0007862, .17896.
4. 24.36, 1.358, .004, and 1632.1.
5. .175, 1.75, 17.5, 175., 1750.
6. 1., .1, .01, .001, 100, 10., 10.1, 100.001.
7. Add 5 tenths; 8063 millionths; 25 hundred-thousandths; 48 thousandths; 17 millionths; 95 ten-millionths; 5, and 5 hundred-thousandths; 17 ten-thousandths.
8. Add $24\frac{3}{4}$, $17\frac{1}{4}$, .0058, $7\frac{1}{8}$, $9\frac{1}{16}$.
9. 32.58, 28963.1, 287.531, 76398.9341.
10. 145., 14.5, 1.45, .145, .0145.

Subtraction of Decimals

To subtract decimals, write the smaller number under the larger with the decimal point of the subtrahend directly under the decimal point of the minuend. Subtract as in integers, and place the point directly under the points above.

EXAMPLE. — Subtract 2.17857 from 4.3257.

4.32570	*Minuend*
2.17857	*Subtrahend*
2.14713	*Remainder*

Write the lesser number under the greater, with the decimal points under each other. Add a 0 to the minuend, 4.3257, to give it the same denominator as the subtrahend. Then subtract as in subtraction of integers. Write the remainder with decimal point under the other two points.

EXAMPLES

Subtract:

1. $59.0364 - 30.8691 = ?$
2. $48.7209 - 12.0039 = ?$
3. $.0625 - .03125 = ?$
4. $.00011 - .000011 = ?$
5. $10 - .1 + .0001 = ?$
6. From one thousand take five thousandths.
7. Take 17 hundred-thousandths from 1.2.
8. From $17.37\frac{1}{2}$ take $14.16\frac{1}{8}$.
9. Prove that $\frac{1}{2}$ and .500 are equal.
10. Find the difference between $\frac{884}{1000}$ and $\frac{884}{10000}$.

Multiplication of Decimals

To multiply decimals proceed as in integers, and give to the product as many decimal figures as there are in both multiplier and multiplicand. When there are not figures enough in the product, prefix ciphers.

Example. — Find the product of 6.8 and .63.

6.8	*Multiplicand*
.63	*Multiplier*
204	
408	
4.284	*Product*

6.8 is the multiplicand and .63 the multiplier. Their product is 4.284 with three decimal figures, the number of decimal figures in the multiplier and multiplicand.

Example. — Find the product of .05 and .3.

.05	*Multiplicand*
.3	*Multiplier*
.015	*Product*

The product of .05 and .3 is .015 with a cipher prefixed to make the three decimal figures required in the product.

EXAMPLES

Find the products:

1. $46.25 \times .125$
2. $8.0625 \times .1875$
3. $.015 \times .05$
4. $25.863 \times 4\frac{1}{8}$

5. 11.11×100
6. $.5625 \times 6.28125$
7. $.326 \times 2.78$
8. $.325 \times 12\frac{1}{2}$
9. $.001542 \times .0052$
10. 1.001×1.01

To multiply by 10, 100, 1000, *etc., remove the point one place to the right for each cipher in the multiplier.*

This can be performed without writing the multiplier.

EXAMPLE. — Multiply 1.625 by 100.

$$1.625 \times 100 = 162.5$$

To multiply by 200, *remove the point to the right and multiply by* 2.

EXAMPLE. — Multiply 86.44 by 200.

$$\begin{array}{r} 86.44. \\ 2 \\ \hline 17{,}288 \end{array}$$

EXAMPLES

Find the product of:

1. 1 thousand by one thousandth.
2. 1 million by one millionth.
3. 700 thousands by 7 hundred-thousandths.
4. 3.894×3000
5. $1.892 \times 2000.$

Division of Decimals

To divide decimals proceed as in integers, and give to the quotient as many decimal figures as the number in the dividend exceeds those in the divisor.

EXAMPLE. — Divide 12.685 by .5.

$$\textit{Divisor}\ .5\overline{)12.685}\ \textit{Dividend}$$
$$25.37\ \textit{Quotient}$$

The number of decimal figures in the quotient, 12.685, exceeds the number of decimal figures in the divisor, .5, by two. So there must be two decimal figures in the quotient.

EXAMPLE. — Divide 399.552 by 192.

```
                2.081  Quotient
Divisor  192)399.552  Dividend
             384
             1555
             1536
               192
               192
```

When the divisor is an integer, the point in the quotient should be placed directly over the point in the dividend, and the division performed as in integers. This may be proved by multiplying divisor by quotient, which would give the dividend.

EXAMPLE. — Divide 28.78884 by 1.25.

```
                 23.031+    Quotient
Divisor 1.25.)28.78.884   Dividend
               250
                378
                375
                  388
                  375
                   134
                   125
                     9  Remainder
```

When the divisor contains decimal figures, move the point in both divisor and dividend as many places to the right as there are decimal places in the divisor, which is equivalent to multiplying both divisor and dividend by the same number and does not change the quotient. Then place the point in the quotient as if the divisor were an integer. In this example, the multiplier of both dividend and divisor is 100.

EXAMPLES

Find the quotients:

1. $.0625 \div .125$
2. $315.432 \div .132$
3. $.75 \div .0125$
4. $125 \div 12\frac{1}{2}$
5. $1000 \div .001$
6. $2.496 \div .136$
7. $28000 \div 16.8$
8. $1.225 \div 4.9$
9. $3.1416 \div 27$
10. $8.33 \div 5$

To divide by 10, 100, 1000, *etc., remove the point one place to the left for each cipher in the divisor.*

To divide by 200, *remove the point two places to the left, and divide by* 2.

EXAMPLES

Find the quotients:

1. $38.64 \div 10$
2. $398.42 \div 1000$
3. $1684.32 \div 1000$
4. $1.155 \div 100$
5. $386.54 \div 2000$
6. $865.45 \div 5000$
7. $38.28 \div 400$
8. $2.5 \div 500$
9. $.5 \div 10$
10. $.001 \div 1000$

Parts of 100 or 1000

1. What part of 100 is $12\frac{1}{2}$? 25? $33\frac{1}{3}$?
2. What part of 1000 is 125? 250? $333\frac{1}{3}$?
3. How much is $\frac{1}{5}$ of 100? Of 1000?
4. How much is $\frac{1}{4}$ of 100? Of 1000?
5. What is $\frac{1}{8}$ of 100? Of 1000?

EXAMPLE. — How much is 25 times 24?

100 times 24 = 2400.
25 times 24 = $\frac{1}{4}$ as much as 100 times 24 = 600. *Ans.*

Short Method in Multiplication

To multiply by

25, multiply by 100 and divide by 4;
$33\frac{1}{3}$, multiply by 100 and divide by 3;
$16\frac{2}{3}$, multiply by 100 and divide by 6;
$12\frac{1}{2}$, multiply by 100 and divide by 8;
9, multiply by 10 and subtract the multiplicand;
11, if more than two figures, multiply by 10 and add the multiplicand;
11, if two figures, place the figure that is their sum between them.

$$63 \times 11 = 693 \qquad 74 \times 11 = 814$$

Note that when the sum of the two figures exceeds nine, the one in the tens place is carried to the figure at the left.

EXAMPLES

Multiply by the short process:

1. 81 by 11 =?	10. 68 by $16\frac{2}{3}$ = ?
2. 75 by $33\frac{1}{3}$ = ?	11. 112 by 11 = ?
3. 128 by $12\frac{1}{2}$ = ?	12. 37 by 11 = ?
4. 87 by 11 = ?	13. 4183 by 11 = ?
5. 19 by 9 = ?	14. 364 by $33\frac{1}{3}$ = ?
6. 846 by 11 = ?	15. 8712 by $12\frac{1}{2}$ = ?
7. 88 by 11 = ?	16. 984 by $16\frac{2}{3}$ = ?
8. 19 by 11 = ?	17. 36 by 25 = ?
9. 846 by $16\frac{2}{3}$ = ?	18. 30 by $333\frac{1}{3}$ = ?

Aliquot Parts of $1.00

The **aliquot parts** of a number are the numbers that are exactly contained in it. The aliquot parts of 100 are 5, 20, $12\frac{1}{2}$, $16\frac{2}{3}$, $33\frac{1}{3}$, etc.

The monetary unit of the United States is the dollar, containing one hundred cents which are written decimally.

$6\frac{1}{4}$ cents = \$ $\frac{1}{16}$	25 cents = \$ $\frac{1}{4}$ = quarter dollar
$8\frac{1}{3}$ cents = \$ $\frac{1}{12}$	$33\frac{1}{3}$ cents = \$ $\frac{1}{3}$
$12\frac{1}{2}$ cents = \$ $\frac{1}{8}$	50 cents = \$ $\frac{1}{2}$ = half dollar
$16\frac{2}{3}$ cents = \$ $\frac{1}{6}$	

10 mills	= 1 cent, ct.	= \$ .01 or \$ 0.01
5 cents	= 1 "nickel"	= \$ .05
10 cents	= 1 dime, *d.*	= \$ .10
10 dimes	= 1 dollar, \$	= \$ 1.00
10 dollars	= 1 eagle, E.	= \$ 10.00

EXAMPLE.—What will 69 drills cost at $16\frac{2}{3}$ cents each?

69 drills will cost 69 × $16\frac{2}{3}$ cts., or 69 × \$ $\frac{1}{6} = \frac{69}{6}$ = \$ $11\frac{3}{6}$ = \$11.50.

EXAMPLE. — At 25¢ a box, how many boxes of nails can be bought for $ 8.00?

$8 \div \frac{1}{4} = 8 \times \frac{4}{1} = 32$ boxes. *Ans.*

Review of Decimals

1. For work on a job one man receives $ 13.75, a second man $ 12.45, a third man $ 14.21, and a fourth man $ 21.85. What is the total amount paid for the work?

2. A pipe has an inside diameter of 3.067 inches and an outside diameter of 3.428 inches. What is the thickness of the metal of the pipe?

3. At $2\frac{1}{2}$ cts. a pound, what will be the cost of 108 castings each weighing 29 lb.?

4. A man receives $ 121.50 for doing a piece of work. He gives $ 12.25 to one of his helpers, and $ 10.50 to another. He also pays $ 75.75 for material. How much does he make on the job?

5. An automobile runs at the rate of $9\frac{1}{2}$ miles an hour. How long will it take it to go from Lowell to Boston, a distance of 26.51 miles?

6. A $\frac{3}{4}''$ square steel bar weighs 1.914 lb. per foot. What will be the cost of 5000 feet of $\frac{3}{4}''$ steel bars, if it costs $ 1.75 per 100 lb.?

7. Which is cheaper, and how much, to have a $13\frac{1}{2}$ cents an hour man take $13\frac{1}{4}$ hours on a piece of work, or hire a $17\frac{1}{2}$ cents an hour man who can do it in $9\frac{1}{2}$ hours?

8. On Monday 1725.25 lb. of coal are used, on Tuesday 2134.43 lb., on Wednesday 1651.21 lb., on Thursday 1821.42 lb., on Friday 1958.82 lb., and on Saturday 658.32 lb. How many pounds of coal were used during the week?

9. If in the example above there were 10,433.91 lb. of coal on hand at the beginning of the week, how much was left at the end of the week?

10. A foot length of $\frac{3}{4}''$ round steel bar weighs 1.503 lb., 10-foot length of $\frac{3}{4}''$ square steel bar weighs 19.140 lb., 1-foot length of $\frac{1}{16}''$ round steel bar weighs 3.017 lb. What is the total weight of the three pieces?

11. An alloy is made of copper and zinc. If .66 is copper and .34 is zinc, how many pounds of zinc and how many pounds of copper will there be in a casting of the alloy weighing 98 lb.?

12. A train leaves New York at 2.10 P.M., and arrives in Philadelphia at 4.15 P.M. The distance is 90 miles. What is the average rate per hour of the train?

13. The weight of a foot of $\frac{9}{16}''$ steel bar is 1.08 lb. Find the weight of a 21-foot bar.

14. A steam pump pumps 3.38 gallons of water to each stroke and the pump makes 51.1 strokes per minute. How many gallons of water will it pump in an hour?

15. At $12\frac{1}{2}$ cents per hour, what will be the pay for $23\frac{1}{2}$ days if the days are 10 hours each?

Compound Numbers

A number which expresses only one kind of concrete units is a **simple number**; as, 5 pk., 4 knives, 6.

A number composed of different kinds of concrete units that are related to each other is a **compound number**; as, 3 bu. 2 pk. 1 qt.

A **denomination** is a name given to a unit of measure or of weight.

A number having one or more denominations is also called a **denominate number.**

Reduction is the process of changing a number from one denomination to another without changing its value.

Changing to a lower denomination is called **reduction descending**; as, 2 bu. 3 pk. = 88 qt.

Changing to a higher denomination is called **reduction ascending**; as, 88 qt. = 2 bu. 3 pk.

Linear Measure is used in measuring lines or distance.

Table

12 in. (in.)	= 1 foot, ft.
3 feet	= 1 yard, yd.
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet	= 1 rod, rd.
40 rods	= 1 furlong, fur.
8 furlongs	= 1 mile, mi.
320 rods, or 5280 feet	= 1 mile.

1 mi. = 320 rd. = 1760 yd. = 5280 ft. = 63,360 in.

Surveyors' Measure is used in measuring land.

Table

7.92 inches	= 1 link, li.
100 links	= 1 chain, ch.
80 chains	= 1 mile, mi.

A chain, or steel measuring tape, 100 feet long, is generally used by engineers. The feet are usually divided into tenths instead of into inches.

Square Measure is used in measuring surfaces.

Table

144 square inches	= 1 square foot, sq. ft.
9 square feet	= 1 square yard, sq. yd.
$30\frac{1}{4}$ square yards / $272\frac{1}{4}$ square feet	= 1 square rod, sq. rd.
160 square rods	= 1 acre, A.
640 acres	= 1 square mile, sq. mi.

1 sq. mi. = 640 A. = 102,400 sq. rd. = 3,097,600 sq. yd.

Cubic Measure is used in measuring volumes or solids.

Table

1728 cubic inches	= 1 cubic foot, cu. ft.
27 cubic feet	= 1 cubic yard, cu. yd.
16 cubic feet	= 1 cord foot, cd. ft.
8 cord feet, or 128 cu. ft.	= 1 cord, cd.

1 cu. yd. = 27 cu. ft. = 46,656 cu. in.

Liquid Measure is used in measuring liquids.

Table

4 gills (gi.)	= 1 pint, pt.
2 pints	= 1 quart, qt.
4 quarts	= 1 gallon, gal.

1 gal. = 4 qt. = 8 pt. = 32 gi.

A gallon contains 231 cubic inches.

The standard barrel is $31\frac{1}{2}$ gal., and the hogshead 63 gal.

Dry Measure is used in measuring roots, grain, vegetables, etc.

Table

2 pints	= 1 quart, qt.
8 quarts	= 1 peck, pk.
4 pecks	= 1 bushel, bu.

1 bu. = 4 pk. = 32 qt. = 64 pints.

The bushel contains 2150.42 cubic inches; 1 dry quart contains 67.2 cu. in. A cubic foot is $\frac{4}{5}$ of a bushel.

Avoirdupois Weight is used in weighing all common articles; as, coal, groceries, hay, etc.

Table

16 ounces (oz.)	= 1 pound, lb.
100 pounds	= 1 hundredweight, cwt.; or cental, ctl.
20 cwt., or 2000 lb.	= 1 ton, T.

1 T. = 20 cwt. = 2000 lb. = 32,000 oz.

The *long ton* of 2240 pounds is used at the United States Custom House and in weighing coal at the mines.

Measure of Time.

Table

60 seconds (sec.)	= 1 minute, min.
60 minutes	= 1 hour, hr.
24 hours	= 1 day, da.
7 days	= 1 week, wk.
365 days	= 1 year, yr.
366 days	= 1 leap year.
100 years	= 1 century.

Counting.

Table

12 things = 1 dozen, doz.
12 dozen = 1 gross, gr.
12 gross = 1 great gross, G. gr.

Paper Measure.

Table

24 sheets = 1 quire	2 reams = 1 bundle
20 quires = 1 ream	5 bundles = 1 bale

Reduction Descending

EXAMPLE. — Reduce 17 yd. 2 ft. 9 in. to inches.

1 yd. = 3 ft.
17 yd. = $17 \times 3 = 51$ ft.
$51 + 2 = 53$ ft.
1 ft. = 12 in.
53 ft. = $53 \times 12 = 636$ in.
$636 + 9 = 645$ in. *Ans.*

EXAMPLES

Reduce to lower denominations:

1. 46 rd. 4 yd. 2 ft. to feet.
2. 4 A. 15 sq. rd. 4 sq. ft. to square inches.
3. 16 cu. yd. 25 cu. ft. 900 cu. in. to cubic inches.
4. 15 gal. 3 qt. 1 pt. to pints.
5. 27 da. 18 hr. 49 min. to seconds.

Reduction Ascending

EXAMPLE. — Reduce 1306 gills to higher denominations.

4)1306 gi.
2)326 pt. + 2 gi.
4)163 qt.
40 gal. + 3 qt.
40 gal. 3 qt. 2 gi. *Ans.*

Since in 1 pt. there are 4 gi., in 1306 gi. there are as many pints as 4 gi. are contained times in 1306 gi., or 326 pt. and 2 gi. remainder.

In the same way the quarts and gallons are found. So there are in 1306 gi., 40 gal. 3 qt. 2 gi.

EXAMPLES

Reduce to higher denominations:

1. Reduce 225,932 in. to miles, etc.

2. Change 1384 dry pints to higher denominations.

3. In 139,843 sq. in. how many square miles, rods, etc.?

4. How many cords of wood in 3692 cu. ft.?

5. How many bales in 24,000 sheets of paper?

A **denominate fraction** is a fraction of a unit of weight or measure.

To reduce denominate fractions to integers of lower denominations.

Change the fraction to the next lower denomination. Treat the fractional part of the product in the same way, and so proceed to the required denomination.

EXAMPLE. — Reduce $\frac{5}{7}$ of a mile to rods, yards, feet, etc.

$\frac{5}{7}$ of 320 rd. $= \frac{1600}{7}$ rd. $= 228\frac{4}{7}$ rd.
$\frac{4}{7}$ of $\frac{11}{2}$ yd. $= \frac{44}{14}$ yd. $= 3\frac{1}{7}$ yd.
$\frac{1}{7}$ of 3 ft. $= 0\frac{3}{7}$ ft.
$\frac{3}{7}$ of 12 in. $= \frac{36}{7}$ in. $= 5\frac{1}{7}$ in.
$\frac{5}{7}$ of a mile = 228 rd. 3 yd. 0 ft. $5\frac{1}{7}$ in.

The same process applies to denominate decimals.

To reduce denominate decimals to denominate numbers.

EXAMPLE. — Reduce .87 bu. to pecks, quarts, etc.

$$\begin{array}{r} .87 \text{ bu.} \\ 4 \\ \hline 3.48 \text{ pk.} \end{array} \qquad \begin{array}{r} .48 \text{ pk.} \\ 8 \\ \hline 3.84 \text{ qt.} \end{array} \qquad \begin{array}{r} .84 \text{ qt.} \\ 2 \\ \hline 1.68 \text{ pt.} \end{array}$$

3 pk. 3 qt. 1.68 pt. *Ans.*

Change the decimal fraction to the next lower denomination. Treat the decimal part of the product in the same way, and so proceed to the required denomination.

EXAMPLES

Reduce to integers of lower denominations :

1. $\frac{5}{6}$ of an acre.
3. $\frac{3}{7}$ of a ton.
2. .3125 of a gallon.
4. .51625 of a mile.
5. Change $\frac{3}{7}$ of a year to months and days.
6. .2364 of a ton.
7. What is the value of $\frac{1}{8}$ of $1\frac{1}{8}$ of a mile?
8. Reduce $\frac{34}{50}$ bu. to integers of lower denominations.
9. .375 of a month.
10. $\frac{9}{14}$ acre are equal to how many square rods, etc.?

Addition of Compound Numbers

EXAMPLE. — Find the sum of 7 hr. 30 min. 45 sec., 12 hr. 25 min. 30 sec., 20 hr. 15 min. 33 sec., 10 hr. 27 min. 46 sec.

hr.	min.	sec.
7	30	45
12	25	30
20	15	33
10	27	46
50	39	34

50 hr. 39 min. 34 sec. *Ans.*

The sum of the seconds = 154 sec. = 2 min. 34 sec. Write the 34 sec. under the sec. column and add the 2 min. to the min. column. Add the other columns in the same way.

Subtraction of Compound Numbers

EXAMPLE. — From 39 gal. 2 qt. 2 pt. 1 gi. take 16 gal. 2 qt. 3 pt. 3 gi.

gal.	qt.	pt.	gi.
39	2	2	1
16	2	3	3
22	3	0	2

22 gal. 3 qt. 2 gi. *Ans.*

As 3 gi. cannot be taken from 1 gi., 4 gi. or 1 pt. are borrowed from the pt. column and added to the 1 gi. Subtract 3 gi. from the 5 gi. and the remainder is 2 gi. Continue in the same way until all are subtracted. Then the remainder is 22 gal. 3 qt. 0 pt. 2 gi.

Multiplication of Compound Numbers

EXAMPLE. — Multiply 4 yd. 2 ft. 8 in. by 8.

yd.	ft.	in.
4	2	8
		8
39	0	4

39 yd. 4 in. *Ans.*

8 times 8 in. = 64 in. = 5 ft. 4 in. Place the 4 in. under the in. column, and add the 5 ft. to the product of 2 ft. by 8, which equals 21 ft. = 7 yd. Add 7 yd. to the product of 4 yd. by 8 = 39 yd.

Division of Compound Numbers

EXAMPLE. — Find $\frac{1}{35}$ of 42 rd. 4 yd. 2 ft. 8 in.

rd. yd. ft. in.	
35)42 4 2 8(1 rd.	
35	
7	
$5\frac{1}{2}$	
$3\frac{1}{2}$	35)$24\frac{1}{2}$(0 ft.
35	12
$38\frac{1}{2}$	294
+ 4	+ 8
35)$42\frac{1}{2}$ yd. (1 yd.	35)302($8\frac{22}{35}$ in.
35	280
$7\frac{1}{2}$	22
3	
$22\frac{1}{2}$ ft.	1 rd. 1 yd. $8\frac{22}{35}$ in. *Ans.*
12	

$\frac{1}{35}$ of 42 rd. = 1 rd.; remainder, 7 rd. = $38\frac{1}{2}$ yd.; add 4 yd. = $42\frac{1}{2}$ yd. $\frac{1}{35}$ of $42\frac{1}{2}$ yd. = 1 yd.; remainder, $7\frac{1}{2}$ yd., = $22\frac{1}{2}$ ft. = $24\frac{1}{2}$ ft. $\frac{1}{35}$ of $24\frac{1}{2}$ ft. = 0 ft. $24\frac{1}{2}$ ft. = 294 in.; add 8 in. = 302 in. $\frac{1}{35}$ of 302 in. = $8\frac{22}{35}$ in.

Difference between Dates

EXAMPLE. — Find the time from Jan. 25, 1842, to July 4, 1896.

1896	7	4	
1842	1	25	
54 yr.	5 mo.	9 da.	*Ans.*

It is customary to consider 30 days to a month. July 4, 1896, is the 1896th yr., 7th mo., 4th da., and Jan. 25, 1842, is the 1842d yr., 1st. mo., 25th da. Subtract, taking 30 da. for a month.

Example. — What is the exact number of days between Dec. 16, 1895, and March 12, 1896?

Dec.	15	Do not count the first day mentioned. There are 15 days in December, after the 16th. January has 31 days, February 29 (leap year), and 12 days in March; making 87 days.
Jan.	31	
Feb.	29	
Mar.	12	
	87 days. *Ans.*	

EXAMPLES

1. How much time elapsed from the landing of the Pilgrims, Dec. 11, 1620, to the Declaration of Independence, July 4, 1776?

2. Washington was born Feb. 22, 1732, and died Dec. 14, 1799. How long did he live?

3. Mr. Smith gave a note dated Feb. 25, 1896, and paid it July 12, 1896. Find the exact number of days between its date and the time of payment.

4. A carpenter earning $2.50 per day commenced Wednesday morning, April 1, 1896, and continued working every week day until June 6. How much did he earn?

5. Find the exact number of days between Jan. 10, 1896, and May 5, 1896.

6. John goes to bed at 9.15 P.M. and gets up at 7.10 A.M. How many minutes does he spend in bed?

To multiply or divide a compound number by a fraction.

To multiply by a fraction, multiply by the numerator, and divide the product by the denominator.

To divide by a fraction, multiply by the denominator, and divide the product by the numerator.

When the multiplier or divisor is a mixed number, reduce to an improper fraction, and proceed as above.

EXAMPLES

1. How much is $\frac{6}{7}$ of 16 hr. 17 min. 14 sec.?

2. A field contains 10 acres 12 sq. rd. of land, which is $\frac{5}{6}$ of the whole farm. Find the size of the farm.

3. If a train runs 60 mi. 35 rd. 16 ft. in one hour, how far will it run in $12\frac{2}{5}$ hr. at the same rate of speed?

4. Divide 14 bu. 3 pk. 6 qt. 1 pt. by $\frac{7}{8}$.

5. Divide 5 yr. 1 mo. 1 wk. 1 da. 1 hr. 1 min. 1 sec. by $3\frac{2}{5}$.

EXAMPLES

1. A time card on a piece of work states that 2 hours and 15 minutes were spent in lathe work, 1 hour and 12 minutes in milling, 2 hours and 45 minutes in planing, and 1 hour and 30 minutes on bench work. What was the number of hours spent on the job?

2. How many castings, each weighing 14 oz., can be obtained from 860 lb. of metal if nothing is allowed for waste?

3. How many feet long must a machine shop be to hold a lathe 8′ 6″, a planer 14′ 4″, a milling machine 4′ 2″, and a lathe 7′ 5″, placed side by side? 3′ 3″ were allowed between the machines and between the walls and the machines.

4. How many gross in a lot of 968 screws?

5. Find the sum of 7 hr. 30 min. 45 sec., 12 hr. 25 min. 30 sec., 20 hr. 15 min. 33 sec., 10 hr. 27 min. 46 sec.

6. If a train is run for 8 hours at the average rate of 50 mi. 30 rd. 10 ft. per hour, how great is the distance covered?

7. A telephone pole is 31 feet long. If 4 ft. 7 in. are under ground, how high (in inches) is the top of the pole above the street?

8. If 100 bars of iron, each $2\frac{3}{4}$′ long, weigh 70 lb., what is the total weight of 2300?

9. If 43 in. are cut from a wire 3 yd. 2 ft. 6 in. long, what is the length of the remaining piece?

10. If a rod of iron 18′ 8″ long is cut into pieces $6\frac{7}{8}''$ long and $\frac{7}{64}''$ is allowed for waste in each cut, how many pieces can be cut? How much remains?

11. I have 84 lb. 14 oz. of salt which I wish to put into packages of 2 lb. 6 oz. each. How many packages will there be?

12. If one bottle holds 1 pt. 3 gi., how many dozen bottles will be required to hold 65 gal. 2 qt. 1 pt.?

13. How many pieces $5\frac{1}{2}''$ long can be cut from a rod 16′ 8″ long, if 5″ is allowed for waste?

14. What is the entire length of a railway consisting of five different lines measuring respectively 160 mi. 185 rd. 2 yd., 97 mi. 63 rd. 4 yd., 126 mi. 272 rd. 3 yd., 67 mi. 199 rd. 5 yd., and 48 mi. 266 rd. 5 yd.?

Percentage

Percentage is a process of solving questions of relation by means of hundredths or per cent (%).

Every question in percentage involves three elements: the rate per cent, the base, and the percentage.

The *rate per cent* is the number of hundredths taken.

The *base* is the number of which the hundredths are taken.

The *percentage* is the result obtained by taking a certain per cent of a number.

Since the percentage is the result obtained by taking a certain per cent of a number it follows that, *the percentage is the product of the base and the rate.* The rate and base are always factors, the percentage is the product.

EXAMPLE.—How much is 8% of \$200?

$$8\% \text{ of } \$200 = 200 \times .08 = \$16. \qquad (1)$$

In (1) we have the three elements: 8 % is the rate, \$200 is the base, and \$16 is the percentage.

Since \$200 $\times$ .08 = \$16, the percentage;
\$16 $\div$ .08 = \$200, the base;
and \$16 $\div$ \$200 = .08, the rate.

If any two of these elements are given, the other may be found:

$$Base \times Rate = Percentage$$
$$Percentage \div Rate = Base$$
$$Percentage \div Base = Rate$$

Per cent is commonly used in the decimal form, but many operations may be much shortened by using the common fraction form.

$1\% = .01 = \frac{1}{100}$	$\frac{1}{2}\% = .00\frac{1}{2}$ or $.005$
$10\% = .10 = \frac{1}{10}$	$33\frac{1}{3}\% = .33\frac{1}{3} = \frac{1}{3}$
$100\% = 1.00 = 1$	$8\frac{1}{4}\% = .08\frac{1}{4} = .0825$
$12\frac{1}{2}\% = .12\frac{1}{2}$ or $.125 = \frac{1}{8}$	$\frac{1}{8}\% = .00\frac{1}{8} = .00125$

There are certain per cents that are used so frequently that we should memorize their equivalent fractions.

$6\frac{1}{4}\% = \frac{1}{16}$	$33\frac{1}{3}\% = \frac{1}{3}$	$66\frac{2}{3}\% = \frac{2}{3}$
$10\% = \frac{1}{10}$	$37\frac{1}{2}\% = \frac{3}{8}$	$75\% = \frac{3}{4}$
$12\frac{1}{2}\% = \frac{1}{8}$	$40\% = \frac{2}{5}$	$80\% = \frac{4}{5}$
$16\frac{2}{3}\% = \frac{1}{6}$	$50\% = \frac{1}{2}$	$83\frac{1}{3}\% = \frac{5}{6}$
$20\% = \frac{1}{5}$	$60\% = \frac{3}{5}$	$87\frac{1}{2}\% = \frac{7}{8}$
$25\% = \frac{1}{4}$	$62\frac{1}{2}\% = \frac{5}{8}$	

EXAMPLES

1. Find 75 % of \$368.
2. Find 15 % of \$412.
3. 840 is $33\frac{1}{3}$ % of what number?
4. 615 is 15 % of what number?
5. What per cent of 12 is 8?

6. What per cent of 245 is 5?

7. What per cent of 195 is 39?

8. What per cent of 640 is 80?

9. What per cent of 750 is 25?

10. What per cent of 819 is 45?

Trade Discount

Merchants and jobbers have a price list. From this list they give special discounts according to the credit of the customer and the amount of supplies purchased, etc. If they give more than one discount it is understood that the first means the discount from the list price, the second denotes the discount from the remainder.

EXAMPLES

1. What is the price of 200 No. 1 cleats at $ 36.68 per M. at 40 % off?

2. Supplies from a hardware store amounted to $ 58.75. If $12\frac{1}{2}$ % were allowed for discount, what was the amount paid?

3. A dealer received a bill amounting to $ 212.75. Successive discounts of 75 %, 15 %, 10 %, and 5 % were allowed. What was amount to be paid?

4. 2 % is usually discounted on bills paid within 30 days. If the following are paid within 30 days, what will be the amounts due?

a.	$ 2816.49	*d.*	$ 1369.99	*g.*	$ 4916.01
b.	399.16	*e.*	2717.02	*h.*	30.19
c.	489.01	*f.*	918.69		

5. What is the price of 20 fuse plugs at $ .07 each, 30 % off?

6. Hardware supplies amounted to $ 127.79 with a discount of 40 and 15 %. What was the net price?

7. Which is better, for a merchant to receive a straight discount of 95 % or a successive discount of 75, 15, 5 %?

8. Twenty per cent is added to the number of workmen in a machine shop of 575 men. What is the number employed after the increase?

9. A steam pressure of 180 lb. per square inch is raised to 225 lb. per square inch. What is the per cent of increase?

10. If 31 out of 595 wheels are rejected because of defects, what per cent is rejected?

11. A clerk's salary was increased $6\frac{1}{4}$ %. If he now receives $ 850, what was his original salary?

12. A company lost $12\frac{1}{2}$ % of its men and had 560 left. How many men were there before?

Simple Interest

Money that is paid for the use of money is called *interest.* The money for the use of which interest is paid is called the *principal,* and the sum of the principal and interest is called the *amount.*

Interest at 6 % means 6 % of the principal for 1 year; 12 months of 30 days each are usually regarded as a year in computing interest.

EXAMPLE. — What is the interest on $ 100 for 3 years at 6 %?

$$\begin{array}{rl} \$100 & \\ .06 & \\ \hline \$6.00 & \text{interest for one year.} \\ 3 & \\ \hline \$18.00 & \text{interest for 3 years. } \textit{Ans.} \end{array}$$

Or, $\frac{6}{100} \times \frac{100}{1} \times \frac{3}{1} = \$18.$ *Ans.*

$$\$100 + \$18 = \$118, \text{ amount.}$$

$$\textit{Principal} \times \textit{Rate} \times \textit{Time} = \textit{Interest.}$$

Example. — What is the interest on $297.62 for 5 yr. 3 mo. at 6 % ?

$$\begin{array}{r} \$297.62 \\ .06 \\ \hline \$17.8572 \\ 5\tfrac{1}{4} \\ \hline 44643 \\ 892860 \\ \hline \$93.7503 \end{array}$$

$93.75. *Ans.*

Or, $\dfrac{\overset{3}{\cancel{6}}}{100} \times \dfrac{\$297.62}{1} \times \dfrac{21}{\underset{2}{\cancel{4}}} = \dfrac{\$18750.06}{200} = \$93.75.$

Note. — Final results should not include mills. Mills are disregarded if less than 5, and called another cent if 5 or more.

EXAMPLES

1. What is the interest on $ 586.24 for 3 months at 6 % ?
2. What is the interest on $ 816.01 for 9 months at 5 % ?
3. What is the interest on $ 314.72 for 1 year at 4 % ?
4. What is the interest on $ 876.79 for 2 yr. 3 mo. at $4\frac{1}{2}$ % ?

The Six Per Cent Method

By the 6 % method it is convenient to find first the interest of $ 1, then multiply it by the principal.

Example. — What is the interest of $ 50.24 at 6 % for 2 yr. 8 mo. 18 da. ?

$$\begin{array}{llll} \text{Interest on \$1 for 2 yr.} & = 2 \times \$.06 & = & \$.12 \\ \text{Interest on \$1 for 8 mo.} & = 8 \times \$.00\tfrac{1}{2} & = & .04 \\ \text{Interest on \$1 for 18 da.} & = 18 \times \$.000\tfrac{1}{6} & = & .003 \\ \hline \text{Interest on \$1 for 2 yr. 8 mo. 18 da.} & & & \$.163 \end{array}$$

Interest on $50.24 is 50.24 times $.163 = $8.19. *Ans.*

Find the interest on $1 for the given time, and multiply it by the principal, considered as an abstract number.

EXAMPLES

Find the interest and amount of the following:

1. $ 2350 for 1 yr. 3 mo. 6 da. at 5 %.
2. $ 125.75 for 2 mo. 18 da. at 7 %.
3. $ 950.63 for 3 yr. 17 da. at $4\frac{1}{2}$ %.
4. $ 625.57 for 1 yr. 2 mo. 15 da. at 6 %.

Exact Interest

When the time includes days, interest computed by the 6% method is not strictly exact, by reason of using only 30 days for a month, which makes the year only 360 days. The day is therefore reckoned as $\frac{1}{360}$ of a year, whereas it is $\frac{1}{365}$ of a year.

To compute exact interest, find the exact time in days, and consider 1 *day's interest as* $\frac{1}{365}$ *of* 1 *year's interest.*

EXAMPLE. — Find the exact interest of $ 358 for 74 days at 7%.

$ 358 × .07 = $25.06, 1 year's interest.
74 days' interest is $\frac{74}{365}$ of 1 year's interest.
$\frac{74}{365}$ of $ 25.06 = $ 5.08. *Ans.*

Or, $\frac{\$358}{1} \times \frac{7}{100} \times \frac{74}{365} = ?$

EXAMPLES

Find the exact interest of:

1. $ 324 for 15 da. at 5 %.
2. $ 253 for 98 da. at 4 %.
3. $ 624 for 117 da. at 7 %.
4. $ 620 from Aug. 15 to Nov. 12 at 6 %.
5. $ 153.26 for 256 da. at $5\frac{1}{2}$ %.
6. $ 540.25 from June 12 to Sept. 14 at 8 %.

Rules for Computing Interest

The following will be found to be excellent rules for finding the interest on any principal for any number of days.

Divide the principal by 100 and proceed as follows:

2 % — Multiply by number of days to run, and divide by 180.

$2\frac{1}{2}$ % — Multiply by number of days, and divide by 144.

3 % — Multiply by number of days, and divide by 120.

$3\frac{1}{2}$ % — Multiply by number of days, and divide by 102.86.

4 % — Multiply by number of days, and divide by 90.
5 % — Multiply by number of days, and divide by 72.
6 % — Multiply by number of days, and divide by 60.
7 % — Multiply by number of days, and divide by 51.43.
8 % — Multiply by number of days, and divide by 45.

Savings Bank Compound Interest Table

Showing the amount of $1, from 1 year to 15 years, with compound interest added semiannually, at different rates.

Per Cent	3	4	5	6	7	8	9
½ year	1 01	1 02	1 02	1 03	1 03	1 04	1 04
1 year	1 03	1 04	1 05	1 06	1 07	1 08	1 09
1½ years	1 04	1 06	1 07	1 09	1 10	1 12	1 14
2 years	1 06	1 08	1 10	1 12	1 14	1 16	1 19
2½ years	1 07	1 10	1 13	1 15	1 18	1 21	1 24
3 years	1 09	1 12	1 15	1 19	1 22	1 26	1 30
3½ years	1 10	1 14	1 18	1 22	1 27	1 31	1 36
4 years	1 12	1 17	1 21	1 26	1 31	1 36	1 42
4½ years	1 14	1 19	1 24	1 30	1 36	1 42	1 48
5 years	1 16	1 21	1 28	1 34	1 41	1 48	1 55
5½ years	1 17	1 24	1 31	1 38	1 45	1 53	1 62
6 years	1 19	1 26	1 34	1 42	1 51	1 60	1 69
6½ years	1 21	1 29	1 37	1 46	1 56	1 66	1 77
7 years	1 23	1 31	1 41	1 51	1 61	1 73	1 85
7½ years	1 24	1 34	1 44	1 55	1 67	1 80	1 93
8 years	1 26	1 37	1 48	1 60	1 73	1 87	2 02
8½ years	1 28	1 39	1 52	1 65	1 79	1 94	2 11
9 years	1 30	1 42	1 55	1 70	1 85	2 02	2 20
9½ years	1 32	1 45	1 59	1 75	1 92	2 10	2 30
10 years	1 34	1 48	1 63	1 80	1 98	2 19	2 41
11 years	1 38	1 54	1 72	1 91	2 13	2 36	2 63
12 years	1 42	1 60	1 80	2 03	2 28	2 56	2 87
13 years	1 47	1 67	1 90	2 15	2 44	2 77	3 14
14 years	1 51	1 73	1 99	2 28	2 62	2 99	3 42
15 years	1 56	1 80	2 09	2 42	2 80	3 24	3 74

EXAMPLES

Solve the following problems according to the tables given above:

1. What is the compound interest of $1 at the end of $8\frac{1}{2}$ years?

2. What is the compound interest of $1 at the end of 11 years?

3. How long will it take $400 to double itself at 4 %, compound interest?

4. How long will it take $580 to double itself at $4\frac{1}{2}$ %, compound interest?

5. How long will it take $615 to double itself at 5 %, simple interest?

6. How long will it take $784 to double itself at $5\frac{1}{2}$ %, simple interest?

7. Find the interest of $684 for 94 days at 3 %.

8. Find the interest of $1217 for 37 days at 4 %.

9. Find the interest of $681.14 for 74 days at $4\frac{1}{2}$ %.

10. Find the interest of $414.50 for 65 days at 5 %.

11. Find the interest of $384.79 for 115 days at 6 %.

Ratio and Proportion

Ratio is the relation between two numbers. It is found by dividing one by the other. The ratio of 4 to 8 is $4 \div 8 = \frac{1}{2}$.

The **terms** of the ratio are the two numbers compared. The first term of a ratio is the *antecedent*, and the second the *consequent*. The sign of the ratio is (:). (It is the division sign with the line omitted.) Ratio may also be expressed fractionally, as $\frac{16}{4}$ or $16:4$; or $\frac{3}{17}$ or $3:17$.

A ratio formed by dividing the consequent by the antecedent is an *inverse ratio:* $12:6$ is the inverse ratio of $6:12$.

The two terms of the ratio taken together form a couplet. Two or more couplets taken together form a *compound ratio.*

Thus, $3 : 6 = 23 : 46$

A compound ratio may be changed to a simple ratio by taking the product of the antecedents for a new antecedent, and the product of the consequents for a new consequent.

$$\textit{Antecedent} \div \textit{Consequent} = \textit{Ratio}$$
$$\textit{Antecedent} \div \textit{Ratio} = \textit{Consequent}$$
$$\textit{Ratio} \times \textit{Consequent} = \textit{Antecedent}$$

To multiply or divide both terms of a ratio by the same number does not change the ratio.

Thus
$$12 : 6 = 2$$
$$3 \times 12 : 3 \times 6 = 2$$
$$\tfrac{12}{3} : \tfrac{6}{3} = 2$$

EXAMPLES

Find the ratio of

1. $20 : 300$
2. 3 bu. : 3 pk.
3. $2\frac{1}{2} : 16$
4. $12 : \frac{1}{4}$
5. $\frac{1}{2} : \frac{2}{3}$
6. $16 : (?) = \frac{1}{2}$

Fractions with a common denominator have the same ratio as their numerators.

7. $\frac{8}{17} : \frac{16}{17}$, $\frac{28}{75} : \frac{7}{75}$, $\frac{15}{11} : \frac{30}{11}$
8. $\frac{3}{4} : \frac{2}{3}$, $\frac{3}{7} : \frac{5}{8}$, $\frac{2}{3} : \frac{5}{6}$

Proportion

An equality of ratios is a **proportion**.

A **proportion** is usually expressed thus: $4 : 2 :: 12 : 6$, and is read 4 *is to* 2 *as* 12 *is to* 6.

A proportion has four terms, of which the first and third are *antecedents* and the second and fourth are *consequents.* The first and fourth terms are called *extremes,* and the second and third terms are called *means.*

The product of the extremes equals the product of the means.

To find an extreme, divide the product of the means by the given extreme.

To find a mean, divide the product of the extremes by the given mean.

EXAMPLES

Supply the missing term:

1. $1 : 836 :: 25 : (\ )$
2. $6 : 24 :: (\ ) : 40$
3. $(\ ) : 15 :: 60 : 6$
4. 10 yd. : 50 yd. :: \$20 : (\$)
5. $\$\frac{3}{4} : \$3\frac{3}{4} :: (\ ) : 5$

Simple Proportion

An equality of two simple ratios is a *simple proportion.*

EXAMPLE. — If 12 bushels of charcoal cost \$4, what will 60 bushels cost?

$12 : 60 :: \$4 : (\$\ )$

$\frac{60 \times 4}{12} = \$20.$ *Ans.*

There is the same relation between the cost of 12 bu. and the cost of 60 bu. as there is between the 12 bu. and the 60 bu. \$4 is the third term. The answer is the fourth term. It must form a ratio of 12 and 60 that shall equal the ratio of \$4 to the answer. Since the third term is less than the required answer, the first must be less than the second, and 12 : 60 is the first ratio. The product of the means divided by the given extreme gives the other extreme, or \$20.

EXAMPLES

Solve by proportion:

1. If 150 fuses cost \$6, how much will 1200 cost?

2. If 250 pounds of lead pipe cost \$15, how much will 1200 pounds cost?

3. If 5 men can dig a ditch in 3 days, how long will it take 2 men?

4. If 4 men can shingle a shed in 2 days, how long will it take 3 men?

5. The ratio of Simon's pay to Matthew's is $\frac{2}{3}$. Simon earns \$18 per week. What does Matthew earn?

6. What will $11\frac{3}{4}$ yards of cambric cost if 50 yards cost \$ 6.75?

7. A spur gear making 210 revolutions per minute is enmeshed with a pinion. The gear has 126 teeth and the pinion has 42 teeth. How many revolutions does the pinion make?

8. In a velocity diagram a line $3\frac{3}{4}''$ long represents 45 ft. What would be the length of a line representing 30 ft. velocity?

9. How many pounds of lead and tin would it take to make 4100 pounds of solder if there are 27 pounds of tin in each 100 pounds of solder?

10. It is necessary to obtain a speed reduction of 7 to 3 by use of gears. If the pinion has 21 teeth, how many teeth must the gear have?

11. A bar of iron $3\frac{1}{4}$ ft. long and $\frac{7}{8}''$ diameter weighs 6.64 pounds. What would a bar $4\frac{1}{8}$ ft. long of the same diameter weigh?

12. In a certain time 15 workmen made 525 pulleys. How many pulleys will 32 men make in the same length of time?

13. When a post 11.5 ft. high casts a shadow on level ground 20.6 ft. long, a telephone pole near by casts a shadow 59.2 ft. long. How high is the pole?

14. The diameter of a driving pulley is 18″. This pulley makes 320 revolutions per minute. What must be the diameter of a driving pulley in order to make 420 revolutions per minute?

15. A ditch is dug in 14 days of 8 hours each. How many days of 10 hours each would it have taken?

16. If in a drawing a tree 38 ft. high is represented by $1\frac{1}{4}''$, what on the same scale will represent the height of a house 47 ft. high?

17. What will be the cost of 21 motors if 15 motors cost \$ 887.509?

18. The main drive pulley of a machine is 6 inches in diameter and makes 756 revolutions per minute. A pulley on the line shaft is belted to a machine. What is the diameter of the line shaft pulley if the line shaft makes 252 revolutions per minute?

19. If a pole 8 ft. high casts a shadow $4\frac{1}{2}$ ft. long, how high is a tree which casts a shadow 48 ft. long?

Involution

The product of equal factors is a **power.**

The process of finding powers is **involution.**

The product of two equal factors is the *second power,* or **square,** of the equal factor.

The product of three equal factors is the *third power,* or **cube,** of the factor.

$4^2 = 4 \times 4$ is 4 to the second power, or the square of 4.
$2^3 = 2 \times 2 \times 2$ is 2 to the third power, or the cube of 2.
$3^4 = 3 \times 3 \times 3 \times 3$ is 3 to the fourth power, or the fourth power of 4.

EXAMPLES

Find the powers:

1. 5^3	3. 1^4	5. $(2\frac{1}{2})^2$	7. 9^3
2. 1.1^2	4. 25^2	6. 2^4	8. $.15^2$

Evolution

One of the *equal* factors of a power is a **root.**

One of *two equal* factors of a number is the **square root.**

One of *three equal* factors of a number is the **cube root** of it.

The square root of 16 = 4. The cube root of 27 = 3.

The radical sign ($\sqrt{}$) placed before a number indicates that its root is to be found. The radical sign alone before a number indicates the square root.

Thus, $\sqrt{9} = 3$ is read, the square root of 9 = 3.

A small figure placed in the opening of the radical sign is called the index of the root, and shows what root is to be taken.

Thus, $\sqrt[3]{8} = 2$ is read, the cube root of 8 is 2.

Square Root

The square of a number composed of tens and units is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.

$$tens^2 + 2 \times tens \times units + units^2$$

EXAMPLE. — What is the square root of 1225?

$$\begin{array}{lcl}
 & & 12'25(\underline{30 + 5 = 35} \\
\textit{Tens}^2,\ 30^2 & = & \underline{900} \\
2 \times \textit{tens} = 2 \times 30 & = 60 & \boxed{325} \\
2 \times \textit{tens} + \textit{units} = 2 \times 30 + 5 & = 65 & \underline{325}
\end{array}$$

Separating into periods of two figures each, by a check mark ($'$), beginning at units, we have 12'25. Since there are two periods in the power, there must be two figures in the root, tens and units.

The greatest square of even tens contained in 1225 is 900, and its square root is 30 (3 tens). Subtracting the square of the tens, 900, the remainder consists of 2 × (tens × units) + units.

325, therefore, is composed of two factors, units being one of them, and 2 × tens – units being the other. But the greater part of this factor is 2 × tens ($2 \times 30 = 60$). By trial we divide 325 by 60 to find the other factor (units), which is 5, if correct. Completing the factor, we have 2 × tens + units = 65, which, multiplied by the other factor, 5, gives 325. Therefore the square root is $30 + 5 = 35$.

The area of every square surface is the product of two equal factors, length, and width.

Finding the square root of a number, therefore, is equivalent to finding the length of one side of a square surface, its area being given.

1. *Length* × *Width* = *Area*
2. *Area* ÷ *Length* = *Width*
3. *Area* ÷ *Width* = *Length*

Short Method

Example. — Find the square root of 1306.0996.

```
     13'06.09'96(36.14
      9
66)  409
     396
 721)1009
      721
 7224)28896
      28896
```

Beginning at the decimal point, separate the number into periods of two figures each, pointing whole numbers to the left and decimals to the right. Find the greatest square in the left-hand period, and write its root at the right. Subtract the square from the left-hand period, and bring down the next period for a dividend.

Divide the dividend, with its right-hand figure omitted, by twice the root already found, and annex the quotient to the root, and to the divisor. Multiply this complete divisor by the last root figure, and bring down the next period for a dividend, as before.

Proceed in this manner till all the periods are exhausted.

When 0 occurs in the root, annex 0 to the trial divisor, bring down the next period, and divide as before.

If there is a remainder after all the periods are exhausted, annex decimal periods.

If, after multiplying by any root figure, the product is larger than the dividend, the root figure is too large and must be diminished. Also the last figure in the complete divisor must be diminished.

For every decimal period in the power, there must be a decimal figure in the root. If the last decimal period does not contain two figures, supply the deficiency by annexing a cipher.

EXAMPLES

Find the square root of:

1. 8836	**5.** $\sqrt{\frac{1}{4} \times \frac{1}{9}}$	**9.** $\sqrt{3.532 \div 6.28}$
2. 370881	**6.** 72.5	**10.** $\sqrt{625 + 1296}$
3. 29.0521	**7.** $.009\frac{9}{16}$	**11.** $\frac{1}{\sqrt{9}} \times \frac{\sqrt{9}}{3}$
4. 46656	**8.** 1684.298431	**12.** $\frac{3969}{5625}$

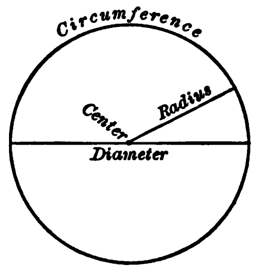
13. What is the length of one side of a square field that has an area equal to a field 75 rd. long and 45 rd. wide?

CHAPTER II

MENSURATION

The Circle

A **circle** is a plane figure bounded by a curved line, called the **circumference**, every point of which is equidistant from the center.



The **diameter** is a straight line drawn from one point of the circumference to another and passing through the center.

The ratio of the circumference to the diameter of any circle is always a constant number, 3.1416+, approximately $3\frac{1}{7}$, which is represented by the Greek letter π (*pi*).

$C =$ Circumference
$D =$ Diameter
$C = \pi D$[1]

The **radius** is a straight line drawn from the center to the circumference.

Any portion of the circumference is an **arc**.

By drawing a number of radii a circle may be cut into a series of figures, each one of which is called a **sector**. The area of each sector is equal to one half the product of the arc and radius. Therefore the *area of the circle* is equal to one half of the product of the circumference and radius.

[1] See Appendix for use of formulas.

$$A = C \times \frac{R}{2}$$

$$A = \pi 2\, R \times \frac{R}{2} = \pi R^2$$

In this formula A equals area, $\pi = 3.1416$, and $R^2 =$ the radius squared.

$$A = \tfrac{1}{2}\, D \times \tfrac{1}{2}\, C$$

In this formula D equals the diameter and C the circumference,

or $$A = \frac{\pi D^2}{4} = \frac{3.1416\, D^2}{4} = .7854\, D^2$$

EXAMPLE. —What is the area of a circle whose radius is 3 ft.?

$$A = \pi R^2 \qquad A = \frac{\pi D^2}{4}$$

$$A = \pi \times 9 \qquad A = \frac{\pi \overset{9}{\cancel{36}}}{\cancel{4}} = \pi\, 9 = 28.27 \text{ sq. ft.} \quad \textit{Ans.}$$

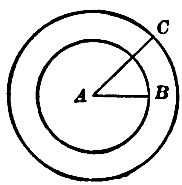
EXAMPLE. —What is the area of a circle whose circumference is 10 ft.?

$$D = \frac{10}{3.1416} \qquad A = \frac{1}{2} D \times \frac{1}{2} C$$

$$\frac{1}{2} \times \frac{10}{3.1416} \times \frac{1}{2} \times 10 = \frac{25}{3.1416} = 7.1 \text{ sq. ft.} \quad \textit{Ans.}$$

Area of a Ring. — On examining a flat iron ring it is clear that the area of one side of the ring may be found by subtracting the area of the inside circle from the area of the outside circle.

Let $D =$ outside diameter
$d =$ inside diameter
$A =$ area of outside circle
$a =$ area of inside circle



(1) $$A = \frac{D^2 \pi}{4} = .7854\, D^2$$

(2) $$a = \frac{d^2\pi}{4} = .7854\ d^2$$

(3) $$A - a = \frac{D^2\pi}{4} - \frac{d^2\pi}{4}$$

Let $B =$ area of circular ring $= A - a$

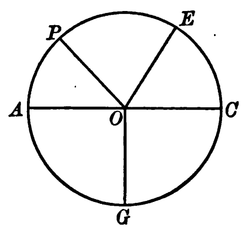
$$B = \frac{D^2\pi}{4} - \frac{d^2\pi}{4} = \frac{\pi}{4}(D^2 - d^2) = .7854\ (D^2 - d^2)$$

EXAMPLE. — If the outside diameter of a flat ring is 9″ and the inside diameter 7″, what is the area of one side of the ring?

$B = .7854\ (D^2 - d^2)$
$B = .7854\ (81 - 49) = .7854 \times 32 = 25.1328$ sq. in. *Ans.*

Angles

Mechanics make two uses of angles: (1) to measure a circular movement, and (2) to measure a difference in direction. A circle contains 360°, and the angles at the center of the circle contain as many degrees as their corresponding arcs on the circumference.



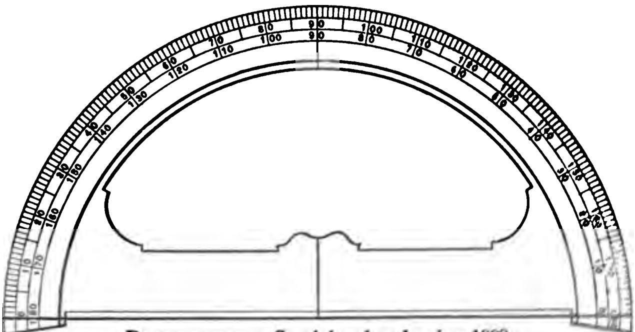
Angle *POE* has as many degrees as arc *PE*.

A **right angle** is measured by a quarter of the circumference of the circle, which is 90°.

The angle *AOG* is a right angle.

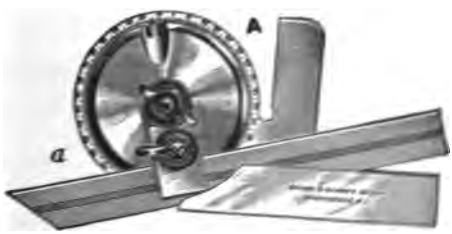
The angle *AC*, made with half the circumference of the circle, is a **straight angle**, and the two right angles, *AOG* and *GOC*, which it contains, are **supplementary** to each other. When the sum of two angles is equal to 90°, they are said to be **complementary angles**, and one is the complement of the other. When the sum of two angles equals 180°, they are **supplementary angles**, and one is said to be the **supplement** of the other.

The number of degrees in an angle may be measured by a *protractor*. The distance around a semicircular protractor is



PROTRACTOR — Semicircular, having 180°.

divided into 180 parts, each division measuring a degree. It is used by placing the center of the protractor on the *vertex* and the base of the protractor on the base of the angle to be



The Protractor.
Aa is a circle divided into degrees.

measured. Where the other side of the angle cuts the circular piece, the size of the angle may be read.

EXAMPLES

1. What is the area of a circular sheet of iron [illegible] in diameter?

2. What is the distance around the edge of a pulley [illegible] in diameter?

3. What is the area of one side of a flat iron ring 14″ inside diameter and 18″ outside diameter?

4. A driving wheel of a locomotive has a wheel center of 56″ in diameter; if the tires are 3″ thick, what is the circumference of the wheel when finished?

5. Find the area of a section of an iron pipe which has an inside diameter of 17″ and an outside diameter of 17$\frac{3}{4}$″.

6. Name the complements of angles of 30°, 45°, 65°, 70°, 85°.

7. Name the supplements of angles of 55°, 140°, 69°, 98° 44′, 81° 19′.

8. What is the diameter of a wheel that is 12′ 6″ in circumference?

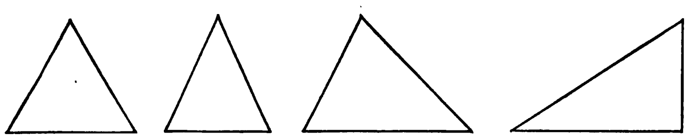
Triangles

A **triangle** is a plane figure bounded by three straight lines. Triangles are classified according to the relative lengths of their sides and the size of their angles.

A triangle having equal sides is called **equilateral**. One having two sides equal is **isosceles**. A triangle having no sides equal is called **scalene**.

If the angles of a triangle are equal, the triangle is **equiangular**.

If one of the angles of a triangle is a right angle, the triangle is a **right triangle**. In a right triangle the side opposite the right angle is called the **hypotenuse** and is the longest side. The other two sides of the right triangle are the legs, and are at right angles to each other.



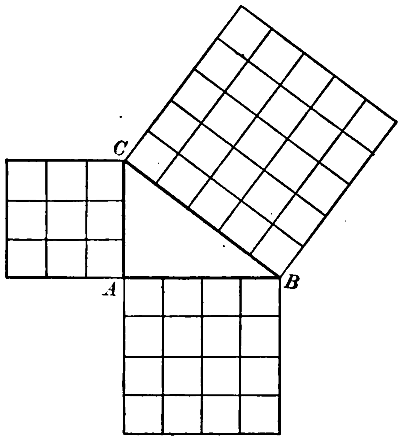
EQUILATERAL ISOSCELES SCALENE RIGHT

Kinds of Triangles

Right Triangles

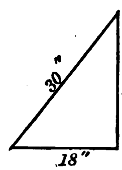
In a right triangle the square of the hypotenuse equals the sum of the squares of the other two sides or legs.

If the length of the hypotenuse and one leg of a right triangle is known, the other side may be found by squaring the hypotenuse and squaring the leg, and extracting the square root of their difference.



Example. — If the hypotenuse of a right angle triangle is $30''$ and the base is $18''$, what is the altitude?

$$30^2 = 30 \times 30 = 900$$
$$18^2 = 18 \times 18 = 324$$
$$900 - 324 = 576$$
$$\sqrt{576} = 24''. \quad \textit{Ans.}$$



Areas of Triangles

The *area of a triangle* may be found when the length of the three sides is given by adding the three sides together, dividing by 2, and subtracting from this sum each side separately. Multiply the four results together and find the square root of their product.

EXAMPLE. — What is the area of a triangle whose sides measure 15, 16, and 17 inches, respectively?

$$\begin{array}{r} 15 \\ 16 \\ \underline{17} \\ 2\overline{)48} \\ 24 - 15 = 9 \\ 24 - 16 = 8 \\ 24 - 17 = 7 \end{array}$$

$$\sqrt{24 \times 9 \times 8 \times 7} = \sqrt{12096}$$

$$\sqrt{12096} = 109.98 \text{ sq. in. } \textit{Ans.}$$

Area of a Triangle $= \frac{1}{2}$ *Base* $\times$ *Altitude*

EXAMPLE. — What is the area of a triangle whose base is 17″ and altitude 10″?

$$A = \frac{1}{\cancel{2}} \times 17 \times \overset{5}{\cancel{10}} = 85 \text{ sq. in. } \textit{Ans.}$$

EXAMPLES

1. A ladder 17 ft. long standing on level ground reached to a window 12 ft. from the ground. If it is assumed that the wall is perpendicular, how far is the foot of the ladder from the base of the wall?

2. Find the area of a triangular sheet of metal having the base 81″ and the height measured from the opposite angle 56″.

3. Find the length of the hypotenuse of a right triangle with equal legs and having an area of 280 sq. in.

4. Find the length of a side of a right triangle with equal legs and an area of 72 sq. in.

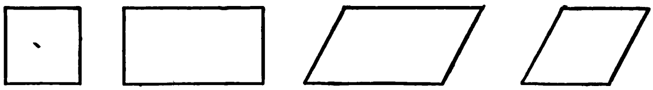
5. Find the hypotenuse of a right triangle with a base of 8″ and the altitude of 7″.

6. What is the area of a triangle whose sides measure 12, 19, and 21 inches?

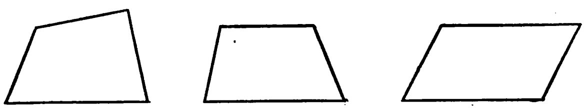
7. What is the altitude of an isosceles triangle having sides 8 ft. long and a base 6 ft. long?

Quadrilaterals

Four-sided plane figures are called **quadrilaterals**. Among them are the *trapezoid, trapezium, rectangle, rhombus, and rhomboid.*



SQUARE RECTANGLE RHOMBOID RHOMBUS



TRAPEZIUM TRAPEZOID PARALLELOGRAM

KINDS OF QUADRILATERALS

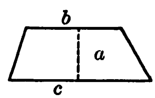
A **rectangle** is a quadrilateral which has its opposite sides parallel and its angles right angles. Its area equals the product of its base and altitude.

$$A = ba$$

A **trapezoid** is a quadrilateral having only two sides parallel. Its area is equal to the product of the altitude by one half the sum of the bases.

$$A = (b + c) \times \tfrac{1}{2} a$$

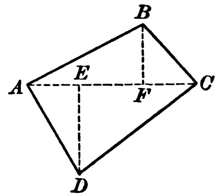
In this formula $c =$ length of longest side
$b =$ length of shortest side
$a =$ altitude



A **trapezium** is a four-sided figure with no two sides parallel. The area of a trapezium is found by dividing the trapezium into triangles by means of a diagonal. Then the area may be found if the diagonal and perpendicular heights of the triangles are known.

EXAMPLE. — In the trapezium $ABCD$ if the diagonal is 43′ and the perpendiculars 11′ and 17′, respectively, what is the area of the trapezium?

$43 \times \frac{11}{2} = \frac{473}{2} = 236\frac{1}{2}$ sq. ft., area of ABC
$43 \times \frac{17}{2} = \frac{731}{2} = 365\frac{1}{2}$ sq. ft., area of ADC
602 sq. ft., total area
Ans.



To find the areas of irregular figures, draw the longest diagonal and upon this diagonal drop perpendiculars from the vertices of the figure. These perpendiculars will form trapezoids and right triangles whose areas may be determined by the preceding rules. The sum of the areas of the separate figures will give the area of the whole irregular figure.

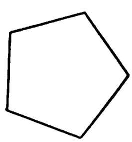
Polygons

A plane figure bounded by straight lines is a **polygon.** A polygon which has equal sides and equal angles is a **regular polygon.**

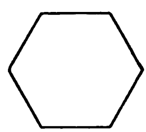
The **apothem** of a regular polygon is the line drawn from the center of the polygon perpendicular to one of the sides.

A five-sided polygon is a **pentagon.**

A six-sided polygon is a **hexagon.**



PENTAGON



HEXAGON

An eight-sided polygon is an **octagon.**

The shortest distance between the flats of a regular hexagon is the perpendicular distance between two opposite sides, and is equal to the diameter of the inscribed circle. The diameter of the circumscribed circle is the long diameter of a regular hexagon.

The **perimeter** of a polygon is the sum of its sides.

The area of a regular polygon equals one half the product of the apothem and the perimeter.

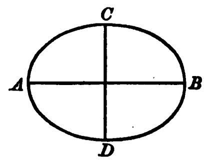
Formula	$A = \frac{1}{2} aP$
In this formula	$P =$ perimeter
	$a =$ apothem

Ellipse

Only the approximate circumference of an ellipse can be obtained.

The circumference of an ellipse equals one half the product of the sum of two diameters and π.

If	$d_1 =$ major diameter
	$d_2 =$ minor diameter
	$C =$ circumference
then	$C = \frac{d_1 + d_2}{2}\pi$



The *area of an ellipse* is equal to one fourth the product of the major and minor diameters by π.

If	$A =$ area
	$d_1 =$ major diameter
	$d_2 =$ minor diameter
then	$A = \pi \frac{d_1 d_2}{4}$

EXAMPLES

1. Find the area of a trapezium if the diagonal is 93′ and the perpendiculars are 19′ and 33′.

2. What is the area of a trapezoid whose parallel sides are 18 ft. and 12 ft., and the altitude 8 ft.?

3. What is the distance around an ellipse whose major diameter is 14″ and minor diameter 8″?

4. In the map of a country a district is found to have two of its boundaries approximately parallel and equal to 276 and 216 miles. If the breadth is 100 miles, what is its area?

5. If the greater and lesser diameters of an elliptical manhole door are 2′ 9″ and 2′ 6″, what is its area?

6. Find the area of a trapezium if the diagonal is 78″ and the perpendiculars 18″ and 27″.

7. The greater diameter of an elliptical funnel is 4 ft. 6 in., and the lesser diameter is 4 ft. (*a*) What is its area? (*b*) How many square feet of iron will it contain if its height is 16 ft., allowing 4″ for the seams?

8. What is the area of a pentagon, whose apothem is $4\frac{1}{2}''$ and whose side is 5″?

Volumes

The *volume of a rectangular-shaped bar* is found by multiplying the area of the base by the length. If the area is in square inches, the length must be in inches.

The *volume of a cube* is equal to the cube of an edge.

The *contents or volume of a cylindrical solid* is equal to the product of the area of the base by the height.

If $S =$ contents or capacity of cylinder
$R =$ radius of base
$H =$ height of cylinder
$\pi = 3.1416^+$ or $\frac{22}{7}$ (approx.)
$S = \pi R^2 H$

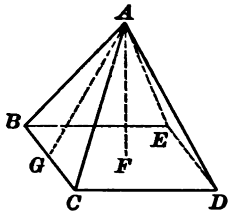
EXAMPLE. — Find the contents of a cylindrical tank whose inside diameter is 14″ and height 6′.

$$S = \pi R^2 H$$
$$H = 6' = 72''$$
$$S = \tfrac{22}{7} \times 7 \times 7 \times 72 = 11{,}088 \text{ cu. in.}$$

The Pyramid

The *volume of a pyramid* equals one third of the product of the area of the base and the altitude.

$$V = \frac{1}{3} ba$$



The *volume of a frustum of a pyramid* equals the product of one third the altitude and the sum of the two bases and the square root of the product of the bases.

$$V = \frac{1}{3} h(b + b^1 + \sqrt{bb^1})$$

The *surface of a regular pyramid* is equal to the product of the perimeter of the bases and one half the slant height.

$$S = P \times \frac{1}{2} sh$$

The Cone

A **cone** is a solid generated by a right triangle revolving on one of its legs as an axis.

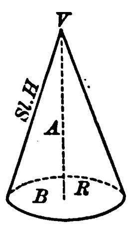
The **altitude** of the cone is the perpendicular distance from the base to the apex.

The *volume of a cone* equals the product of the area of the base and one third of the altitude.

$$V = \frac{1}{3} \pi R^2 H$$

or

$$V = .2618\ D^2 H$$



EXAMPLE. — What is the volume of a cone $1\frac{1}{2}''$ in diameter and $4''$ high?

$$\text{Area of base} = .7854 \times \frac{9}{4}$$

$$= \frac{7.0686}{4} = 1.7671 \text{ sq. in.}$$

$$V = .2618\,D^2H$$

$$= .2618 \times \frac{9}{4} \times 4 = 2.3562 \text{ cu. in.} \quad \textit{Ans.}$$

The *lateral surface of* a *cone* equals one half the product of the perimeter of the base by the slant height.

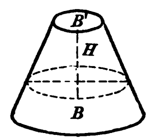
EXAMPLE. — What is the surface of a cone having a slant height of 36 in., and a diameter of 14 in.?

$$\pi = 3\tfrac{1}{7}$$
$$C = \pi D = 14 \times \tfrac{22}{7} = 44''$$
$$\frac{44 \times 36}{2} = 792 \text{ sq. in.} \quad \textit{Ans.}$$

Frustum of a Cone

The **frustum** of a cone is the part of a cone included between the base and a plane or upper base which is parallel to the lower base.

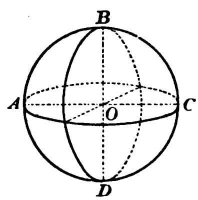
The *volume of a frustum of a cone* equals the product of one third of the altitude and the sum of the two bases and the square root of their product.



When $H =$ altitude
$B^1 =$ upper base
$B =$ lower base
$V = \frac{1}{3} H(B + B^1 + \sqrt{BB^1})$

The *lateral surface of a frustum of a cone* equals one half the product of the slant height and the sum of the perimeters of the bases.

The Sphere



The *volume of a sphere* is equal to

$$V = \frac{4\pi R^3}{3}$$

where R is the radius.

The *surface of a sphere* is equal to

$$S = 4\pi R^2$$

The Barrel

To find the cubical contents of a barrel, (1) multiply the square of the largest diameter by 2, (2) add to this product

the square of the head diameter, and (3) multiply this sum by the length of the barrel and that product by .2618.

EXAMPLE.—Find the cubical contents of a barrel whose largest diameter is 21″ and head diameter 18″, and whose length is 33″.

$$\begin{array}{lr} 21^2 = 441 \times 2 = & 882 \\ 18^2 = 324 & \underline{324} \\ & 1206 \\ & \underline{33} \\ & 3618 \\ & \underline{3618} \\ & 39798 \end{array}$$

$$V = (D^2 \times 2) + d^2 \times L \times .2618$$

$$\begin{array}{r} 39798 \\ \underline{.2618} \\ 10419.11 \text{ cu. in.} \end{array}$$

$$\frac{10419.11}{231} = 24.10 \text{ gal.} \quad \textit{Ans.}$$

Similar Figures

Similar figures are figures that have exactly the same shape.

The *areas of similar figures* have the same ratio as the squares of their corresponding dimensions.

EXAMPLE.—If two boilers are 15′ and 20′ in length, what is the ratio of their surfaces?

$\frac{15}{20} = \frac{3}{4}$, ratio of lengths

$\frac{3^2}{4^2} = \frac{9}{16}$, ratio of surfaces

One boiler is $\frac{9}{16}$ as large as the other. *Ans.*

The *volumes of similar figures* are to each other as the cubes of their corresponding dimensions.

EXAMPLE.—If two iron balls have 8″ and 12″ diameters, respectively, what is the ratio of their volumes?

$\frac{8}{12} = \frac{2}{3}$, ratio of diameters

$= \frac{8}{27}$, ratio of their volumes. *Ans.*

One ball weighs $\frac{8}{27}$ as much as the other.

EXAMPLES

1. Find the volume of a rectangular iron bar 8″ by 10″ and 4′ long.

2. Find the weight of a rectangular steel bar 31″ × 49″ × 3″ thick, if the metal weighs .28 lb. per cubic foot.

3. The radius of the small end of a bucket is 4 in. Water stands in the bucket to a depth of 9 in., and the radius of the surface of the water is 6 in. (1) Find the volume of the water in cubic inches. (2) Find the volume of the water in gallons if a cubic foot contains 7.48 gal.

4. What is the volume of a steel cone $2\frac{1}{2}$″ in diameter and 6″ high?

5. Find the contents of a barrel whose largest diameter is 22″, head diameter 18″, and height 35″.

6. What is the volume of a sphere 8″ in diameter?

7. What is the volume of a pyramid with a square base, 4″ on a side and 11″ high?

8. What is the surface of a steel cone with a 6″ diameter and 14″ slant height?

9. Find the surface of a pyramid with a perimeter of 18″ and a slant height of 11″.

10. Find the volume of a cask whose height is $3\frac{1}{2}$″ and the greatest radius 16″ and the least radius 12″, respectively.

11. What is the weight of a cast-iron cylinder 2.75″ in diameter and $12\frac{3}{4}$″ long, if cast iron weighs 450 lb. per cu. ft.?

12. How many gallons of water will a round tank hold which is 4 ft. in diameter at the top, 5 ft. in diameter at the bottom, and 8 ft. deep? (231 cu. in. = 1 gal.)

13. What is the volume of a cylindrical ring having an outside diameter of $6\frac{1}{2}$″, an inside diameter of $5\frac{1}{16}$″, and a height of $3\frac{3}{8}$″? What is its outside area?

14. A sphere has a circumferenee of 8.2467″. (*a*) What is its area? (*b*) What is its volume?

15. If it is desired to make a conical oil can with a base 3.5″ in diameter to contain $\frac{1}{4}$ pint, what must the approximate height be?

16. What is the area of one side of a flat ring if the inside diameter is $2\frac{1}{2}$″ and the outside diameter $4\frac{7}{8}$″?

17. There are two balls of the same material with diameters 4″ and 1″, respectively. If the smaller one weighs 3 lb., how much does the larger one weigh?

18. If the inside diameter of a ring must be 5 in., what must the outside diameter be if the area of the ring is 6.9 sq. in.?

19. How much less paint will it take to paint a wooden ball 4″ in diameter than one 10″ in diameter?

20. What is the weight of a brass ball $3\frac{1}{2}$″ in diameter if brass weighs .303 lb. per cubic inch?

21. A cube is 19″ on its edge. (*a*) Find its total area. (*b*) Its volume.

22. If the area of a $\frac{1}{4}$″ pipe is .049 sq. in., what will be the diameter of a pipe having 8 times the area?

23. What is the weight of a cast-iron cylinder 2.75″ in diameter and $12\frac{3}{4}$″ long, if the cast iron weighs 450 lb. per cubic foot?

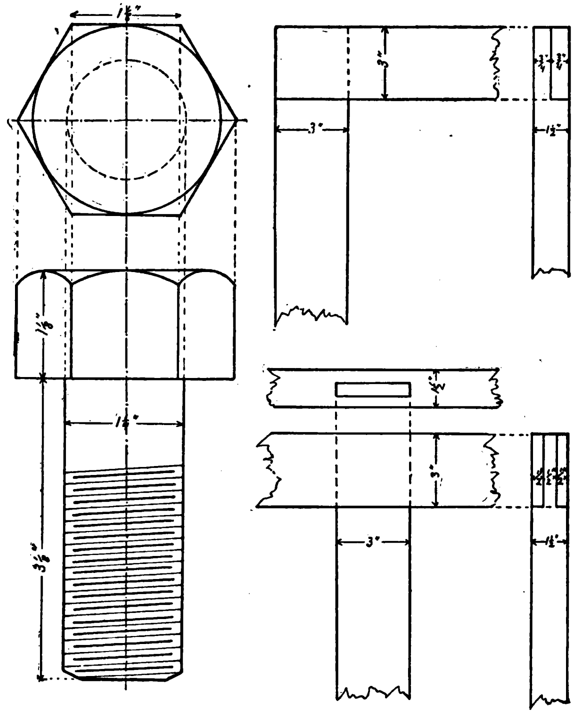
24. A conical funnel has an inside diameter of 19.25″ at the base and is 43″ high inside. (*a*) Find its total area. (*b*) Find its cubical contents.

25. If a bar 3″ in diameter weighs 24.03 lb. per foot of length, what must be the weight per foot of a bar 3″ square of the same material?

CHAPTER III

Reading a Blue Print

Every skilled worker in wood or metal must know how to read a "blue print," which is the name given to working plans



Screw *Table Frame*

Simple Blue Prints or Working Drawings

and drawings with white lines upon a blue background. The blue print is the language which the architect uses to the

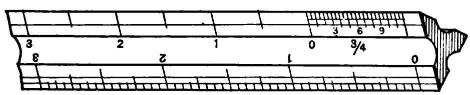
builder, the machinist to the pattern maker, the engineer to the foreman of construction, and the designer to the workman. Through following the directions of the blue print the carpenter, metal worker, and mechanic are able to produce the object wanted by the employer and his designer or draftsman.

Two views are usually necessary in every working drawing, one the plan or top view obtained by looking down upon the object, and the other the elevation or front view. When an object is very complicated, a third view, called an end or profile view, is shown.

All the information, such as dimensions, etc., necessary to construct whatever is represented by the blue print, must be supplied on the drawing. If the blue print represents a machine it is necessary to show all the parts of the machine put together in their proper places. This is called an assembly drawing. Then there must be a drawing for each part of the machine, giving information as to the size, shape, and number of the pieces. Then if there are interior sections, these must be represented in section drawings.

Drawing to Scale

As it is impossible to draw most objects full size on paper, it is necessary to make the drawings proportionately smaller. This is done by making all the dimensions of the drawing a certain fraction of the true dimensions of the object. A drawing made in this way is called *drawn to scale.*



TRIANGULAR SCALE

The dimensions on the drawing are designated the actual size of the object — not of the drawing. If a drawing were made of an iron bolt 25 inches long, it would be inconvenient to represent the actual size of the bolt, and the drawing might be made half or quarter the size of the bolt, but the length would read on the drawing 25 inches.

In making a drawing "to scale," it becomes very tedious to be obliged to calculate all the small dimensions. In order to obviate this work a triangular scale is used. It is a ruler with the different scales marked on it. By practice the student will be able to use the scale with as much ease as the ordinary ruler.

QUESTIONS AND EXAMPLES

1. Tell what is the scale and the length of the drawing of each of the following:

   a. An object 14″ long drawn half size.

   b. An object 26″ long drawn quarter size.

   c. An object 34″ long drawn one third size.

   d. An object 41″ long drawn one twelfth size.

2. If a drawing made to the scale of $\frac{3}{8}'' = 1$ ft. is reduced $\frac{1}{3}$ in size, what will the new scale be?

3. The scale of a drawing is made $\frac{1}{8}$ size. If it is doubled, how many inches to the foot will the new scale be?

4. On the $\frac{1}{16}''$ scale, how many feet are there in 18 inches?

5. On the $\frac{1}{8}''$ scale, how many feet are there in 26 inches?

6. On the $\frac{1}{4}''$ scale, how many feet are there in 27 inches?

7. If the drawing of a bolt is made $\frac{1}{3}$ size and the length of the drawing is $8\frac{1}{2}''$, what will it measure if made to scale $3'' = 1$ ft.?

8. What will be the dimensions of the drawing of a machine shop 582′ by 195′ if it is made to a scale of $\frac{1}{16}'' = 1$ ft.?

Arithmetic and Blue Prints. — Mechanics are obliged to read from blue prints. In order to verify the necessary dimensions in the detail of the work, it becomes necessary to do more or less addition and subtraction of the given dimensions. This involves ability to add and subtract mixed numbers and fractions.

Methods of Solving Examples

Every mechanical problem or operation has two distinct sides: the collecting of data and the solving of the problem. The first part, the collecting of data, demands a knowledge of the materials and conditions under which the problem is given, and calls for considerable judgment as to the necessary accurateness of the work.

There are three ways by which a problem may be solved:

1. Exact method.
2. Rule of thumb method, by the use of a two-foot rule or a slide scale.
3. By means of tables.

The *exact method* of solving a problem in arithmetic is the one usually taught in school and is the method obtained by analysis. Every one should be able to solve a problem by the exact method.

The *rule of thumb method.*—Many of the problems that arise in industrial life have been met before and very careful judgment has been exercised in solving them. As the result of this experience and the tendency to abbreviate and devise shorter methods that give sufficiently accurate results, we find many rule of thumb methods used by the mechanics in daily life. The exact method would involve considerable time and the use of pencil and paper, whereas in cases that are not too complicated the two-foot rule or the slide scale will give a quick and accurate result.

In solving problems involving the addition and subtraction of fractions, by the rule of thumb, use the two-foot rule or steel scale to carry on the computation. To illustrate: if we desire to add $\frac{1}{8}$ and $\frac{1}{16}$, place the thumb on $\frac{1}{8}$ division, then slide (move) the thumb along a division corresponding to $\frac{1}{16}$, and then read the number of divisions passed over by the thumb. In this case the result is $\frac{3}{16}$. For fractions involving $\frac{1}{32}$, $\frac{1}{64}$, $\frac{1}{10}$, and $\frac{1}{100}$ use the steel scale. The majority of machinists, carpenters, etc., use this method of sliding the thumb over the rule, in adding and subtracting inches and fractions of inches.

The use of tables. — In the commercial and industrial world the tendency is to do a thing in the quickest and the most economical way. To illustrate: hand labor is more costly than machine work, so wherever possible, machine work is substituted for hand labor. The same condition applies to the calculations that are used in the shop. The methods of performing calculations are the most economical — that is, the quickest and most accurate — that the ordinary mechanic is able to perform. Since a great many of the problems in calculation that arise in the daily experience of the mechanic are about standardized pieces of metal and repeat themselves often, it is not necessary to work them each time if results are kept on file when they are once solved. This filing is done by means of tables that are made from these problems.

See pages 105, 117, and 121, for tables used in this book.

PART II — MATHEMATICS FOR CARPENTERING AND BUILDING

CHAPTER IV

MEASURING LUMBER

THE carpenter or builder is often required to give an estimate of the cost of the work to be done for his prospective customers. People who contemplate building have several estimates submitted to them by different builders and generally give the work to the lowest bidder. An architect usually draws plans of the building and from these plans the contractor or builder makes his estimate of the cost. In doing repairing or cabinet-making the carpenter makes his own plans and estimates. In order to make a proper estimate of the cost, one must know the market price of materials, the cost of labor, the amount of material needed, and the length of time required to do the work.

Preparation of Wood for Building Purposes

In winter the forest trees are cut and in the spring the logs floated down the rivers to sawmills, where they are sawed into boards of different thickness. To square the log, four slabs are first sawed off. After these slabs are off, the remainder is sawed into boards.

As soon as the boards or planks are sawed from the logs, they are piled on prepared foundations in the open air to season. Each layer is separated from the one above by a crosspiece, called a strap, in order to allow free circulation of air about each board to dry it quickly and evenly. If lumber were to be piled up without the strips, one board upon another, the ends of the pile would dry and the center would rot. This seasoning or drying out of the sap usually lasts several months. "Air dried" lumber is used for most building purposes except in buildings or places where there is a warm, dry atmosphere.

Wood that is to be subject to a warm atmosphere has to be artificially dried. This artificially dried or kiln-dried lumber has to be dried to a point in excess of that of the atmosphere in which it is to be placed after being removed from the kiln. This process of drying must be done gradually and evenly or the boards may warp and then be unmarketable.

Definitions

Board Measure. — *A board one inch or less in thickness is said to have as many board feet as there are square feet in its surface.* If it is more than one inch thick, the number of board feet is found by multiplying the number of square feet in its surface by its thickness measured in inches and fractions of an inch.

The number of board feet = length (in feet) × width (in feet) × thickness (in inches).

Board measure is used for plank measure. A plank 2″ thick, 10″ wide, and 15′ long, contains twice as many square feet (board measure) as a board 1″ thick of the same width and length.

To measure a board that tapers, the width is taken at the middle, where it is one half the sum of the widths of the ends.

Boards are sold at a certain price per hundred (C) or per thousand (M) board feet.

The term **lumber** is applied to pieces not more than four inches thick; **timber** to pieces more than four inches thick; but a large amount taken together often goes by the general name of *lumber*. A piece of lumber less than an inch and a half thick is called a **board** and a piece from one inch and a half to four inches thick a **plank**.

Rough Stock is lumber the surface of which has not been dressed or planed.

The standard lengths of pieces of lumber are 10, 12, 14, 16, 18 feet, etc.

In measuring and marking large lots of lumber in which there are a number of pieces containing a fraction of a foot, in case of one half foot or more, 1 is added, and in case of less than one half foot, it is disregarded. This is especially true with boards.

A board 1″ × 4″ × 16′ contains $5\frac{1}{3}$ feet (board measure). Two boards would be marked 5 ft., and every third board would be marked 6 ft. So two boards may be exactly the same length and one marked 5 ft. and the other 6 ft. In the purchasing of a single board there might be a small undercharge or overcharge, but in large lots the average would be struck.

EXAMPLES

1. How many board feet in a board 1 in. thick, 15 in. wide, and 15 ft. long?

2. How many board feet of 2-inch planking will it take to make a walk 3 feet wide and 4 feet long?

3. A plank 19′ long, 3″ thick, 10″ wide at one end and 12″ wide at the other, contains how many board feet?

4. Find the cost of 7 2-inch planks 12 ft. long, 16 in. wide at one end, and 12 in. at the other, at $0.08 a board foot.

5. At $12 per M, what will be the cost of 2-inch plank for a 3 ft. 6 in. sidewalk on the street sides of a rectangular corner lot 56 ft. by 106 ft. 6 in.?

Quick Method for Measuring Boards

To measure boards 1″ thick, multiply length in feet by width in inches and divide by 12, and the result will be the board measure in feet.

For boards $1\frac{1}{4}$″ thick, add one quarter of the quotient to the result as above.

For boards $1\frac{1}{2}$″ thick, add one half of the quotient to the result as found above.

For plank 2″ thick, divide by 6 instead of 12.

For plank 3″ thick, divide by 4 instead of 12.

For plank 4″ thick, divide by 3 instead of 12.

For timber 6″ thick, divide by 2 instead of 12.

EXAMPLES

1. Find the number of board feet in 10 planks, 3″ thick, 12″ wide, 14′ long.

2. Find the number of board feet in 4 timbers, 8″ thick, 10″ wide, 17′ long.

3. Find the number of board feet in 18 joists, 2″ thick, 4″ wide, 14′ long.

4. Find the number of board feet in 16 beams, 10″ thick, 12″ wide, 11′ long.

5. Find the number of board feet in 112 boards, $\frac{7}{8}$″ thick, 8″ wide, 14′ long.

NOTE. — Ordinarily fractions of a foot less than one half are omitted, but when the fraction is one half or larger it is reckoned as a foot. This is sufficiently accurate for all practical purposes.

Board measure of one lineal foot of timber may be found from the following table:

CONTENTS (BOARD MEASURE) OF ONE LINEAL FOOT OF TIMBER

Width in Inches	Thickness in Inches												
	2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
18	3.	4.5	6.	7.5	9.	10.5	12.	13.5	15.	16.5	18	19.5	21.
17	2.83	4.25	5.66	7.08	8.5	9.92	11.33	12.75	14.17	15.58	17	18.42	19.83
16	2.67	4.	5.33	6.67	8.	9.33	10.67	12.	13.33	14.67	16	17.3	18.66
15	2.5	3.75	5.	6.25	7.5	8.75	10.	11.25	12.5	13.75	15	16.25	17.5
14	2.33	3.5	4.67	5.83	7.	8.17	9.33	10.5	11.67	12.83	14	15.17	16.66
13	2.17	3.25	4.33	5.42	6.5	7.58	8.67	9.75	10.83	11.92	13	14.08	
12	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12		
11	1.83	2.75	3.67	4.58	5.5	6.42	7.33	8.25	9.17	10.08			
10	1.67	2.5	3.33	4.17	5.	5.83	6.67	7.5	8.33				
9	1.5	2.25	3.	3.75	4.5	5.25	6.	6.75					
8	1.33	2.	2.67	3.33	4.	4.67	5.33						
7	1.17	1.75	2.33	2.92	3.5	4.08							
6	1.	1.5	2.	2.5	3.								
5	.83	1.25	1.67	2.08									
4	.67	1.	1.33										
3	.5	.75											
2	.33												

To ascertain the contents of a piece of timber, find in the table the contents of one foot and multiply by the length in feet of the piece.

EXAMPLES

By means of the above table find the board measure of the following:

1. One timber 8″ × 9″, 14′ long.
2. One timber 8″ × 11″, 13′ long.
3. One timber 9″ × 10″, 11′ long.
4. One timber 8″ × 10″, 16′ long.
5. One timber 7″ × 9″, 11′ long.
6. Two planks 2″ × 3″, 10′ long.
7. Two planks 4″ × 4″, 13′ long.
8. Two timbers 10″ × 11″, 15′ long.
9. Two timbers 10″ × 18″, 14′ long.
10. Two timbers 8″ × 16″, 13′ long.

The weight per cubic foot of different woods can be readily seen from the following table:

WEIGHT OF ONE CUBIC FOOT OF TIMBER

Wood	Weight per Cu. Ft.	Wood	Weight per Cu. Ft.
White Pine	28 lb.	Whitewood	30 lb.
Georgia Pine	38 lb.	Ash	45 lb.
Hemlock	24 lb.	Hickory	48 lb.
Cypress	33 lb.	Chestnut	35 lb.
Spruce	28 lb.	Cedar	39 lb.
White Oak	48 lb.	Birch	41 lb.
Red Oak	46 lb.	Ebony	76 lb.
Maple	42 lb.	Boxwood	70 lb.

Lumber is bought and sold in the log by cubic measure. Lumber used for framing buildings and for building bridges, docks, and ships is also sold by the cubic foot.

The rule that is most extensively used for computing the contents in board feet of a log is as follows:

RULE. — *Subtract* 4 *inches from the diameter of the log at the small end, square one quarter of the remainder, and multiply the result by the length of the log in feet.*

EXAMPLES

Find the board feet of lumber in the following logs:

	1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**
Diameter in inches at small end:	12	13	14	15	16	17	18	20	24
Length in feet:	10	12	14	16	18	20	16	20	24

CHAPTER V

CONSTRUCTION

Excavations. — After the plans for a building are drawn by the architect and the work given to the contractor on a bid, usually the lowest, the work of excavating begins. In estimating excavations the cubic yard, or 27 cubic feet, is used.

EXAMPLES

1. What will it cost to excavate a cellar that is $32' \times 28'$ and $5'$ deep at 34 cents per cubic yard?

2. What will the cost be of excavating a lot $112'$ by $58'$ and averaging $12'$ deep at $1.65 per yard?

3. In excavating a tunnel 374,166 cubic feet of earth were removed. If the length of the tunnel was 492 ft. and the width 39 ft., what was the height of the tunnel?

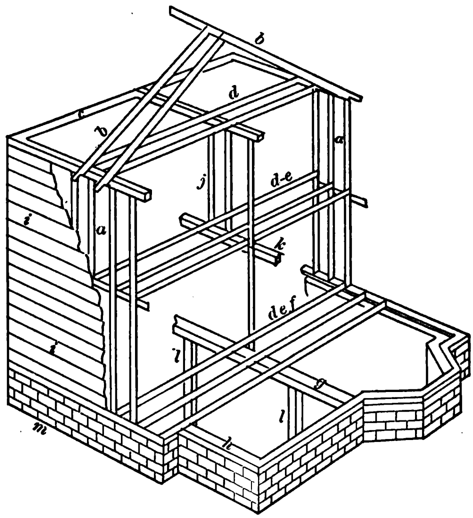
4. How many cubic yards of earth must be removed to build a cellar for a house when the measurements inside the wall are $28'$ long and $16'$ wide, the wall being $1'\ 8''$ thick and $8'$ deep, with $2'$ of the wall above the ground level?

5. In making a bid on some excavating a contractor notes that the excavation is in the shape of a rectangle $8'$ deep, $11'$ wide at the top and bottom, and $483'$ long. What will it cost him to excavate it at 29 cents per cubic yard? What must his bid be to make 10 % profit?

Frame and Roof

After the excavation is finished and the foundation laid, the construction of the building itself is begun. On the top of the foundation a large timber called a **sill** is placed. The timbers

running at right angles to the front sill are called side sills; The sills are joined at the corners by a half-lap joint and held together by spikes.

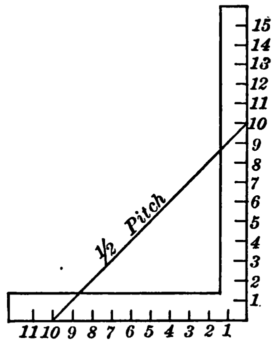


a.	Outside studding	*de.*	Second floor joists	*i.*	Sheathing
b.	Rafters	*def.*	First floor joists	*j.*	Partition studs
c.	Plates	*g.*	Girder or cross sill	*k.*	Partition heads
d.	Ceiling joists	*h.*	Sills	*l.*	Piers[1]

Then the building has its walls framed by placing corner posts of 4″ by 6″ on the four corners. Between these corner posts there are placed smaller timbers called **studding**, 2″ by 4″, 16″ apart. Later the **laths**, 4′ long, are nailed to this studding. The upright timbers are mortised into the sills at the bottom. When these uprights are all in position a timber called a **plate** is placed on the top of them and they are spiked together.

On the top of the plate is placed the roof. The principal timbers of the roof are the **rafters**. Different roofs have a dif-

[1] If made of brick or stone, "shones" or "supports" if made of wood.



ferent pitch or slope — that is, form different angles with the plate. To get the desired pitch the carpenter uses the steel square.

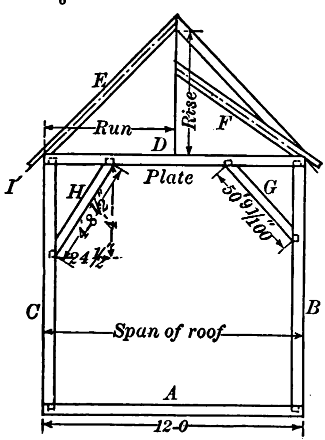
When we speak of the pitch of a roof we mean the slope or slant of the roof. A roof with one half pitch means that the height of the ridge of the roof above the level of the plate is equal to one half of the width of the roof, or $\frac{W}{2} = R$.

If the width of a building is 16 feet and the roof is one quarter pitch, it means that the height of the ridge of the roof above the plate is 4 feet, or $\frac{W}{4} = R$.

EXAMPLES

Give the height of the ridge of the roof above the level of the plate of the following buildings:

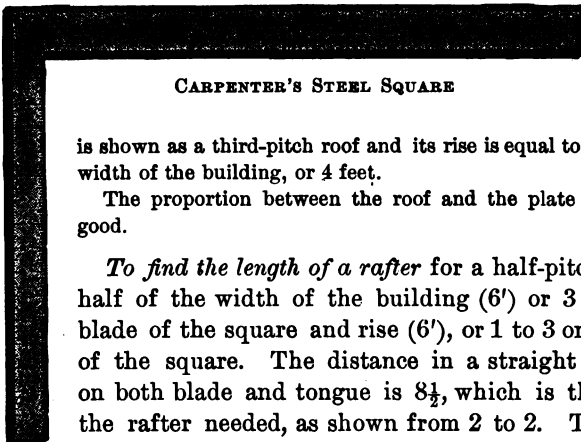
Pitch	Width of Building	Pitch	Width of Building
1. $\frac{1}{2}$	32 feet	5. $\frac{3}{4}$	36 feet
2. $\frac{1}{4}$	40 feet	6. $\frac{2}{3}$	48 feet
3. $\frac{1}{3}$	36 feet	7. $\frac{3}{4}$	28 feet
4. $\frac{1}{6}$	48 feet	8. $\frac{1}{2}$	34 feet



The height of the ridge of the roof above the level of the plate is the **rise** of rafter. The distance from where the rafter intersects the outer edge of the plate to a point on the plate directly beneath the peak is called the **run** of rafter.

The figure represents a frame, *A*, *B*, *C*, *D*, with rafters *E* and *F*, braces *G* and *H*. The frame is 12 feet high and 12 feet wide.

Rafter *E* is shown as a half-pitch roof, meaning that its rise is equal to



CARPENTER'S STEEL SQUARE

half the width of the building, or 6 feet.

Rafter F is shown as a third-pitch roof and its rise is equal to one third the width of the building, or 4 feet.

The proportion between the roof and the plate always holds good.

To find the length of a rafter for a half-pitch roof, take half of the width of the building (6′) or 3 to 1 on the blade of the square and rise (6′), or 1 to 3 on the tongue of the square. The distance in a straight line from 6 on both blade and tongue is $8\frac{1}{2}$, which is the length of the rafter needed, as shown from 2 to 2. The measure must be taken on the line indicated by the arrow I (see diagram on page 91), not on the top of the rafter. It is usual to cut away a piece 2 inches thick from the part of the rafter that projects past the side walls, — the part to which the cornice is nailed, or 2 inches down from the top, — which is the line to measure for the length of the rafter.

To find the cut (bevel) of the top and bottom of the rafter, lay the square flat on the side of the rafter with the blades toward the bottom and the tongue toward the top, with the figure 6 on the blade and the figure 4 on the tongue at the edge of the rafter. Mark along the blade for the bottom and cut along the tongue for the top cut. This gives the bevel for the top and bottom cuts for a third-pitch roof. With 6 on both blade and tongue you get the bevel for a half-pitch roof or a miter cut for any square purpose.

Mark the run on the blade and the rise on the tongue and then measure across from one figure to the other and you have the length of any rafter or brace. Take the brace H in the diagram on page 91, for example. It has 4-foot rise and 2-foot 6-inch run. Its length is found in the same way. By measuring

from 4 on the blade to $2\frac{1}{2}$ on the tongue you find 4 feet $8\frac{1}{2}$ inches the length of the brace.

In measuring a brace you must mark the length on the outer edge instead of 2 inches inside (like the rafter), for the **tenon** is usually made to extend to the extreme point. The bevel at each end is obtained by the square in the same manner as with the rafter.

Lathing

Laths are thin pieces of wood, 4 ft. long and $1\frac{1}{2}$ in. wide, upon which the plastering of a house is laid. They are usually put up in bundles of one hundred. They are nailed $\frac{3}{8}$ in. apart and fifty will cover about 30 sq. ft.

EXAMPLES

1. At 30 cents per square yard what will it cost to lath and plaster a wall 12 ft. by 15 ft.?

2. At 45 cents per square yard what will it cost to lath and plaster a wall 18 ft. by 16 ft.?

3. What will it cost to lath and plaster a room (including walls and ceiling) 16 ft. square by 12 ft. high, allowing 34 square feet for windows and doors, at 40 cents per square yard?

4. What will it cost to lath and plaster the following rooms at $42\frac{1}{2}$ cents per square yard?

a. $16' \times 14' \times 11'$ high with a door $8' \times 2\frac{1}{2}'$ and 2 windows $2\frac{1}{2}' \times 5'$.
b. $18' \times 15' \times 11'$ high with a door $10' \times 3'$ and 4 windows $2\frac{1}{2}' \times 5'$.
c. $20' \times 18' \times 12'$ high with a door $11' \times 3'$ and 4 windows $2\frac{1}{2}' \times 4'$.
d. $28' \times 32' \times 16'$ high with a door $10' \times 3'$ and 4 windows $3' \times 5'$.
e. $28' \times 30' \times 15'$ high with a door $10' \times 3'$ and 3 windows $3' \times 5'$.

CHAPTER VI

BUILDING MATERIALS

Besides wood many materials enter into the construction of buildings. Among these materials are mortar, cement, stone, bricks, marble, slate, etc.

Mortar is a paste formed by mixing lime with water and sand in the correct proportions. (Common mortar is generally made of 1 part of lime to 5 parts of sand.) It is used to hold bricks, etc., together and when stones or bricks are covered with this paste and placed together, the moisture in the mortar evaporates and the mixture "sets" by the absorption of the carbon dioxide from the air. Mortar is strengthened by adding cow's hair when it is used to plaster a house; in such mortar there is sometimes half as much lime as sand.

Plaster is a mixture of a cheap grade of gypsum (calcium sulphate), sand, and hair. When the plaster is mixed with water, the water combines with the gypsum and the minute crystals in forming interlace and cause the plaster to "set."

When masons plaster a house they estimate the amount of work to be done by the square yard. Nearly all masons use the following rule: Calculate the total area of walls and ceilings and deduct from this total area one half the area of openings such as doors and windows. A bushel of mortar will cover about 3 sq. yd. with two coats.

Example. — How many square yards of plastering are necessary to plaster walls and ceiling of a room 28′ by 32′ and 12′ high?

Areas of the front and back walls are	$28 \times 12 \times 2 =$	672 sq. ft.
Areas of the side walls are	$32 \times 12 \times 2 =$	768 sq. ft.
Area of the ceiling is	$28 \times 32 =$	896 sq. ft.
		2336 sq. ft.

$\frac{2336}{9}$ sq. ft. $= 259\frac{5}{9}$ sq. yd.

260 sq. yd. *Ans.*

EXAMPLES

1. What will it cost to plaster a wall 10 ft. by 13 ft. at $0.30 per square yard?

2. What will it cost to plaster a room 28′ 6″ by 32′ 4″ and 9′ 6″ high, at 29 cents a square yard, if one half their area is allowed for openings and there are two doors 8′ by $3\frac{1}{2}$′ and three windows 6′ by 3′ 3″?

3. What will it cost to plaster an attic 22′ 4″ by 16′ 8″ and 9′ 4″ high, at a cost of 32 cents a square yard?

Bricks used in Building

Brickwork is estimated by the thousand, and for various thicknesses of wall the number required is as follows:

$8\frac{1}{4}$-inch wall, or 1 brick in thickness, 14 bricks per superficial foot;
$12\frac{3}{4}$-inch wall, or $1\frac{1}{2}$ bricks in thickness, 21 bricks per superficial foot;
17-inch wall, or 2 bricks in thickness, 28 bricks per superficial foot;
$21\frac{1}{2}$-inch wall, or $2\frac{1}{2}$ bricks in thickness, 35 bricks per superficial foot.

EXAMPLES

From the above table solve the following examples:

1. How much brickwork is in a 17″ wall (that is, 2 bricks in thickness) 180′ long by 6′ high?

2. How many bricks in an $8\frac{1}{4}$″ wall, 164′ 6″ long by 6′ 4″?

3. How many bricks in a 17″ wall, 48′ 3″ long by 4′ 8″?

4. How many bricks in a $21\frac{1}{2}$″ wall, 36′ 4″ long by 3′ 6″?

5. How many bricks in a $12\frac{3}{4}$″ wall, 38′ 3″ long by 4′ 2″?

6. At $19 per thousand find the cost of bricks for a building 48′ long, 31′ wide, 23′ high, with walls $12\frac{3}{4}$″ thick. There are 5 windows (7′ × 3′) and 4 doors (4′ × $8\frac{1}{2}$′).

To estimate the number of bricks in a wall it is customary to find the number of cubic feet and then multiply by 22, which is the number of bricks in a cubic foot with mortar.

EXAMPLES

1. How many bricks are necessary to build a partition wall 36′ long, 22′ wide, and 18″ thick?

2. How many bricks will be required for a wall 28′ 6″ long, 16′ 8″ wide, and 6′ 5″ high?

3. How many cubic yards of masonry will be necessary to build a wall 18′ 4″ long and 12′ 2″ wide and 4″ thick?

4. At $19 per thousand, how much will the bricks cost to build an $8\frac{1}{4}''$, or one-brick wall, 28′ 4″ long and 8′ 3″ high?

5. At $20.50 per thousand, how much will the bricks cost to build a $12\frac{3}{4}''$ wall, 52′ 6″ long and 14′ 8″ high?

6. A house is 45′ × 34′ × 18′ and the walls 1 foot thick, the windows and doors occupy 368 cu. ft.; how many bricks will be required to build the house?

7. What will it cost to lay 250,000 bricks, if the cost per thousand is $8.90 for the bricks; $3 for mortar; laying, $8; and staging, $1.25?

Stone Work

Stone work, like brick work, is measured by the cubic foot or by the perch ($16\frac{1}{2}' \times 1\frac{1}{2}' \times 1'$) or cord. Practical men usually consider 24 cubic feet to the perch and 120 cubic feet to the cord. The cord or perch is not much used.

The usual way is to measure the distance around the cellar on the outside for the length. This includes the corners twice, but owing to the extra work in making corners this is considered proper. No allowance is made for openings unless they are very large, when one half is deducted.

The four walls may be considered as one wall with the same height.

Example. — If the outside dimensions of a wall are 44′ by 31′, 10′ 6″ high and 8″ thick, find the number of cubic feet.

$$\begin{array}{r} 44 \\ 31 \\ \hline 75 \\ 2 \\ \hline 150 \end{array} \text{ ft. length.} \qquad \overset{25}{\cancel{150}} \times \frac{21}{\cancel{2}} \times \frac{\overset{2}{\cancel{8}}}{\underset{4}{\cancel{12}}} = 1050 \text{ cu. ft. } \textit{Ans.}$$

Cement

Some buildings have their columns and beams made of concrete. Wooden forms are first set up and the concrete is poured into them. The concrete consists of Portland cement, sand, and broken stone, usually in the proportion of 1 part cement to 2 parts sand and 4 parts stone. The average weight of this mixture is 150 pounds per cubic foot. After the concrete has "set," the wooden boxes or forms are removed.

Within a few years twisted steel rods have been placed in the forms and the concrete poured around them. This is called reënforced concrete and makes a stronger and safer combination than the whole concrete. It is used in walls, sewers, and arches. It takes a long time for the concrete to reach its highest compressive and tensile strength.

Cement is also used for walls and floors where a waterproof surface is desired. When the cement "sets" it forms a layer like stone, through which water cannot pass. If the cement is inferior or not properly made, it will not be waterproof and water will press through it and in time destroy it.

EXAMPLES

1. If one bag (cubic foot) of cement and one bag of sand will cover $2\frac{3}{8}$ sq. yd. one inch thick, how many bags of cement and of sand will be required to cover 30 sq. yd. $2\frac{1}{2}$″ thick?

2. How many bags of cement and of sand will be required to lay a foundation 1″ thick on a sidewalk 20′ by 8′?

3. How many bags of cement and of sand will it take to cover a walk, 34′ by 8′ 6″, $\frac{3}{4}$″ thick?

4. If one bag of cement and two of sand will cover $5\frac{1}{3}$ sq. yd. $\frac{3}{4}''$ thick, how much of each will it take to cover 128 sq. ft.?

5. How much of a mixture of one part cement, two parts sand, and four parts cracked stone will be needed to cover a floor 28′ by 32′ and 8″ deep? How much of each will be used?

Shingles

Shingles for roofs are figured as being 16″ by 4″ and are sold by the thousand. The widths vary from 2″ upwards. They are put in bundles of 250 each. When shingles are laid on the roof of a building they overlap so that only part of them is exposed to the weather.

One thousand shingles laid 4″ to the weather will cover 100 square feet, or ten shingles to every square foot. If 6″ are exposed to the weather 600 shingles will be necessary.

1 *square* = 100 *square feet.*

Table for Number and Weight of Pine Shingles, 4″ Wide, for One Square of Roof

Inches exposed to the weather	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$	6	The number of shingles per square is for common gable roofs. For hip roofs add five per cent to these figures.
Number of shingles for one square of roof	1000	800	750	655	600	
Weight in lb. of shingles on one square of roof	240	192	180	157	144	The weights per square are based on the number per square: 1000 4-inch shingles weigh 240 lb.

There are several methods for finding the number of shingles required to cover a roof. One is, first to find the number of squares in the roof; divide this number by $1\frac{1}{4}$ and multiply the result by 1000.[1]

[1] This rule refers to shingles laid $4\frac{1}{2}$ inches to the weather. For other conditions refer to the above table.

EXAMPLE. — Find the number of shingles required to cover a roof 40 ft. long and 25 ft. wide, laid $4\frac{1}{2}$ inches to the weather.

$$40 \times 25 = 1000 \text{ or } 10 \text{ squares}$$
$$\frac{10}{1\frac{1}{4}} \times 1000 = 10 \times \frac{4}{5} \times 1000 = 8000. \textit{ Ans.}$$

Another method, used when the roof is straight, is to find the number of courses and multiply this by the number of shingles in a course.

EXAMPLE. — The ridge of a roof is 25 ft. long and the rafters are 15 ft. on each side. How many shingles will be required to cover this roof? The shingles are 4″ wide. Each shingle is 5″ to the weather.

$$25 \div \tfrac{1}{3} = 75 \qquad 30 \div \tfrac{5}{12} = 72 \qquad 75 \times 72 = 5{,}400 \textit{ Ans.}$$

This gives the exact number but a few more should be added for waste, etc. Verify answer by use of table on page 98.

Another method is: Since it takes approximately 3 bunches (250 shingles each) laid 5″ to the weather to cover a square, multiplying the number of squares in the roof by 3 will give the number of bunches required.

EXAMPLE. — If a roof contains 50 squares, how many shingles will the roof need to cover it?

$$50 \times 3 = 150 \text{ bunches} = 37{,}500 \text{ shingles. } \textit{Ans.}$$

EXAMPLES

1. How much will it cost for shingles to shingle a roof 50 ft. by 40 ft., if 1000 shingles are allowed for 125 square feet and the shingles cost $1.18 per bundle?

2. Find the cost of shingling a roof 38 ft. by 74 ft., 4″ to the weather, if the shingles cost $1.47 a bundle, and a pound and a half of cut nails at $.06 a pound are used with each bundle.

3. How many shingles would be needed for a roof having four sides, two in the shape of a trapezoid with bases 30 ft. by 10 ft., and altitude 15 ft., and two (front and back) in the shape of a triangle with base 20 ft. and altitude 15 ft.? (1000 shingles will cover 120 sq. ft.)

Slate Roofing

Slates make a good-looking and durable roof. They are put on with nails similarly to shingles. Estimates for slate roofing are made on 100 sq. ft. of the roof.

The following are typical data for building a slate roof:

A square of No. 10 × 20 Monson slate costs about $8.35.
Two pounds of galvanized nails cost $.16 per pound.
Labor, $3 per square.
Tar paper, at 2¾ cents per pound, 1½ lb. per square yard.

EXAMPLES

Using the above data, give the cost of making slate roofs for the following:

1. What is the cost of laying a square of slate?
2. What is the cost of laying slate on a roof 112′ by 44′?
3. What is the cost of laying slate on a roof 156′ by 64′?
4. What is the cost of laying slate on a roof 118′ by 52′?
5. What is the cost of laying slate on a roof 284′ by 78′?

Weight of Roof Coverings

Every builder should know the weight of different roof coverings in order to make a roof strong enough to support its covering. Besides, allowance must be made for ice and snow. The weight in pounds of roof covering is usually expressed in pounds per 100 sq. ft., or square of roof.

Approximate Weight of Roof Coverings

Name	Weight per 100 sq. ft.
Sheathing, Pine 1 inch thick, yellow northern	300
Sheathing, Pine 1 inch thick, yellow southern	400
Spruce, 1 inch thick	200
Sheathing, Chestnut or Maple, 1 inch thick	400
Sheathing, Ash, Hickory, or Oak, 1 inch thick	500
Sheet iron, $\frac{1}{16}$ inch thick	300
Sheet iron, $\frac{1}{16}$ inch thick, and laths	500
Shingles, Pine	200
Slates, $\frac{1}{4}$ inch thick	900
Skylights (Glass, $\frac{3}{16}$ to $\frac{1}{2}$ inch thick)	250–700
Sheet Lead	500–800
Cast Iron Plates, $\frac{3}{8}$ inch thick	1500
Copper	80–125
Felt and Asphalt	100
Felt and Gravel	800–1000
Iron, Corrugated	100–375
Iron, Galvanized Flat	100–350
Lath and Plaster	900–1000
Thatch	650
Tin	70–125
Tiles, Flat	1500–2000
Tiles (Grooves and Fillets)	700–1000
Tiles, Pan	1000
Tiles, with Mortar	2000–3000
Zinc	100–200

EXAMPLES

What is the approximate weight of:

1. 800 sq. ft. of $\frac{1}{4}$ inch slate?
2. 1645 sq. ft. of flat tiles?
3. 2332 sq. ft. of tiles with mortar?
4. 3184 sq. ft. of lath and plaster?
5. 2789 sq. ft. of sheathing pine 1″ thick?
6. 1841 sq. ft. of pine shingles?

7. 1794 sq. ft. of thatch?

8. 3279 sq. ft. of felt and gravel?

9. 1973 sq. ft. of asphalt?

10. 1589 sq. ft. of skylight glass $\frac{1}{2}$ in. thick?

Clapboards

Clapboards are used to cover the outside walls of frame buildings. Most clapboards are 4 ft. long and 6 in. wide. They are sold in bundles of twenty-five. Three bundles will cover 100 square feet if they are laid 4″ to the weather.

To find the number of clapboards required to cover a given area, find the area in square feet and divide by $1\frac{1}{8}$. One quarter the area should be deducted to allow for openings.

EXAMPLES

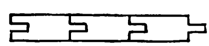
1. How many clapboards will be required to cover an area of 40 ft. by 30 ft.?

2. How many clapboards will be necessary to cover an area of 38′ by 42′ if 56 sq. ft. are allowed for doors and windows?

3. How many clapboards will a barn 60 ft. by 50 ft. require if 10% is allowed for openings and the distance from foundation to the plate is 17 ft. and the gable 10 ft. high?

Flooring

Most floors in houses are made of oak, maple, birch, or pine. This flooring is grooved so that the boards fit closely together without cracks between them.



The accompanying figure shows the ends of pieces of matched flooring. Matched boards are also used for ceilings and walls. In estimating for matched flooring enough stock must be added to make up for what is cut away from the width in matching. This amount varies from $\frac{1}{4}''$ to $\frac{3}{4}''$ on each board according to its size. Some is also wasted in squaring ends, cutting up, and

fitting to exact lengths. A common floor is made of unmatched boards and is usually used as an under floor. Not more than $\frac{1}{4}$ is allowed for waste.

EXAMPLE. — A room is 12 ft. square and is to have a floor laid of matched boards $1\frac{1}{2}''$ wide; one third is to be added for waste. What is the number of square feet in the floor? What is the number of board feet required for laying the floor?

$12 \times 12 = 144$ sq. ft. = area.	$144 \times \frac{1}{3} = 48$
144 *Ans.*	144
	192 board measure for matched floor.
	192 *Ans.*

EXAMPLES

1. How much $\frac{7}{8}$ in. matched flooring 3″ wide will be required to lay a floor 16 ft. by 18 ft.? One fourth more is allowed for matching and 3 % for squaring ends.

2. How much hard pine matched flooring $\frac{7}{8}''$ thick and $1\frac{1}{2}''$ wide will be required for a floor 13′ 6″ × 14′ 10″? Allow $\frac{1}{8}$ for matching and add 4 % for waste.

3. An office floor is 10′ 6″ wide at one end and 9′ 6″ wide at the other (trapezoid) and 11′ 7″ long. What will the material cost for a maple floor $\frac{7}{8}''$ thick and $1\frac{1}{2}''$ wide at $60 per M, if 4 sq. ft. are allowed for waste?

4. How many square feet of sheathing are required for the outside, including the top, of a freight car 34′ long, 8′ wide, and $7\frac{1}{2}'$ high, if $12\frac{1}{2}$% is allowed for waste and overhang?

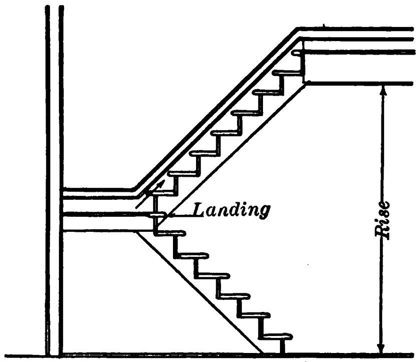
5. In a room 50′ long and 20′ wide flooring is to be laid; how many feet (board measure) will be required if the stock is $\frac{7}{8}'' \times 3''$ and $\frac{1}{4}$ allowance for waste is made?

Stairs

The perpendicular distance between two floors of a building is called the **rise** of a flight of stairs. The width of all the steps is called the **run.** The perpendicular distance between steps is called the **width of rises. Nosing** is the slight projec-

tion on the front of each step. The board on each step is the **tread**.

To find the number of stairs necessary to reach from one floor to another: Measure the rise first. Divide this by six inches,[1] which is the most comfortable rise for stairs. The run should be $8\frac{1}{2}$ inches or more to allow for a tread of $9\frac{3}{4}$ inches with a nosing of $1\frac{1}{4}$ inches.



STAIRS

EXAMPLE. — How many steps will be required, and what will be the rise, if the distance between floors is 118 inches?

$$118 \div 8 = 14\tfrac{3}{4} \text{ or } 15 \text{ steps.}$$
$$118 \div 15 = 7\tfrac{13}{15} \text{ inches each rise. } Ans.$$

EXAMPLES

1. How many steps will be required, and what will be the rise, (a) if the distance between floors is 8′? (b) If the distance is 9 feet?

2. How many steps will be required, and what will be the rise, (a) if the distance between floors is 12′? (b) If the distance is 8′ 8″?

Carpenters' Table of Wages

To find the amount due at any rate from 30 cents to 55 cents per hour, look at the column containing the rate per hour and the column opposite that containing the number of hours, and the amount will be shown. Time and a half is counted for overtime on regular working days, and double time for Sundays and holidays.

[1] Other distances may also be used if required.

Hours	Rate per Hr.	Reg. Time	Overtime	Dbl. Time	Rate per Hr.	Reg. Time	Overtime	Dbl. Time	Rate per Hr.	Reg. Time	Overtime	Dbl. Time
½...	$0 30	$0 15	$0 22½	$0 30	$0 32½	$0 16¼	$0 24⅜	$0 32½	$0 45	$0 22½	$0 33¾	$0 45
1...	30	30	45	60	32½	32½	48¾	65	45	45	67½	90
2...	30	60	90	1 20	32½	65	97½	1 30	45	90	1 35	1 80
3...	30	90	1 35	1 80	32½	97½	1 46¼	1 95	45	1 35	2 02½	2 70
4...	30	1 20	1 80	2 40	32½	1 30	1 95	2 60	45	1 80	2 70	3 60
5...	30	1 50	2 25	3 00	32½	1 62½	2 43¾	3 25	45	2 25	3 37½	4 50
6...	30	1 80	2 70	3 60	32½	1 95	2 92½	3 90	45	2 70	4 05	5 40
7...	30	2 10	3 15	4 20	32½	2 27½	3 41¼	4 55	45	3 15	4 72½	6 30
8...	30	2 40	3 60	4 80	32½	2 60	3 90	5 20	45	3 60	5 40	7 20
9...	30	2 70	4 05	5 40	32½	2 92½	4 38¾	5 85	45	4 05	6 07½	8 10
10...	30	3 00	4 50	6 00	32½	3 25	4 87½	6 50	45	4 50	6 75	9 00
½...	$0 47½	$0 23¾	$0 35⅝	$0 47½	$0 50	$0 25	$0 37½	$0 50	$0 55	$0 27½	$0 41¼	$0 55
1...	47½	47½	71¼	95	50	50	75	1 00	55	55	82½	1 10
2...	47½	95	1 42½	1 90	50	1 00	1 50	2 00	55	1 10	1 65	2 20
3...	47½	1 42½	2 13¾	2 85	50	1 50	2 25	3 00	55	1 65	2 47½	3 30
4...	47½	1 90	2 85	3 80	50	2 00	3 00	4 00	55	2 20	3 30	4 40
5...	47½	2 37½	3 56¼	4 75	50	2 50	3 75	5 00	55	2 75	4 12½	5 50
6...	47½	2 85	4 27½	5 70	50	3 00	4 50	6 00	55	3 30	4 95	6 60
7...	47½	3 32½	4 98¾	6 65	50	3 50	5 25	7 00	55	3 85	5 77½	7 70
8...	47½	3 80	5 70	7 60	50	4 00	6 00	8 00	55	4 40	6 60	8 80
9...	47½	4 27½	6 41¼	8 55	50	4 50	6 75	9 00	55	4 96	7 42½	9 90
10...	47½	4 75	7 12½	9 50	50	5 00	7 50	10 00	55	5 50	8 25	11 00

EXAMPLES

1. Find the amount due a carpenter who has worked 8 hours regular time and 2 hours overtime at 55 cents per hour.

2. A carpenter worked on Sunday from 8 to 11 o'clock. If his regular wages are 45 cents per hour, how much will he receive?

3. A carpenter received 55 cents an hour. How much money is due him for working July 4th from 8–12 A.M. and 1–4.30 P.M.?

4. A carpenter works six days in the week; every morning from 7.30 to 12 M.; three afternoons from 1 to 4.30 P.M.; two afternoons from 1 to 5.30; and one from 1 until 6 P.M. What will he receive for his week's wage at 50 cents per hour?

Painting

Paint, which is composed of dry coloring matter or pigment mixed with oil, drier, etc., is applied to the surface of wood by means of a brush to preserve the wood. The paint must be composed of materials which will render it impervious to water, or rain would wash it from the exterior of houses. It should thoroughly conceal the surface of whatever it is applied to. The unit of painting is one square yard. In painting wooden houses two coats are usually applied.

It is often estimated that one pound of paint will cover 4 sq. yd. for the first coat and 6 sq. yd. for the second coat. Some allowance is made for openings ; usually about one half of the area of openings is deducted, for considerable paint is used in painting around them.

TABLE

1 gallon of paint will cover on concrete . .	300 to 375 superficial feet
1 gallon of paint will cover on stone or brick work	190 to 225 superficial feet
1 gallon of paint will cover on wood	375 to 525 superficial feet
1 gallon of paint will cover on well-painted surface or iron	600 superficial feet
1 gallon of tar will cover on first coat . . .	90 superficial feet
1 gallon of tar will cover on second coat . .	160 superficial feet

EXAMPLES

1. How many gallons of paint will it take to paint a fence 6′ high and 50′ long, if one gallon of paint is required for every 350 sq. ft.?

2. What will the cost be of varnishing a floor 22′ long and 16′ wide, if it takes a pint of varnish for every four square yards of flooring and the varnish costs $2.65 per gal.?

3. What will it cost to paint a ceiling 36′ by 29′ at 21 cents per square yard?

4. What will be the cost of painting a house which is 52′ long, 31′ wide, 21′ high, if it takes one gallon of paint to cover 300 sq. ft. and the paint costs $1.65 per gallon? (House has a hip roof.)

PART III — SHEET AND ROD METAL WORK

CHAPTER VII

Blanking or Cutting Dies

MANY kinds of receptacles are pressed or cut from different kinds of sheet metal, such as copper, tin, and aluminum. Cans, pots, parts of metal boxes, and all sorts of metal novelties are punched out of sheet metal most economically by the punch and die operated by the hydraulic press. So skillfully can die makers produce dies and punches to cut out articles that thousands of everyday necessities in the household are made by this method. Parts of watches, parts of automobiles, and parts of machinery are punched out. Some presses that operate the

"BLANKING" OR "CUTTING" DIES

Cutting dies consist of an upper "male" die or "punch," and the lower or "female" die. Circumstances determine whether any or how much "shear" shall be given to the cutting edge. For ordinary work in tin, brass, etc., a moderate amount of shear is desirable. Ordinarily, the steel cutting rings are welded to wrought-iron plates, after which they are hardened, carefully tempered, and ground on special machinery. In some cases it is preferable to fasten the steel dies in cast-iron chucks or die-beds by means of keys

or screws. This applies more particularly to small dies. Cutting dies may be made to fit any size and style of press. For cutting thick iron, steel, brass, and other heavy metals, both the die and punch should be hard and provided with strippers.

PUNCH AND DIE WITHOUT STRIPPER

PUNCH AND DIE WITH STRIPPER

punch and die are run by foot power, but those most generally used are run by electricity.

A blanking or cutting die is a metal plate or disk having an opening in the center used in a punching machine or press which is supplied with ample power and which also supports the metal from which pieces are punched. Dies are made in almost any size and shape for cutting flat blanks in tin, iron, steel, aluminum, brass, copper, zinc, silver, paper, leather, etc.

Holes are punched in thick sheets of metal or in heavy plates by means of great pressure exerted by a hydraulic press. This pressure, in pounds, is usually about 60,000 times the area (expressed in square inches) of the surface cut out, or in other words about 60,000 lb. per square inch.

EXAMPLES

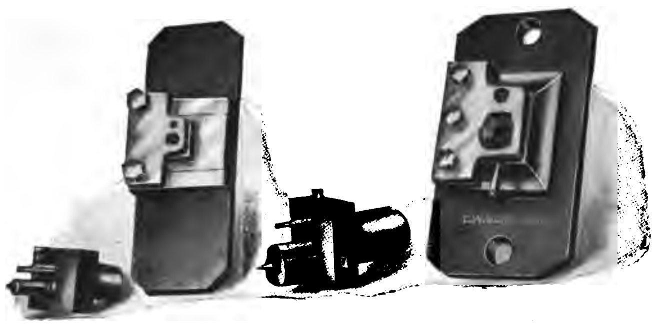
1. How much pressure will be necessary for a hydraulic press to exert on a sheet of boiler plate $\frac{7}{16}''$ thick, if it is desired to punch holes $\frac{1}{2}''$ in diameter?

2. How much pressure will it be necessary to use in order to punch holes $\frac{3}{4}''$ in diameter out of $\frac{1}{2}''$ boiler plate?

3. How much pressure will it be necessary to use in order to punch $\frac{5}{16}''$ holes out of $\frac{7}{16}''$ boiler plate?

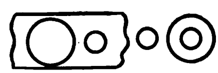
Combination Dies

Double dies for blanking and perforating are extensively used in the manufacture of washers, key blanks, electrical instruments, hardware, etc. The blanking and perforating punches act simultaneously. At a single stroke of the press

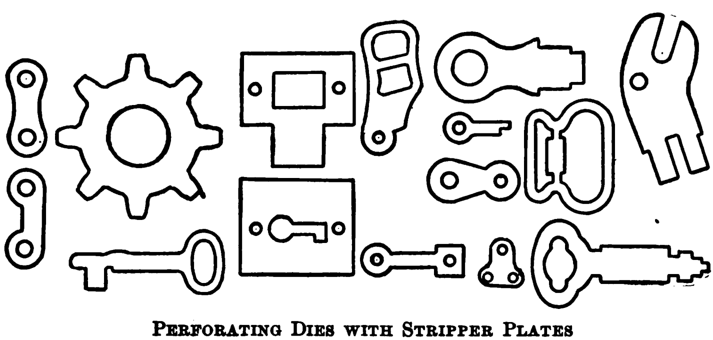


"Double" Dies for Blanking and Perforating

The perforating and blanking punches act simultaneously in such a manner that the holes are punched first, whereupon the strip or sheet being fed forward, the blank is cut out around them and the holes for the next blank perforated at the same stroke. In this manner a blank is completely perforated at every stroke of the press.



The same principle may be extended so as to punch a number of perforated blanks at a time.



Perforating Dies with Stripper Plates

one punch perforates the holes and the other punch cuts out the metal between the holes punched at the previous stroke. These dies are usually so arranged that the finished article is automatically pushed out from the dies by the action of the springs. An expert operator can punch many thousands of pieces in a day.

EXAMPLE. — How much metal will be required for 2000 blacking box covers, 6″ in diameter, and $\frac{3}{4}$″ deep?

$6'' + \frac{3}{4}'' + \frac{3}{4}'' = 7\frac{1}{2}''$, diameter of one cover.
$7.5 \times 7.5 = 44.1786$ sq. in., area.
$44.1786 \times 2000 = 88,357.2$ sq. in. $= 613.6$ sq. ft. *Ans.*

EXAMPLES

1. How large must the blank be cut for a pail cover that is to be (*a*) 5″ in diameter and $\frac{7}{8}$″ deep? (*b*) 7″ in diameter and $1\frac{1}{8}$″ deep? (*c*) 6″ in diameter and 1″ deep? (*d*) 8″ in diameter and $1\frac{1}{8}$″ deep?

2. How large must the blank be cut for the pail bottom in each of the above examples (*a*, *b*, *c*, and *d*)?

The blank must be $\frac{1}{4}$″ larger in diameter than the diameter of the part in order to allow $\frac{1}{8}$″ all around for forming into the sides.

3. How large must the piece be to form the sides of the pail in each of the above examples? Pail (*a*) to be 6″ high, (*b*) 9″ high, (*c*) 8″ high, and (*d*) 10″ high.

Allow $\frac{1}{4}$″ in height, and for the lock seam allow $\frac{3}{8}$″ in circumference.

4. How much metal will be necessary for 850 complete pails in example (*a*) above?

5. How much metal will be necessary for 1500 complete pails in example (*b*) above?

6. How much metal will be necessary for 840 complete pails in example (*c*) above?

7. How much metal will be necessary for 1000 complete pails in example (*d*) above?

MACHINE FOR USING DIES

8. How many square feet of sheet copper will be required to make a rectangular tank 7′ long, 3′ wide, $1\frac{1}{2}$′ deep, allowing 10 % extra for waste? The tank is to be open on top.

9. If the sheet copper weighs 12 lb. per sq. ft. and costs 25 cents per lb., what will be the cost of the material used?

10. Find the amount of material in a straight piece of copper pipe 24″ diameter and 8′ long. The pipe is formed from sheet copper weighing 15.4 lb. per sq. ft. What does it weigh?

Allowance for seam, $1\frac{1}{4}$″.

11. If the cost of copper in Example 10 is 27 cents per lb. and the amount of copper wasted in making the pipe is equal to $\frac{1}{7}$ the finished weight of the pipe, what is the total cost of material used?

12. What is the cost per pound of finished pipe for the material used in Examples 10 and 11?

13. If the labor to make the pipe in Example 10 costs $ 36, what is the cost per lb. for labor?

14. What will be the total cost in Example 10 of all material and labor per pound of finished pipe?

Circles

Review the paragraphs on the Circle on pages 62 and 63, and on the Cylinder, page 72.

EXAMPLES

To find circumference of a cylindrical tank, when the diameter is given.

1. What is the circumference of a tin plate (*a*) 8″ in diameter? (*b*) 14″ in diameter? (*c*) 1′ 6″ in diameter? (*d*) 2′ 3″ in diameter? (*e*) 1′ 9″ in diameter?

2. What is the diameter of a copper plate containing (*a*) 25 sq. in.? (*b*) 43 sq. ft.? (*c*) 349 sq. in.? (*d*) 8840 sq. in.? (*e*) 616 sq. in.? (*f*) 340 sq. in.?

3. What is the diameter of an iron tank (*a*) 6′ high, containing 40 gallons? (*b*) $5\frac{1}{2}$′ high, containing 360 gallons? (*c*) 6′ high, containing 120 gallons? (*d*) 12′ high, containing 141 gal-

lons? (*e*) 8′ high, containing 181 gallons? (*f*) 4′ high, containing 241 gallons? (231 cu. in. = 1 gal.)

Two circles are formed by a plane passing through a hollow cylinder of metal, as it has an outside and an inside circumference, due to the thickness of the metal. A circle the circumference of which is midway between these two circumferences is said to be the **neutral** circle or ring, and its diameter is the **neutral** diameter.

To illustrate: the neutral diameter of a ring 8″ outside diameter and 6″ inside diameter is 7″.

4. What is the length of a smoke arch ring whose outside diameter is 60″ and a section of which measures $2\frac{1}{2}'' \times 2\frac{1}{2}''$?

The length of the ring is the length or circumference of a circle whose diameter is the neutral diameter of the ring.

5. A smoke stack 32″ inside diameter is made from $\frac{3}{16}''$ iron sheet stock. What would be the length of the sheet, allowing 2″ for lapping?

Use the neutral diameter.

6. If we desire to make a close fit over the above sheet and allow 2″ for overlapping, what size sheet will we use?

7. A blacksmith desires to place a band around a hub that is 17″ in diameter. If the band is 2″ square and if no allowance is made for shrinking, how long a piece of iron is necessary to do the job?

8. If the above band averages $\frac{1}{4}$ lb. per cu. in., what is the weight of the band?

9. A flat iron ring casting has the following dimensions: 18″ outside diameter, and 9″ inside diameter, and 3″ thick. What will it cost at 3 cents a pound? What is the cost of a cubic foot of the iron? (1 cu. in. iron = .26 lb.)

10. The neutral diameter of an engine wheel tire is 76″. If the steel weighs .28 lb. per cu. in., what would be the weight of the foregoing, if its section was a trapezoid whose dimensions were 6″ wide on the face, $3\frac{3}{4}''$ thick on one side, and $4\frac{7}{8}''$ on the other?

11. What will be the weight per sq. ft. of a steel plate $\frac{1}{2}''$ thick if a steel plate $\frac{1}{2}''$ thick weighs 20 lb. per sq. ft.?

12. What is the weight of a steel bar 17′ long having a cross section area of $6\frac{3}{4}$ sq. in.? (1 cu. ft. steel = 490 lb.)

13. A "1226 class" draft pan sheet was laid out in the form of a rectangle, the length of which measured $62\frac{1}{2}''$ and the width 22″. Find its area in square feet.

14. If the above sheet were $\frac{1}{4}''$ thick and weighed 9 lb. per sq. ft., what would be its weight after deducting 7 lb. for rivet holes?

15. An iron plate is divided into four sections; the first contains $29\frac{3}{4}$ sq. in., the second contains $50\frac{5}{8}$ sq. in., the third contains 41 sq. in., and the fourth $69\frac{3}{16}$ sq. in. How many square inches in the plate? If 2″ thick, what does it weigh?

Calculate 480 lb. to a cubic foot.

16. What is the area of the surface of a boiler plate 3′ 8″ by 1′ 6″?

17. How many square pieces of zinc 6″ by 6″ can be cut from a zinc plate 3′ by 6′?

18. How many pieces of zinc 4″ by 8″ can be cut from a zinc plate 3′ by 6′?

19. What is the value of the copper in a copper tank measuring $4\frac{3}{4}' \times 3'\ 6'' \times 2'\ 3''$ and made of copper weighing 12 lb. per sq. ft., if the copper costs 25 cents per lb. and no account is made of laps and seams and waste? Tank open at top.

20. If a sheet of copper 30″ by 60″ weighs 25 lb., what is its weight per sq. ft.?

21. What would be the length of a 40 lb. sheet of the same thickness of copper, 16″ wide?

Use the result from Example 20.

22. What is the capacity of a cylindrical tank 9′ long and 5′ diameter, inside measurements?

23. What would be the dimensions of a cubical tank containing the same amount?

24. On a base 4′ × 5′, how high shall a tank be made to contain 14 tons of water which weighs 64 lb. per cu. ft.?

25. How many sq. ft. of sheet metal will be required to form the bottom and sloping sides of a pan, the bottom $2\frac{1}{2}''$ square, the slant height of the sides 8″, and the opening across the open top $3\frac{1}{4}''$ square? Do not consider any allowances for waste, laps, etc.

26. What is the area of one side of a sheet ring that is 3 inches inside diameter and 4 inches outside diameter?

27. The circular basin of a washing machine is 20 feet in diameter. What will it cost to copper line the bottom at $ 1.20 per sq. ft.?

28. A piece of steel shafting 10′ 3″ long is rough machined to a diameter of $11\frac{3}{4}''$. When finished to a diameter of $11\frac{1}{2}''$ and with a 1″ diameter hole running through its entire length, how much has it been reduced in weight?

29. If the average weight of wrought iron is 480 lb. per cu. ft., what is the weight of a piece of bar iron 1″ square, 9′ long? What would be the area of the cross section of a bar 6′ long weighing 72 lb.?

30. A solid cast iron cone pulley is $3'\,2\frac{1}{2}''$ long. The diameter of one end of the pulley is $4\frac{7}{8}''$ and the other end of the pulley is $10\frac{1}{2}''$. A hole 2″ in diameter runs through the entire length of the pulley. What is its weight? (Cast iron weighs 450 lb. per cu. ft.)

31. How many square feet of sheet copper will be required to make a rectangular tank 7′ long, 3′ wide, $1\frac{1}{2}'$ deep, allowing 10% extra for waste? Tank open on top.

32. If the sheet copper weighs 12 lb. per sq. ft. and costs 29 cents per lb., what will be the cost of the material used?

33. The average weight of wrought iron is 480 lb. per cu. ft. A bar 4″ square and 3′ long weighs how many pounds?

34. A cast iron pulley is 4′ $3\frac{1}{2}$″ long. The diameter of one end of the pulley is $5\frac{3}{8}$″ and of the other end of the pulley is $11\frac{1}{4}$″. A hole $1\frac{3}{4}$″ in diameter runs through the entire length of the pulley. What is its weight?

35. How many bosom pieces $15\frac{7}{8}$″ long can be made from a bar of angle iron 25′ long, and how much waste will there be?

36. How many clips $9\frac{5}{16}$″ long can be made from a bar 25′ long, and what will be the waste?

Standard Gauge for Sheet Metal and Wire

In order to measure the thickness of a piece of wire or sheet metal, a gauge has been made. The U. S. standard gauge is a circular instrument $3\frac{1}{4}$″ in diameter and about $\frac{1}{8}$″ thick. The gauge numbers, which run from 0 to 36, are those adopted by Congress, March 3, 1893. The gauge is made of hardened and tempered steel.

U. S. Standard Gauge

To measure wire or sheet metal, the wire or metal is placed in a perforation that it fits, and the number of this perforation is called the number of the gauge of the wire or metal.

To find the size of a wire or piece of sheet metal in decimal parts of an inch when the gauge has been determined, the following table should be used:

Standards for Wire Gauge

Dimensions of Sizes in Decimal Parts of an Inch

Number of Wire Gauge	American or Brown & Sharpe	Birmingham, or Stubs Iron Wire	Washburn & Moen Mfg. Co.	Imperial Wire Gauge	Stubs Steel Wire	U. S. Standard for Plate	Number of Wire Gauge
000000	....	....	....	.464	....	.46875	000000
00000	....	....	....	.432	....	.4375	00000
0000	.46	.454	.3938	.400	....	.40625	0000
000	.40964	.425	.3625	.372	....	.375	000
00	.3648	.38	.3310	.348	....	.34375	00
0	.32486	.34	.3065	.324	....	.3125	0
1	.2893	.3	.2830	.300	.227	.28125	1
2	.25763	.284	.2625	.276	.219	.265625	2
3	.22942	.259	.2437	.252	.212	.25	3
4	.20431	.238	.2253	.232	.207	.234375	4
5	.18194	.22	.2070	.212	.204	.21875	5
6	.16202	.203	.1920	.192	.201	.203125	6
7	.14428	.18	.1770	.176	.199	.1875	7
8	.12849	.165	.1620	.160	.197	.171875	8
9	.11443	.148	.1483	.144	.194	.15625	9
10	.10189	.134	.1350	.128	.191	.140625	10
11	.090742	.12	.1205	.116	.188	.125	11
12	.080808	.109	.1055	.104	.185	.109375	12
13	.071961	.095	.0915	.092	.182	.09375	13
14	.064084	.083	.0800	.080	.180	.078125	14
15	.057068	.072	.0720	.072	.178	.0703125	15
16	.05082	.065	.0625	.064	.175	.0625	16
17	.045257	.058	.0540	.056	.172	.05625	17
18	.040303	.049	.0475	.048	.168	.05	18
19	.03589	.042	.0410	.040	.164	.04375	19
20	.031961	.035	.0348	.036	.161	.0375	20
21	.028462	.032	.03175	.032	.157	.034375	21
22	.025347	.028	.0286	.028	.155	.03125	22
23	.022571	.025	.0258	.024	.153	.028125	23
24	.0201	.022	.0230	.022	.151	.025	24
25	.0179	.02	.0204	.020	.148	.021875	25
26	.01594	.018	.0181	.018	146	.01875	26
27	.014195	.016	.0173	.0164	.143	.0171875	27
28	.012641	.014	.0162	.0149	.139	.015625	28
29	.011257	.013	.0150	.0136	.134	.0140625	29
30	.010025	.012	.0140	.0124	.127	.0125	30
31	.008928	.01	.0132	.0116	.120	.0109375	31
32	.00795	.009	.0128	.0108	.115	.01015625	32
33	.00708	.008	.0118	.0100	.112	.009375	33
34	.006304	.007	.0104	.0092	.110	.00859375	34
35	.005614	.005	.0095	.0084	.108	.0078125	34
36	.005	.004	.0090	.0076	.106	.00703125	36
37	.004453	....	....	.0068	.103	.006640625	37
38	.003965	....	....	.0060	.101	.00625	38
39	.003531	....	....	.0052	.099	......	39
40	.003144	....	....	.0048	.097	......	40

The American or B. & S. gauge is the standard for sheet brass, copper, or German silver, and for wire of the same material.

The Birmingham gauge is used for soft iron wire or rods.

The Washburn & Moen gauge is used for iron or copper telegraph and telephone wire.

Stubs Steel Wire gauge is the standard for Stubs drill rods. It is not the same as Stubs Iron Wire gauge.

The U. S. Standard gauge is recognized as standard for sheet iron and steel.

EXAMPLES

1. Find the thickness of wire of (*a*) No. 10 gauge; (*b*) No. 28 gauge; (*c*) No. 15 gauge; (*d*) No. 5 gauge; (*e*) No. 3 gauge.

2. Find the thickness of sheet metal No. 7 gauge.

3. Find the thickness of iron of (*a*) No. 24 gauge; (*b*) No. 18 gauge.

4. Find the thickness of steel of (*a*) No. 29 gauge; (*b*) No. 16 gauge.

5. Find the weight of 120 sq. ft. of (*a*) No. 4 sheet iron; (*b*) 28 sq. ft. of No. 19 sheet steel; (*c*) 35 sq. ft. of No. 16 sheet iron; (*d*) 79 sq. ft. of No 5 sheet steel.

See page 120 and the tables on pages 121–123.

6. Find the weight of $38\frac{1}{2}$ sq. ft. of sheet steel whose thickness is .125.

7. Find the weight of $69\frac{1}{2}$ sq. ft. of sheet iron whose thickness is .04375.

8. Find the weight of $128\frac{1}{2}$ sq. ft. of No. 8 sheet iron.

9. Find the weight of $250\frac{1}{2}$ sq. ft. of No. 6 sheet steel.

10. What number gauge wire (B. & S.) is .090742?

11. What number gauge wire (Stubs Steel Wire) is .01594?

Additional Tables for Sheet Metal Workers

TIN PLATE

	Thickness Stub's Gauge	No. Sheets in Box	Net Weight of Box 14 × 20 Sheets
Taggers	38 (34)	225 (150)	112 lb.
IC	30	112	107 lb.
IX	28	112	135 lb.
IXX	27	112	156 lb.
IXXX	26	112	176 lb.
IXXXX	25	112	196 lb.

SHEET ZINC — M. & H. GAUGE

No. 1 = 0.002 in.	No. 11 = 0.024 in.	No. 21 = 0.080 in.
No. 2 = 0.004 in.	No. 12 = 0.028 in.	No. 22 = 0.090 in.
No. 3 = 0.006 in.	No. 13 = 0.032 in.	No. 23 = 0.100 in.
No. 4 = 0.008 in.	No. 14 = 0.036 in.	No. 24 = 0.125 in.
No. 5 = 0.010 in.	No. 15 = 0.040 in.	No. 25 = 0.250 in.
No. 6 = 0.012 in.	No. 16 = 0.045 in.	No. 26 = 0.375 in.
No. 7 = 0.014 in.	No. 17 = 0.050 in.	No. 27 = 0.500 in.
No. 8 = 0.016 in.	No. 18 = 0.055 in.	No. 28 = 1.000 in.
No. 9 = 0.018 in.	No. 19 = 0.060 in.	
No. 10 = 0.020 in.	No. 20 = 0.070 in.	

AMERICAN RUSSIA IRON

No. 7 = .015 in.	No. 12 = .021 in.
No. 8 = .016 in.	No. 13 = .024 in.
No. 9 = .017 in.	No. 14 = .025 in.
No. 10 = .018 in.	No. 15 = .027 in.
No. 11 = .020 in.	No. 16 = .030 in.

1. What is the net weight of 4 boxes tin plate IC?
2. What is the net weight of 8 boxes tin plate IXX?
3. What is the net weight of 5 boxes tin plate IXXXX?
4. What is the net weight of 7 boxes tin plate IXXX?
5. What is the net weight of 12 boxes tin plate IXX?
6. How many sheets in 6 boxes tin plate IXX?
7. How many sheets in 8 boxes tin plate IXXXX?
8. How many sheets in 7 boxes tin plate IXXX?
9. How many sheets in 10 boxes tin plate IX?
10. How many sheets in 14 boxes tin plate IC?

Weights and Areas

Iron and steel bars are sold in round, square, or hexagonal shape. When it is necessary to know the area or weight of a bar, look at the left column in tables similar to the following for the number corresponding to the thickness or diameter of the bar, then in the same line in the columns to the right the weight or area is found.

EXAMPLE. — What is the weight of a round bar of steel 12′ long and $\frac{5}{16}''$ in diameter?

According to the table on page 122, the

Wt. of $\frac{5}{16}''$ bar per in. length $= .0218$ lb.
Wt. of $\frac{5}{16}''$ bar per ft. length $= .2616$ lb.
Wt. of $\frac{5}{16}''$ bar 12 ft. in length $= 3.1392$ lb. *Ans.*

To obtain the weight of a certain size sheet steel or iron one should look in the table for the weight per square foot corresponding to the size that it is desired to know and when the weight per square foot is known, any number of feet can be found by multiplying the weight per foot by the number of feet.

Thickness and Weight of Sheet Steel and Iron

Adopted by U. S. Government July 1, 1893

Weight of 1 cu. ft. is assumed to be 487.7 lb. for steel plates and 480 lb. for iron plates.

Number of Gauge	Approximate Thickness		Weight per Sq. Ft.		Overweight
	Fractions	Decimals	Steel	Iron	Up to 75 in. Wide
0000000	**1-2**	.5	20.320	20.00	5 per cent.
000000	15-32	.46875	19.050	18.75	
00000	7-16	.4375	17.780	17.50	6 per cent.
0000	13-32	.40625	16.510	16.25	
000	**3-8**	.375	15.240	15.00	7 per cent.
00	11-32	.34375	13.970	13.75	
0	5-16	.3125	12.700	12.50	8 per cent.
1	9-32	.28125	11.430	11.25	
2	17-64	.26562	10.795	10.625	Up to 50 in. wide
3	**1-4**	.25	10.160	10.00	
4	15-64	.23437	9.525	9.375	7 per cent.
5	7-32	.21875	8.890	8.75	
6	13-64	.20312	8.255	8.125	
7	3-16	.1875	7.620	7.5	
8	11-64	.17187	6.985	6.875	8½ per cent.
9	5-32	.15625	6.350	6.25	
10	9-64	.14062	5.715	5.625	10 per cent.
11	**1-8**	.125	5.080	5.00	
12	7-64	.10937	4.445	4.375	
13	3-32	.09374	3.810	3.75	
14	5-64	.07812	3.175	3.125	
15	9-128	.07031	2.857	2.812	
16	**1-16**	.0625	2.540	2.50	
17	9-160	.05625	2.286	2.25	
18	1-20	.05	2.032	2.	
19	7-160	.04375	1.778	1.75	
20	3-80	.0375	1.524	1.50	
21	11-320	.03437	1.397	1.375	
22	**1-32**	.03125	1.270	1.25	
23	9-320	.02812	1.143	1.125	
24	1-40	.025	1.016	1.	
25	7-320	.02187	1.389	.875	
26	3-160	.01875	.762	.75	
27	11-640	.01718	.698	.687	
28	**1-64**	.01562	.635	.623	
29	9-640	.01406	.571	.562	
30	1-80	.0125	.508	.5	
31	7-640	.01093	.694	.437	
32	13-1280	.01015	.413	.406	
33	**3-320**	.00937	.381	.375	
34	11-1280	.00859	.349	.343	
35	5-640	.00781	.317	.312	
36	9-1280	.00703	.285	.281	
37	17-2560	.00664	.271	.265	
38	1-160	.00625	.254	.25	

Weights and Areas of Round, Square, and Hexagon Steel

Weight of one cubic inch = .2836 lb. Weight of one cubic foot = 490 lb.

Thickness or Diameter	Area = Diam.^{2} × .7854			Area = Side^2 × 1		Area = Diam.^{2} × .866	
	Round			*Square*		*Hexagon*	
	Weight Per Inch	Area Square Inches	Circumference Inches	Weight Per Inch	Area Square Inches	Weight Per Inch	Area Square Inches
1-32	.0002	.0008	.0981	.0003	.0010	.0002	.0008
1-16	.0009	.0031	.1963	.0011	.0039	.0010	.0034
3-32	.0020	.0069	.2995	.0025	.0088	.0022	.0076
1-8	.0035	.0123	.3927	.0044	.0156	.0038	.0135
5-32	.0054	.0192	.4908	.0069	.0244	.0060	.0211
3-16	.0078	.0276	.5890	.0101	.0352	.0086	.0304
7-32	.0107	.0376	.6872	.0136	.0479	.0118	.0414
1-4	.0139	.0491	.7854	.0177	.0625	.0154	.0540
9-32	.0176	.0621	.8835	.0224	.0791	.0194	.0686
5-16	.0218	.0767	.9817	.0277	.0977	.0240	.0846
11-32	.0263	.0928	1.0799	.0335	.1182	.0290	.1023
3-8	.0313	.1104	1.1781	.0405	.1406	.0345	.1218
13-32	.0368	.1296	1.2762	.0466	.1651	.0405	.1428
7-16	.0426	.1503	1.3744	.0543	.1914	.0470	.1658
15-32	.0489	.1726	1.4726	.0623	.2197	.0540	.1903
1-2	.0557	.1963	1.5708	.0709	.2500	.0614	.2161
17-32	.0629	.2217	1.6689	.0800	.2822	.0693	.2444
9-16	.0705	.2485	1.7671	.0897	.3164	.0777	.2743
19-32	.0785	.2769	1.8653	.1036	.3526	.0866	.3053
5-8	.0870	.3068	1.9635	.1108	.3906	.0959	.3383
21-32	.0959	.3382	2.0616	.1221	.4307	.1058	.3730
11-16	.1053	.3712	2.1598	.1340	.4727	.1161	.4093
23-32	.1151	.4057	2.2580	.1465	.5166	.1270	.4474
3-4	.1253	.4418	2.3562	.1622	.5625	.1382	.4871
25-32	.1359	.4794	2.4543	.1732	.6103	.1499	.5286
13-16	.1470	.5185	2.5525	.1872	.6602	.1620	.5712
27-32	.1586	.5591	2.6507	.2019	.7119	.1749	.6165
7-8	.1705	.6013	2.7489	.2171	.7656	.1880	6631
29-32	.1829	.6450	2.8470	.2329	.8213	.2015	.7112
15-16	.1958	.6903	2.9452	,2492	.8789	.2159	.7612
31-32	.2090	.7371	3.0434	.2661	.9384	.2305	.8127
1	.2227	.7854	3.1416	.2836	1.0000	.2456	.8643
1 1-16	.2515	.8866	3.3379	.3201	1.1289	.2773	.9776
1 1-8	.2819	.9940	3.5343	.3589	1.2656	.3109	1.0973
1 3-16	.3141	1.1075	3.7306	.4142	1.4102	.3464	1.2212
1 1-4	.3480	1.2272	3.9270	.4431	1.5625	.3838	1.3531
1 5-16	.3837	1.3530	4.1233	.4885	1.7227	.4231	1.4919
1 3-8	.4211	1.4849	4.3197	.5362	1.8906	.4643	1.6373
1 7-16	.4603	1.6230	4.5160	.5860	2.0664	.5076	1.7898
1 1-2	.5012	1.7671	4.7124	.6487	2.2500	.5526	1.9485
1 9-16	.5438	1.9175	4.9087	.6930	2.4414	.5996	2.1143
1 5-8	.5882	2.0739	5.1051	.7489	2.6406	.6480	2.2847
1 11-16	.6343	2.2365	5.3014	.8076	2.8477	.6994	2.4662
1 3-4	.6821	2.4053	5.4978	.8685	3.0625	.7521	2.6522
1 13-16	.7317	2.5802	5.6941	.9316	3.2852	.8069	2.8450
1 7-8	.7831	2.7612	5.8905	.9970	3.5156	.8635	3.0446
1 15-16	.8361	2.9483	6.0868	1.0646	3.7539	.9220	3.2509
2	.8910	3.1416	6.2832	1.1342	4.0000	.9825	3.4573

Weights and Areas of Round, Square, and Hexagon Steel. — *Continued*

Weight of one cubic inch = .2836 lb. Weight of one cubic foot = 490 lb.

Thickness or Diameter	Area = Diam.² × .7854			Area = Side² × 1		Area = Diam.² × .866	
	Round			*Square*		*Hexagon*	
	Weight Per Inch	Area Square Inches	Circumference Inches	Weight Per Inch	Area Square Inches	Weight Per Inch	Area Square Inches
2 1-16	.9475	3.3410	6.4795	1.2064	4.2539	1.0448	3.6840
2 1-8	1.0058	3.5466	6.6759	1.2806	4.5156	1.1091	3.9106
2 3-16	1.0658	3.7583	6.8722	1.3570	4.7852	1.1753	4.1440
2 1-4	1.1276	3.9761	7.0686	1.4357	5.0625	1.2434	4.3892
2 5-16	1.1911	4.2000	7.2649	1.5165	5.3477	1.3135	4.6312
2 3-8	1.2564	4.4301	7.4613	1.6569	5.6406	1.3854	4.8849
2 7-16	1.3234	4.6664	7.6575	1.6849	5.9414	1.4593	5.1454
2 1-2	1.3921	4.9087	7.8540	1.7724	6.2500	1.5351	5.4126
2 5-8	1.5348	5.4119	8.2467	1.9541	6.8906	1.6924	5.9674
2 3-4	1.6845	5.9396	8.6394	2.1446	7.5625	1.8574	6.5493
2 7-8	1.8411	6.4918	9.0321	2.3441	8.2656	2.0304	7.1590
3	2.0046	7.0686	9.4248	2.5548	9.0000	2.2105	7.7941
3 1-8	2.1752	7.6699	9.8175	2.7719	9.7656	2.3986	8.4573
4 1-4	2.3527	8.2958	10.2102	2.9954	10.5625	2.5918	9.1387
3 3-8	2.5371	8.9462	10.6029	3.2303	11.3906	2.7977	9.8646
3 1-2	2.7286	9.6211	10.9956	3.4740	12.2500	3.0083	10.6089
3 5-8	2.9269	10.3206	11.3883	3.7265	13.1407	3.2275	11.3798
3 3-4	3.1323	11.0447	11.7810	3.9880	14.0625	3.4539	12.1785
3 7-8	3.3446	11.7932	12.1737	4.2582	15.0156	3.6880	13.0035
4	3.5638	12.5664	12.5664	4.5374	16.0000	3.9298	13.8292
4 1-8	3.7900	13.3640	12.9591	4.8254	17.0156	4.1792	14.7359
4 1-4	4.0232	14.1863	13.3518	5.1223	18.0625	4.4364	15.6424
4 3-8	4.2634	15.0332	13.7445	5.4280	19.1406	4.7011	16.5761
4 1-2	4.5105	15.9043	14.1372	5.7426	20.2500	4.9736	17.5569
4 5-8	4.7645	16.8002	14.5299	6.0662	21.3906	5.2538	18.5249
4 3-4	5.0255	17.7205	14.9226	6.6276	22.5625	5.5416	19.5397
4 7-8	5.2935	18.6655	15.3153	6.7397	23.7656	5.8371	20.5816
5	5.5685	19.6350	15.7080	7.0897	25.0000	6.1403	21.6503
5 1-8	5.8504	20.6290	16.1007	7.4496	26.2656	6.4511	22.7456
5 1-4	6.1392	21.6475	16.4934	7.8164	27.5624	6.7697	23.8696
5 3-8	6.4351	22.6905	16.8861	8.1930	28.8906	7.0959	25.0198
5 1-2	6.7379	23.7583	17.2788	8.5786	30.2500	7.4298	26.1971
5 5-8	7.0476	24.8505	17.6715	8.9729	31.6406	7.7713	27.4013
5 3-4	7.3643	25.9672	18.0642	9.3762	33.0625	8.1214	28.6361
5 7-8	7.6880	27.1085	18.4569	9.7883	34.5156	8.4774	29.8913
6	8.0186	28.2743	18.8496	10.2192	36.0000	8.8420	31.1765
6 1-4	8.7007	30.6796	19.6350	11.0877	39.0625	9.5943	33.8291
6 1-2	9.4107	33.1831	20.4204	11.9817	42.2500	10.3673	36.5547
6 3-4	10.1485	35.7847	21.2058	12.9211	45.5625	11.1908	39.4584
7	10.9142	38.4845	21.9912	13.8960	49.0000	12.0351	42.4354
7 1-2	12.5291	44.1786	23.5620	15.9520	56.2500	13.8158	48.7142
8	14.2553	50.2655	25.1328	18.1497	64.0000	15.7192	55.3169

Pupils should practice the use of tables in order to obtain accurate results quickly. Additional tables like these may be obtained from such standard handbooks as Kents. Multiply above weights by .998 for wrought iron, .918 for cast iron, 1.0331 for cast brass, 1.1209 for copper, and 1.1748 for phos. bronze.

EXAMPLES

By means of the table of weights and areas of round, square, and hexagonal steel, solve the following problems:

1. Find the circumference of a steel bar (*a*) $\frac{3}{32}''$ in thickness; (*b*) $\frac{13}{32}''$ in thickness; (*c*) $\frac{17}{32}''$ in thickness; (*d*) $1\frac{11}{16}''$ in thickness; (*e*) $3\frac{5}{8}''$ in thickness.

2. Find the area of a steel bar (*a*) $\frac{41}{32}''$ in diameter; (*b*) $1\frac{9}{16}''$ in diameter; (*c*) $3\frac{3}{8}''$ in diameter; (*d*) $4\frac{1}{8}''$ in diameter; (*e*) $5\frac{3}{4}''$ in diameter.

3. Find the weight per inch of a steel bar (*a*) $\frac{25}{32}''$ in diameter; (*b*) $1\frac{9}{16}''$ in diameter; (*c*) $2\frac{5}{16}''$ in diameter.

4. Find the area of a square bar (*a*) $\frac{3}{4}''$ per side; (*b*) $1\frac{3}{8}''$ per side; (*c*) $5\frac{1}{4}''$ per side; (*d*) $3\frac{3}{8}''$ per side.

5. Find the weight per inch of a square bar (*a*) $\frac{3}{16}''$ in thickness; (*b*) $\frac{31}{32}''$ in thickness; (*c*) $1\frac{13}{16}''$ in thickness.

6. Find the area of a hexagonal bar (*a*) $\frac{19}{32}''$ in thickness; (*b*) $1\frac{11}{16}''$ in thickness; (*c*) $3\frac{7}{8}''$ in thickness.

7. Find the weight per in. of a hexagonal bar (*a*) $\frac{23}{32}''$ in thickness; (*b*) $\frac{31}{32}''$ in thickness; (*c*) $2\frac{3}{16}''$ in thickness.

8. Find the weight of a round bar of steel (*a*) 7′ long $\frac{9}{16}''$ in diameter; (*b*) 11′ long $1\frac{5}{16}''$ in diameter; (*c*) 16′ long $2\frac{5}{16}''$ in diameter; (*d*) 11′ long $2\frac{3}{4}''$ in diameter; (*e*) 13′ long $4\frac{1}{8}''$ in diameter.

9. Find the weight of a square bar of steel (*a*) 8′ long $2\frac{3}{8}''$ in diameter; (*b*) 14′ long $1\frac{5}{8}''$ in diameter; (*c*) 11′ long $1\frac{13}{16}''$ in diameter.

10. Find the weight of a hexagonal bar of steel (*a*) 8′ long $3\frac{3}{8}''$ in diameter; (*b*) 7′ long $2\frac{7}{8}''$ in diameter; (*c*) 9′ long $1\frac{11}{16}''$ in diameter.

11. What is the weight of a square bar of wrought iron 16′ long $4\frac{1}{2}''$ in diameter?

12. What is the weight of a hexagonal bar of wrought iron 18′ long $2\frac{1}{16}''$ in diameter?

13. What is the weight of a round bar of cast iron 14′ long $3\frac{1}{8}''$ in diameter?

14. What is the weight of a square bar of cast iron 13′ long $1\frac{5}{8}''$ in diameter?

15. What is the weight of a square bar of cast brass 15′ long $1\frac{15}{16}''$ in diameter?

16. What is the weight of a hexagonal bar of cast brass 8′ long $\frac{13}{16}''$ in diameter?

17. What is the weight of a hexagonal bar of copper 7′ long $2\frac{1}{16}''$ in diameter?

18. What is the weight of a round bar of copper 6′ long $1\frac{1}{16}''$ in diameter?

PART IV — BOLTS, SCREWS, AND RIVETS

CHAPTER VIII

BOLTS

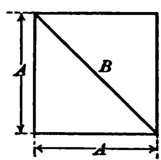
THE most common forms of fastenings are bolts, screws, rivets, pins, and nails. These are turned out in large numbers, usually by feeding long lengths of iron or steel rod into automatic machines. In order to make these fasteners of the right size one has to be familiar with the problems that are connected with them.

The common bolt is made in many different sizes and is usually held in place by a nut screwed on the end. There are many different kinds of bolts for the different uses to which they are put.

Rough Bolts

The small diameter of a rough bolt head, that is, the distance across the flats, is $1\frac{1}{2}$ times the diameter of the bolt, plus $\frac{1}{8}$ inch; or, it may be stated as follows: The diameter of a rough bolt head $= 1\frac{1}{2}D$ plus $\frac{1}{8}$ inch, D being diameter of the bolt.

Sometimes bolts or nuts are made from round stock and cut either square or hexagon. In such cases it is necessary to find the proper diameter to which the stock must be turned in order that it may be milled to size. Let A represent the distance across the flats on the head of a flat bolt, and B the diameter of the round stock required to make a square-head bolt.



SQUARE HEAD

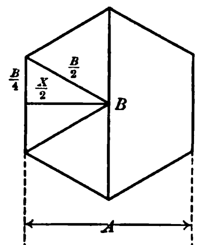
When the dimension A is given and it is necessary to turn a piece that will mill down to this size and leave full corners, multiply A by 1.414. The product is the desired size B.

$$B = \sqrt{A^2 + A^2} \qquad \text{Why ?}$$
$$B = \sqrt{2\,A^2} = A\sqrt{2} = 1.414\,A$$

When A, the distance across the flats, is given, and it is necessary to turn a piece that will mill down to this size and leave full corners, multiply A by 1.155. The product is the desired size (B) of the hexagon head.

Let $\frac{A}{2} = \frac{X}{2}$

$$\left(\frac{B}{2}\right)^2 = \left(\frac{B}{4}\right)^2 + \left(\frac{X}{2}\right)^2 \qquad \text{Why ?}$$
$$\frac{B^2}{4} = \frac{B^2}{16} + \frac{X^2}{4}$$
$$4\,B^2 = B^2 + 4\,X^2$$
$$3\,B^2 = 4\,X^2$$
$$B^2 = \frac{4}{3}X^2$$
$$B = \frac{2\,X}{\sqrt{3}} = \frac{2\,X\sqrt{3}}{3} = 1.155\,X$$



Hexagonal Head

Example. — To what diameter should a piece of stock be turned in order that it may have full corners when milled down six-sided to $1\frac{1}{2}''$ across the flats?

$1\frac{1}{2}'' \times 1.155 = 1.7325''$, diameter of the blank. *Ans.*

EXAMPLES

1. What is the size of stock required to make the following bolts having hexagonal heads? (*a*) Body $\frac{1}{2}''$ diam. (*b*) Body $\frac{3}{4}''$ diam. (*c*) Body $\frac{7}{8}''$ diam. (*d*) Body $1''$ diam.

2. What is the size of stock required to make the following bolts with square heads? (*a*) Body $\frac{1}{2}''$ diam. (*b*) Body $\frac{5}{8}''$ diam. (*c*) Body $\frac{3}{4}''$ diam. (*d*) Body $\frac{7}{8}''$ diam.

EXAMPLES

1. To what diameter should a piece of stock be turned so that it may have full corners when milled down square to $1\frac{3}{4}''$ across the flats?

2. To what diameter should a piece of stock be turned in order that it may have full corners when milled down six-sided to $1\frac{3}{4}''$ across the flats?

SIZES OF STANDARD HEXAGON HEAD BOLTS

DIAM. OF BOLT IN.	THICKNESS OF HEADS IN.	SIZE OF HEXAGON OR DISTANCE ACROSS THE FLATS IN.	ACROSS CORNERS IN.	THREADS PER INCH	TAP DRILL IN.
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{9}{16}$	20	$\frac{3}{16}$
$\frac{5}{16}$	$\frac{19}{64}$	$\frac{19}{32}$	$\frac{11}{16}$	18	C
$\frac{3}{8}$	$\frac{11}{32}$	$\frac{11}{16}$	$\frac{25}{32}$	16	N
$\frac{7}{16}$	$\frac{25}{64}$	$\frac{25}{32}$	$\frac{29}{32}$	14	S
$\frac{1}{2}$	$\frac{7}{16}$	$\frac{7}{8}$	1	13	$\frac{13}{32}$
$\frac{9}{16}$	$\frac{31}{64}$	$\frac{31}{32}$	$1\frac{7}{64}$	12	$\frac{29}{64}$
$\frac{5}{8}$	$\frac{17}{32}$	$1\frac{1}{16}$	$1\frac{7}{32}$	11	$\frac{33}{64}$
$\frac{3}{4}$	$\frac{5}{8}$	$1\frac{1}{4}$	$1\frac{7}{16}$	10	$\frac{5}{8}$
$\frac{7}{8}$	$\frac{23}{32}$	$1\frac{7}{16}$	$1\frac{21}{32}$	9	$\frac{47}{64}$
1	$\frac{13}{16}$	$1\frac{5}{8}$	$1\frac{7}{8}$	8	$\frac{27}{32}$
$1\frac{1}{8}$	$\frac{29}{32}$	$1\frac{13}{16}$	$2\frac{3}{32}$	7	$\frac{61}{64}$
$1\frac{1}{4}$	1	2	$1\frac{5}{16}$	7	$1\frac{5}{64}$
$1\frac{3}{8}$	$1\frac{3}{32}$	$2\frac{3}{16}$	$2\frac{1}{2}$	6	$1\frac{11}{64}$
$1\frac{1}{2}$	$1\frac{3}{16}$	$2\frac{3}{8}$	$2\frac{3}{4}$	6	$1\frac{19}{64}$
$1\frac{5}{8}$	$1\frac{9}{32}$	$2\frac{9}{16}$	$2\frac{15}{16}$	$5\frac{1}{2}$	$1\frac{25}{64}$
$1\frac{3}{4}$	$1\frac{3}{8}$	$2\frac{3}{4}$	$3\frac{3}{16}$	5	$1\frac{1}{2}$
$1\frac{7}{8}$	$1\frac{15}{32}$	$2\frac{15}{16}$	$3\frac{13}{32}$	5	$1\frac{5}{8}$
2	$1\frac{9}{16}$	$3\frac{1}{8}$	$3\frac{5}{8}$	$4\frac{1}{2}$	$1\frac{23}{32}$
$2\frac{1}{4}$	$1\frac{3}{4}$	$3\frac{1}{2}$	$4\frac{1}{16}$	$4\frac{1}{2}$	$1.962 = 1\frac{31}{32}$
$2\frac{1}{2}$	$1\frac{15}{16}$	$3\frac{7}{8}$	$4\frac{1}{2}$	4	$2.176 = 2\frac{3}{16}$
$2\frac{3}{4}$	$2\frac{1}{8}$	$4\frac{1}{4}$	$4\frac{29}{32}$	4	$2.426 = 2\frac{7}{16}$
3	$2\frac{5}{16}$	$4\frac{5}{8}$	$5\frac{3}{8}$	$3\frac{1}{2}$	$2.629 = 2\frac{41}{64}$

Notice that size of hexagon is equal to diameter of bolt + $\frac{1}{2}$ diameter of bolt + $\frac{1}{8}$ of an inch, and also that thickness of head is $\frac{1}{2}$ of hexagon in every case. The thickness of nut is equal to the diameter of bolt.

3. To what diameter should a piece of stock be turned so that it may have full corners when milled down square to $2\frac{3}{4}''$ across the flats?

4. To what diameter should a piece of stock be turned so that it may have full corners when milled down six-sided to $2\frac{1}{4}''$ across the flats?

SIZE ACROSS CORNERS OF SQUARES

SIZE OF SQUARE, IN.	DIAGONAL	SIZE OF SQUARE, IN.	DIAGONAL
$\frac{1}{8}$	.177	1	1.4141
$\frac{3}{16}$	.265	$1\frac{1}{8}$	1.590
$\frac{1}{4}$	.354	$1\frac{1}{4}$	1.768
$\frac{5}{16}$	.442	$1\frac{3}{8}$	1.945
$\frac{3}{8}$	.530	$1\frac{1}{2}$	2.121
$\frac{7}{16}$	.619	$1\frac{5}{8}$	2.298
$\frac{1}{2}$	.707	$1\frac{3}{4}$	2.475
$\frac{9}{16}$	.796	$1\frac{7}{8}$	2.652
$\frac{5}{8}$	.884	2	2.828
$\frac{11}{16}$	.972	$2\frac{1}{8}$	3.005
$\frac{3}{4}$	1.061	$2\frac{1}{4}$	3.182
$\frac{13}{16}$	1.149	$2\frac{1}{2}$	3.535
$\frac{7}{8}$	1.237	$2\frac{3}{4}$	3.889
$\frac{15}{16}$	1.326	3	4.243

EXAMPLES

Use the table to obtain the size of bolts.

1. Find the distance across the corners of a hexagon head bolt (*a*) with a diameter of $1\frac{7}{8}''$; (*b*) with a diameter of $2\frac{3}{4}''$; (*c*) with a diameter of $1\frac{1}{4}''$.

2. Find the diagonal distance across a square bolt with size of square (*a*) $\frac{3}{16}''$; (*b*) $\frac{5}{16}''$.

3. (*a*) If a bolt-heading machine has the following daily output, 2330, 2060, 1950, 2420, 2310, 2030, what is the average

daily output? (*b*) What would be the daily wage at 13 cents per 100 bolts? (*c*) The weekly wage?

4. A blacksmith requires six pieces of steel of the following lengths: $1\frac{3}{4}''$, $2\frac{7}{8}''$, $2\frac{9}{16}''$, $3\frac{11}{16}''$, $1\frac{17}{32}''$, $2''$. How long a piece of steel will be necessary to make them, if $\frac{1}{8}''$ is allowed for each in finishing?

5. A machinist has to make five bolts from the same size bar. One bolt is to be $1\frac{7}{8}''$ over all, another $2\frac{1}{4}''$, another $2\frac{3}{32}''$, another $3\frac{3}{64}''$, and the last $2\frac{5}{8}''$. How much stock will he need if he allows $\frac{1}{4}''$ for each cut-off?

Rivets

One of the simplest and most efficient metal fastenings which has been extensively used is the rivet. It resembles the bolt, but it can be removed only by chipping off the head, while the bolt can be taken off by removing the nut. Rivets, like bolts and nails, are quickly turned out by the thousand with the aid of automatic machines.

EXAMPLES

1. What must be the length of a bolt under the head, to go through $9\frac{3}{32}''$ thickness of plank and allow $1\frac{7}{8}''$ outside for taking a nut?

2. What must be the length of a bolt under the head, to go through $7\frac{5}{32}''$ thickness of plank and allow $1\frac{3}{8}''$ outside for taking a nut?

3. (*a*) How many rivets can be made in a bolt machine, from a round iron rod 6′ long, if each rivet requires $2\frac{1}{2}''$ of bar? (*b*) If the bar weighs $\frac{2}{3}$ lb. per ft., how many rivets weighing $\frac{1}{8}$ lb. apiece can be made from it? (*c*) How much waste will there be in each case?

4. What is the total thickness of three plates riveted together, each plate $\frac{11}{16}''$ thick? What would be the total thickness of five such plates?

5. What is the thickness of a steel plate that is only $\frac{3}{4}$ the thickness of a $\frac{5}{16}''$ plate?

6. A blacksmith and his helper made 192 bolt dogs in $18\frac{1}{4}$ hours. They received $6\frac{1}{2}$ cents apiece for them. (*a*) How much did they both receive per hour? (*b*) If the blacksmith received $66\frac{2}{3}\%$ of the money, how much did each receive per hour?

7. How many feet of round iron weighing 2.67 lb. per foot will be required to make 87 rivets weighing $2\frac{1}{2}$ lb. apiece, not counting waste? Give answer in feet and decimals of foot.

8. How many rivets weighing $7\frac{1}{2}$ ounces each can be made from 15′ of round iron weighing 1.5 lb. per foot? How much waste will there be?

9. If the drawing of an armor bolt is made $\frac{1}{8}$ size and the length of the drawing measures $7\frac{1}{4}''$, what will it measure if made to the scale of $3'' = 1'$? What is the actual length of the bolt?

10. If the drawing of an armor bolt is made $\frac{1}{4}$ size and the length of the drawing measures $9\frac{3}{4}''$, what will it measure if made to the scale of $4'' = 1'$? What is the actual length of the bolt?

11. The over-all length of a threaded bolt is $7\frac{3}{8}''$, the thickness of the head is $\frac{7}{8}''$, and the other end of the bolt is threaded for a distance of $2\frac{1}{2}''$; what is the length of the shank between the under side of the head and the threaded part of the bolt?

Nails

Wooden objects are often held together by means of nails. There are two kinds of nails: cut and wire. The wire nails are more commonly used, as they penetrate the wood without splitting it as the cut nails do. They have different kinds of heads, according to the use for which they are intended.

The origin of the common terms "sixpenny," "tenpenny," etc., as applied to nails, though not generally known, is involved in no mystery. Nails have been made a certain number of pounds to the thousand for many years and are still reckoned in that way in England, a tenpenny being a thousand nails to ten pounds, a sixpenny a thousand nails to six pounds, a twentypenny weighing twenty pounds to the thousand; and in ordering buyers call for the three-pound, six-pound, or ten-pound variety, etc., until, by the Englishmen's abbreviation of "pun" for "pound," the abbreviation has been made to stand for penny, instead of pound, as originally intended.

Length and Number of Cut Nails to the Pound

Size	Length	Common	Clinch	Fence	Finishing	Fine	Barrel	Casing	Brads	Tobacco	Cut Spikes
$\frac{3}{4}$	$\frac{3}{4}$ in.						800				
$\frac{7}{8}$	$\frac{1}{8}$ in.						500				
2d	1 in.	800			1100	1000	376				
3d	$1\frac{1}{4}$ in.	480			720	760	224				
4d	$1\frac{1}{2}$ in.	288			523	368	180	398			
5d	$1\frac{1}{4}$ in.	200			410					130	
6d	2 in.	168	95	84	268			224	126	96	
7d	$2\frac{1}{4}$ in.	124	74	64	188				98	82	
8d	$2\frac{1}{2}$ in.	88	62	48	146			128	75	68	
9d	$2\frac{3}{4}$ in.	70	53	36	130			110	65		
10d	3 in.	58	46	30	102			91	55		28
12d	$3\frac{1}{4}$ in.	44	42	24	76			71	40		
16d	$3\frac{1}{2}$ in.	35	38	20	62			54	27		22
20d	4 in.	23	33	16	54			40			$14\frac{1}{2}$
30d	$4\frac{1}{2}$ in.	18	20					33			$12\frac{1}{2}$
40d	5 in.	14						27			$9\frac{1}{2}$
50d	$5\frac{1}{2}$ in.	10									8
60d	6 in.	8									6
	$6\frac{1}{2}$ in.										$5\frac{1}{2}$
	7 in.										$4\frac{1}{2}$
	8 in.										$2\frac{1}{2}$

EXAMPLES

1. How many $\frac{3}{4}$d nails are there in 3 pounds?
2. How many $\frac{7}{8}$d nails are there in 4 pounds?
3. How many 2d nails are there in 2 pounds?
4. How many 7d nails are there in 8 pounds?
5. How many 16d nails are there in 16 pounds?
6. How long is (*a*) an 8d nail? (*b*) a 40d? (*c*) a 16d? (*d*) an 8d?
7. How long is (*a*) a 7d nail? (*b*) a 5d? (*c*) a 12d?

Tacks

Tacks are used to fasten thin pieces of material to wood. They vary in form and size. The size is represented by a number, as 16 oz. tacks, 24 oz. tacks; a No. 1 tack is called a one-ounce tack.

NUMBER OF TACKS IN A POUND

TITLE	LENGTH	NO. PER LB.	TITLE	LENGTH	NO. PER LB.
1 ounce	$\frac{3}{16}$ inch	16,000	10 ounce	$\frac{11}{16}$ inch	1,600
$1\frac{1}{2}$ ounce	$\frac{7}{32}$ inch	10,666	12 ounce	$\frac{3}{4}$ inch	1,332
2 ounce	$\frac{1}{4}$ inch	8,000	14 ounce	$\frac{13}{16}$ inch	1,143
$2\frac{1}{2}$ ounce	$\frac{5}{16}$ inch	6,400	16 ounce	$\frac{7}{8}$ inch	1,000
3 ounce	$\frac{3}{8}$ inch	5,332	18 ounce	$\frac{15}{16}$ inch	888
4 ounce	$\frac{7}{16}$ inch	4,000	20 ounce	1 inch	800
6 ounce	$\frac{9}{16}$ inch	2,666	22 ounce	$1\frac{1}{16}$ inch	727
8 ounce	$\frac{5}{8}$ inch	2,000	24 ounce	$1\frac{1}{8}$ inch	666

EXAMPLES

1. How many tacks are there in 8 lb. of 1 oz. or No. 1 tacks?
2. How many tacks are there in 13 lb. of $2\frac{1}{2}$ oz. or No. $2\frac{1}{2}$ tacks?
3. How many tacks are there in 7 lb. of 8 oz. or No. 8 tacks?

4. How many tacks are there in 6 lb. of 12 oz. or No. 12 tacks?

5. How many tacks are there in 17 lb. of 20 oz. or No. 20 tacks?

Screws

Pieces of wood and of metal are often fastened together by means of screws instead of by nails, especially if it is desirable to separate the parts at any time. Screws are made of iron, steel, or brass and have either a flat, fillister, or round head. When it is desired to have the head of the screw flush with the surface, the flat head type is used. Screws are made on automatic screw machines into which wire of various sizes may be fed. The machines are so constructed that they turn out large numbers of screws all complete in a very short time.

FLAT HEAD ROUND HEAD FILLISTER HEAD

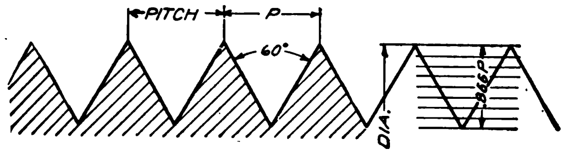
IRON MACHINE SCREWS

Screw threads are divided into two classes: first, those used for fastening; and second, those used in large machines for communicating motion. The screw threads used for communicating motion with which the mechanic has to deal are produced by a cutting process in which the thread is formed from the solid piece of stock by means of a single pointed cutting tool in a lathe. Screws used for fastening are made by means of taps and dies. The tap is a tool used to produce internal threads, and the die is a tool used to cut the external threads. The screw thread is applied in many ways, but the most common use is that of fastening together the various parts of machines, etc.

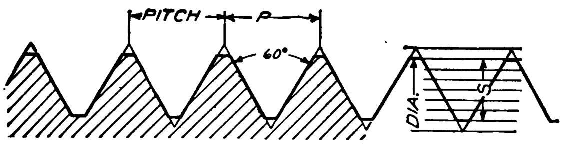
We find the mechanic using many different forms of bolts and screws to meet the needs of industries. In order to

specify a particular grade of bolt or screw it is necessary to mention (*a*) shape or form of head, (*b*) pitch or number of threads to the inch, (*c*) shape of thread, (*d*) outline of body, barrel or stem, (*e*) size of diameter, (*f*) direction of thread, as right or left hand, (*g*) length, (*h*) material, as brass, iron, etc.

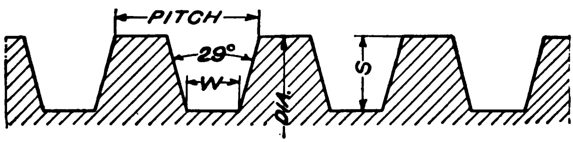
There are four different-shaped threads in common use in the United States: 1. The V thread; 2. The U. S. standard; 3. The Acme standard or worm; 4. The square thread.



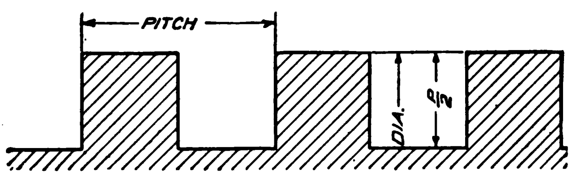
SHARP V THREAD



U. S. STANDARD THREAD



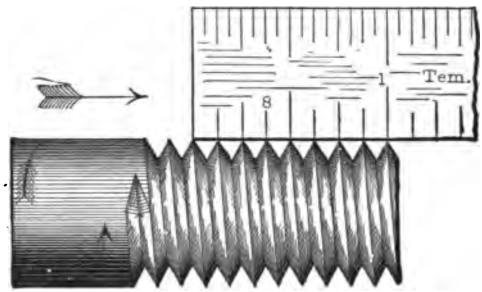
ACME STANDARD OR WORM THREAD[1]



SQUARE THREAD

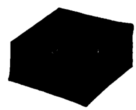
[1] Sometimes called modified square thread.

The different screws are formed by cutting a spiral groove around a cylinder. The projecting stock between the grooves is called the *land* or *thread*. There may be any number of threads; to every groove there is an accompanying thread.



RIGHT-HAND, SINGLE V-THREAD, 8 THREADS TO THE INCH

A screw that has one thread is called single-threaded; one having two threads, double-threaded; three threads, triple-threaded; etc. The cutting of the spiral groove or grooves is called cutting or threading a screw. A *nut* is a piece of iron or steel with a threaded hole which goes over a screw, and will turn off and on the screw.



NUT

Lead of a Screw. — The distance that a thread advances in one turn is called the *lead of the screw*. In a single-threaded screw the lead is equal to the distance occupied by one thread; and when the nut has made one complete turn, it has advanced one thread upon the screw. In a double-threaded screw the nut advances two threads with each complete turn; when the lead is three threads, the nut advances three threads in one turn. In general, the lead can be divided by any number of threads, the advance of any one of these threads in one turn being always equal to the lead.

One complete revolution of a single-threaded screw or the *lead* of a screw, if the screw had twelve threads to the inch, would be one twelfth of an inch.

Threads to an Inch. — By placing a scale upon a screw as in the figures on page 136, the number of thread-windings or coils in an inch can be counted. These windings or coils are called threads, and the number of coils to an inch is called the *number of threads to an inch.* The thread commences at the root or bottom of the screw. To measure screws for the number of threads per inch, the measurement must begin at the point of the thread. Place the end of the scale in line with this portion and then count the number of threads within the one-inch line.

Pitch. — *The distance from the center of one thread to the center of the next thread, measured in a line parallel to the axis, is the pitch of the thread, or the thread-pitch.* Divide 1″ by the number of threads to 1″ and the quotient is the thread-pitch. The threads to 1″ and the thread-pitch are reciprocals of each other.

In a single-threaded screw the pitch is equal to the lead. In a double-threaded screw the pitch is half the lead; in a triple-threaded screw the pitch is $\frac{1}{3}$ the lead; and so on. When the thread inclines so as to be nearer the right hand at the under side, or clockwise, it is a *right-hand thread.* When the under side is toward the left, or counter clockwise, the thread is left-handed. Again, when a right-hand screw turns in a direction to move its upper side away from the eye, the thread appears to move toward the right; while a left-hand thread moves toward the left.

The term *turns to an inch* means the number of times a screw must be turned around to advance one inch. If a screw makes four turns in advancing one inch, its lead is $\frac{1}{4}$, and it has 4 turns to the inch. Divide one inch by the lead, and the quotient is the number of turns that the screw makes in advancing one inch. If a screw does not advance exactly an inch in a whole number of turns, or if it does not advance some whole number of inches in one turn, it is said to have a fractional thread. In any screw divide any number of turns by

the number of inches occupied by these turns and the quotient will be the turns to an inch. Thus, a screw that turns 96 times in 12.005 inches, turns 96 ÷ 12.005, or 7.9967 turns in one inch.

EXAMPLES

1. How many turns to the inch has (*a*) a screw that advances 25 inches in 200 turns? (*b*) a single-threaded screw of 9 threads to the inch? (*c*) a double-threaded screw with 8 threads to the inch? (*d*) a double-threaded screw with 6 threads to the inch? (*e*) a single-threaded screw with 24 threads to the inch?

2. What is the lead of a single-threaded screw if it has (*a*) five threads to the inch? (*b*) eight threads to the inch? (*c*) fifteen threads to the inch? (*d*) twenty-eight threads to the inch?

3. What is the lead of a double-threaded screw if it has eight threads to the inch?

4. What is the lead of a single-threaded screw if it has twenty-two threads to the inch?

5. What is the lead of a single-threaded screw if it has twenty-four threads to the inch?

6. A jackscrew has three threads to the inch; how far does it move in $\frac{1}{4}$ of a revolution?

7. A jackscrew has three threads to the inch; how far does it move in $\frac{1}{2}$ of a revolution?

8. What is the thread pitch of a single-threaded screw that has 18 threads to the inch?

9. What is the thread pitch of a double-threaded screw that has 8 threads to the inch?

10. Let each pupil have five different kinds of screws, and tell the number of threads to the inch of each screw.

The Micrometer

Accurate mathematical work in measuring diameter is done with the *micrometer caliper.* With this instrument thousandths of an inch may easily be found. The micrometer is easily adjusted, finely graduated, and has stamped on its yoke the fractions and decimal equivalents which may be needed in close measuring. The parts of the micrometer best known are the *screw,* the *hub,* the *thimble.*

MICROMETER CALIPER

The screw of the micrometer is covered by the thimble to protect it from dust and wear. By turning the thimble we move the screw back and forward, increasing or decreasing the distance between the measuring points of the micrometer and so opening or closing the instrument for larger or smaller diameters. One complete revolution of the thimble changes the opening of the caliper .025, and as the pitch of the screw in the caliper is 40 per inch and the circumference of the thimble graduated into 25ths, the turn from one of these to the next makes the caliper opening .001.

The heel is graduated in a straight line parallel with the screw length and conforms to the pitch of the screw, each division being .025 inch, and the fourth division, which is .100, is made on the frame with the figure 1, the eighth, with 2, etc. When the thimble is turned one complete revolution,

the screw advances one fortieth of an inch and one twenty-fifth of one fortieth is .001. In using the micrometer care must be taken to get the proper touch with the instrument or it may be crowded over with an error of one half thousandth more or less than the actual size of the work required. The micrometer is a very delicate instrument and must be kept away from excessive heat or cold as expansion or contraction of the metal will cause it to become inaccurate. In close work the heat of the hand, when it is held too long, will change a micrometer reading.

EXAMPLES

1. If the screw of a micrometer has 40 threads to the inch, how far will it move in (*a*) one complete revolution; (*b*) $\frac{1}{2}$ revolution; (*c*) $\frac{1}{4}$ revolution; (*d*) $\frac{3}{4}$ revolution; (*e*) $\frac{1}{10}$ of a revolution?

2. What is the lead of the screw in the above micrometer?

3. If the screw of a micrometer has 60 threads to the inch, what is the lead?

4. In example 3, how far will it advance in (*a*) $\frac{1}{2}$ revolution; (*b*) $\frac{1}{4}$ revolution; (*c*) $\frac{3}{4}$ revolution?

V-Shaped Thread

The common V-shaped thread is a thread having its sides at an angle of 60 degrees to each other, and perfectly sharp, top and bottom. The objections to using this thread are that the top is so sharp that it is injured by the slightest accident and in using the taps and dies in making it the fine, sharp edge is quickly lost, causing constant variation in fitting.

Formula:

$$P = \text{Pitch} = \frac{1}{\text{No. of threads per inch}}$$

$$D = \text{Depth} = P \times .8660$$

The diameter of the root (effective diameter) of the thread is found by multiplying the product above by 2, and then subtracting this double depth from the diameter of the screw.

A formula is used to find the size of a tap drill to use in connection with a tap of a given size which has a given number of threads per inch, as:

Let T = diameter of the tap, or size of the thread the nut is to fit;

N = number of threads per inch;

S = size at root of the thread, or size of the tap drill.

$$S = T - \frac{1.733}{N}$$

EXAMPLE. — What must the size be of a tap drill for a 1-inch V-thread tap or 1-inch bolt having 8 threads per inch?

According to the formula:

$$S = 1 - \frac{1733}{8} \text{ or } S = 1 - .216 \text{ or } S = .784 \text{ inch, } Ans.$$

By referring to the Table of Decimal Equivalents[1] the drill nearest in size to .784 is $\frac{25}{32}$ inch, which will cut a trifle larger and therefore will be right.

United States Standard Thread

The United States Standard Thread has its sides also at an angle of 60 degrees to each other, but the top is cut off to the extent of one eighth of its pitch and the same quantity filled in at the bottom. The advantages claimed for this thread are that it is not so easily injured, that the taps and dies retain their size longer, and that the bolts and screws made with this thread are stronger and have a better appearance. This system has been recommended by the Franklin Institute of Philadelphia and it is often called the Franklin Institute Standard. Although the V-shaped thread is the strongest

[1] See Appendix for Table of Decimal Equivalents.

form of screw thread, yet as the thrust between the screw and the nut is parallel to the axis of the screw, there is a tendency to burst the nut. So this form of thread is unsuitable for transmitting power.

As $\frac{1}{8}$ of the height of the U. S. standard thread is taken from the top and $\frac{1}{8}$ from the bottom, the thread is only $\frac{3}{4}$ as deep as the V-form, so in the formula for finding the diameter at the root of the thread we use a numerator which is but $\frac{3}{4}$ of the numerator used for the V-thread: $\frac{3}{4}$ of 1.733 is 1.3. So the formula is

$$S = T - \frac{1.3}{N}$$

Example. — What should the size of a tap drill be for a one-inch U. S. S. tap?

As the U. S. standard is not only a thread of a certain form but also of a given pitch for each diameter of screw, by referring to the table of United States Standard Screw Threads we find that one-inch screws have eight threads to the inch.

$$S = 1 - \frac{1.3}{8} \text{ or } S = 1 - .1625 \text{ or } S = .8375$$

In the Table of Decimal Equivalents .8375 has no common fraction equivalent among the sizes given, but as $\frac{27}{32}$ is only .006 larger we would select a drill of that size.

Formula: $P = \text{Pitch} = \dfrac{1}{\text{No. of threads per inch}}$

$D = \text{Depth} = P \times .6495$

$F = \text{Flat} = \dfrac{P}{8}$

EXAMPLES

Solve the following examples by the use of the table.

1. How many threads are there per inch of the U. S. S. screw with (*a*) $\frac{3}{8}''$ diameter? (*b*) $\frac{7}{8}''$ diameter?

2. What is the diameter of a screw with the U. S. S. thread with (*a*) 5 threads per inch? (*b*) $4\frac{1}{2}$ threads per inch? (*c*) 13 threads per inch?

TABLE OF UNITED STATES STANDARD SCREW THREADS

DIAM. OF SCREW	THREADS PER INCH	DIAM. OF SCREW	THREADS PER INCH
$\frac{1}{4}$ in.	20	$1\frac{3}{8}$ in.	6
$\frac{5}{16}$ in.	18	$1\frac{1}{2}$ in.	6
$\frac{3}{8}$ in.	16	$1\frac{5}{8}$ in.	$5\frac{1}{2}$
$\frac{7}{16}$ in.	14	$1\frac{3}{4}$ in.	5
$\frac{1}{2}$ in.	13	$1\frac{7}{8}$ in.	5
$\frac{9}{16}$ in.	12	2 in.	$4\frac{1}{2}$
$\frac{5}{8}$ in.	11	$2\frac{1}{4}$ in.	$4\frac{1}{2}$
$\frac{3}{4}$ in.	10	$2\frac{1}{2}$ in.	4
$\frac{7}{8}$ in.	9	$2\frac{3}{4}$ in.	4
1 in.	8	3 in.	$3\frac{1}{2}$
$1\frac{1}{8}$ in.	7		
$1\frac{1}{4}$ in.	7		

Acme Standard Thread

The Acme standard thread is an adaptation of the most commonly used worm thread and is intended to take the place of the square thread. It is more shallow than the worm thread but was the same depth as the square thread and is much stronger than the latter.

The worm thread has sides with an angle of 29°; the top is flat, .335 P, and the bottom is .31 P. The depth is .6866 P, the double depth being 1.3732 P; $d = D - \frac{1.3732''}{N}$, that is, the diameter at the bottom of a worm thread is equal to the diameter of the worm minus 1.3732″ divided by the number of threads to one inch.

Square Thread

A square thread has parallel sides; the thickness of the thread and its depth are each one half the pitch: $d = D - \frac{1}{N}$, that is, the diameter at the bottom of a square thread is equal to the diameter of the screw minus one inch divided by the number of threads to one inch. The thrust on the square-threaded screw is parallel to the axis of the screw; consequently the frictional losses are not so great in this form as in the V form, but it is not so strong in the base as the V thread.

EXAMPLES

1. What is the diameter of the root of a $\frac{3}{8}''$ V-threaded bolt with 20 threads to the inch?

2. What is the diameter of the root of a $\frac{3}{8}''$ U. S. S. threaded bolt with 20 threads to the inch?

3. What is the thread pitch of a V-threaded screw with 20 threads to an inch?

4. What is the thread pitch of an Acme worm screw with 20 threads to an inch?

5. What is the thread pitch of a square-threaded screw with 14 threads to an inch?

6. What is the diameter of the root of a $\frac{7}{16}''$ V-threaded bolt with 18 threads to the inch?

7. What is the diameter of the root of a $\frac{5}{8}''$ V-threaded bolt with 14 threads to the inch?

8. A U. S. S. screw has 20 threads. What is its thread pitch? What is its depth?

9. What is the diameter of the root of a $\frac{3}{8}''$ U. S. S. screw with 20 threads to the inch?

10. What is the thread pitch of a square-threaded screw with 8 threads to the inch?

11. A $2\frac{1}{4}''$ diameter bolt has a diameter of $1.962''$ at the root of the thread. What is the depth of the thread? If the bolt has $4\frac{1}{2}$ threads per inch, what is the pitch of the threads?

12. What is the depth of a U. S. S. threaded screw of 7 threads?

13. What is the diameter of the root of a V-threaded $\frac{3}{4}''$ screw with 11 threads to the inch?

14. What is the depth of a V-threaded screw with 11 threads to the inch?

15. What is the diameter of the root of a U. S. S. threaded bolt of 11 threads?

16. What is the diameter of the root of a U. S. S. $\frac{7}{8}''$ threaded bolt of 8 threads?

17. What is the thread pitch of a U. S. S. threaded screw of 9 threads?

18. What is the depth of a worm-threaded screw with 16 threads to the inch?

19. What is the depth of a square-threaded screw with 8 threads to the inch?

20. What is the depth of a U. S. S. threaded screw with 8 threads to the inch?

21. What is the depth of a U. S. S. $1\frac{1}{8}''$ threaded bolt of 9 threads?

22. What is the diameter of the root of a U. S. S. threaded screw of 9 threads?

23. What is the depth of a V-threaded screw with 14 threads to the inch?

24. What is the thread pitch of a V-threaded screw with 16 threads to the inch?

25. A U. S. S. threaded bolt has 9 threads to the inch. What is its thread pitch?

PART V — SHAFTS, PULLEYS, AND GEARING

CHAPTER IX

SHAFTS AND PULLEYS

IN a machine shop one notices at once the revolution of the shafting. There is one long cylindrical bar called the *main line* attached to the ceiling. The power that drives the machinery is taken from this main line by means of pulleys and belts to smaller shafts called countershafts. The machinery is driven directly from the countershafts while the main line is driven from a motor or engine and flywheel.

Shafts of different sizes are used according to the horse power required. To determine the horse power (H. P.) of a shaft, multiply the speed (revolutions per minute) by the cube of the diameter of the shaft, and divide the product by 84 for a steel shaft or by 160 for an iron shaft, and the quotient is the H. P. (For further discussion of horse power see pages 188 and 225.)

EXAMPLES

1. What is the H.P. of a $2\frac{1}{16}''$ steel shaft having 280 revolutions per minute?

2. What is the H.P. of a $2\frac{1}{4}''$ iron shaft having 245 revolutions per minute?

3. What is the H.P. of a $2\frac{5}{8}''$ steel shaft having 290 revolutions per minute?

4. What is the H.P. of a $2\frac{5}{16}''$ iron shaft having 350 revolutions per minute?

Belting

Belts for transmitting power are divided into two general classes: leather belts and canvas belts. Both are sold by the foot. The material of the belt, the thickness, and width determine its value. Coils of belting need not be stretched out to measure their length (L). To do this first count the number of coils (N), measure the diameter of the hole in the center of the coil (d), and the outside diameter of the roll (D).

Then $$L = 0.1309\,N(D+d)$$

In this formula L = length in feet, and D and d diameter in inches.

This rule is used in estimating.

The formula is obtained as follows:

$$\frac{D+d}{2} = \textit{average} \text{ diameter of coil}$$

The length L = circumference of average diameter

$$C = \pi\frac{D+d}{2}$$

$$L = \frac{\pi\frac{D+d}{2}N}{12}$$

$$\frac{\pi}{24} = 0.1309$$

Substituting this in the formula $\frac{\pi N(D+d)}{24} = 0.1309\,N(D+d)$

EXAMPLES

1. How many feet of belting are there in a coil that has a diameter of 18″, if the hole is 3″ in diameter and there are 36 coils?

2. How many feet of belting are in a coil that has a 16″ diameter, if the hole is $2\frac{1}{2}$″ in diameter and there are 38 coils?

Length of Belting on Pulleys

To find the length of belting on pulleys, add together the diameters of the pulleys in inches and divide the sum by 2. Multiply this quotient by 3.25. Add this product to twice the distance in inches between the centers of the pulleys, and divide by 12. The final quotient is the length in feet of the belting on the pulleys. This is an approximation and applies to open belts only.

EXAMPLE.—The diameters of two pulleys are 24″ and 12″ respectively, and the distance between their centers is 108 inches. Find the length of the belting.

$24'' + 12'' = 36''$	$18'' \times 3.25 = 58.50''$	$\frac{274.5''}{12} = 22.8$ feet
$36'' \div 2 = 18''$	$58.5'' + 216'' = 274.5''$	$22.8' = 22'\ 11''$. *Ans.*

EXAMPLES

1. What is the length of the belting connecting two pulleys having diameters of 18″ and 12″ respectively, if the distance between their centers is 92″?

2. What is the length of the belting on two pulleys having diameters of 16″ and 22″ respectively, if the distance between their centers is 86″?

3. Find how many feet of belting are needed to make a belt to run over two pulleys each 30″ in diameter if the distance between their centers is 13′.

Arc of Contact

In setting up machinery where there are pulleys, it is sometimes desirable to find the arc of contact on the smaller pulley. The Boston Belting Company gives this rule, which holds when the pulleys are nearly of the same diameter: Divide the difference between the diameters of the two pulleys by the distance between the centers of the shafts, both being in the same

denomination; multiply the quotient by 57, and subtract this product from 180; the result will be the number of degrees in the arc of contact.

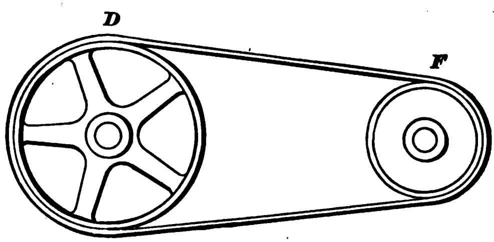
Multiply the entire circumference of the smaller pulley in feet by the degrees of the arc of contact as above, divide by 360, and the result will be the number in feet of the arc of contact of the belt on the smaller pulley.

EXAMPLES

1. Two pulleys, one 18″ and the other 24″ in diameter, are connected by a belt. If the distance between the centers of the shafts is 8′ 6″, what is the number of feet in the arc of contact of the belt on the smaller pulley?

2. Find the number of lineal inches that a belt touches a pulley when the arc of contact is 240°, if the diameter of the pulley is 4 feet.

3. If the distance between the centers of two shafts is 9′ 8″ and the pulleys are 22″ and 16″, what is the number of feet of arc of contact of the belt on the smaller pulley?



A common way for one shaft to drive another is by means of a belt running upon two pulleys, one on the driving shaft and the other on the driven, as in the above figure. In solving problems the pulley on the driving shaft is called the *driver* and the one on the driven shaft the *driven.*

In order to install machinery and have it run at the proper speed, the relations between the driving and the driven pulleys

and the different methods of transmitting power from one shaft to another must be thoroughly understood. To make the shafts and pulleys run at the proper speed the correct diameter and circumference of the driving and the driven pulleys must be known.

At every revolution of the driver the belt is pulled through a distance equal to the circumference of the driver; in moving a distance equal to the circumference of the driven pulley, the belt turns the driven pulley one revolution. When two pulleys are connected by a belt, their rim speeds are equal. If we divide the distance the belt has moved by the circumference of the pulley, the quotient gives the number of revolutions of the pulley. The smaller pulley revolves at the higher speed, a fact that is usually stated mathematically by saying that the revolutions of the pulleys are inversely proportionate to their circumferences.

Example. — A pulley 12 inches in diameter makes 300 revolutions per minute. How fast is the rim traveling in feet per minute?

The circumference equals the diameter multiplied by 3.1416, or approximately $3\frac{1}{7}$.

$$12 \times 3.1416 = 37.6992 \text{ inches circumference}$$

$$\frac{12 \times 3.1416}{12} = 3.1416 \text{ feet}$$

Since it is running 300 revolutions per minute,

$$300 \times 3.1416 = 942.48 \text{ feet per minute}$$

It is possible to express the operations of the above in a formula by letters:

Let D = diameter of the pulley in inches
C = circumference of the pulley in inches
R = revolutions per minute (abbreviated R. P. M.)
$\pi = 3.1416$
F = feet per minute that the rim travels (circumference speed)

Feet traveled per minute is equal to the circumference in inches, multiplied by revolutions per minute, and divided by 12. That is, $F = \frac{CR}{12}$, or substituting for C its equal πD,

$$F = \frac{\pi DR}{12}$$

If we know the value of any two of the quantities F, D, or R, we can find the others:

(1) $F = \frac{\pi DR}{12}$ (2) $D = \frac{12F}{\pi R}$ (3) $R = \frac{12F}{\pi D}$

EXAMPLES

1. If a ten-inch pulley is making 300 revolutions per minute, how fast is a point on the rim traveling in feet per minute?

2. The rim of a 14″ pulley is running 1048 feet per minute. How many revolutions are made per minute?

3. A pulley 18″ in diameter makes 375 revolutions per minute. How fast is the rim traveling in feet per minute?

4. What is the diameter of a pulley making 286 revolutions per minute if a point on the rim is traveling 984 feet per minute?

5. A 9″ pulley is making 198 revolutions per minute. How fast is a point on the rim traveling?

6. A pulley 32″ in diameter is making 198 revolutions per minute. How fast is the rim traveling?

Speed

The speed of any machine from the driving shaft or motor may be traced as follows:

The circumference of a pulley equals its diameter multiplied by 3.1416.

This may be abbreviated when C stands for the circumference and D for the diameter:

$$C = 3.1416\ D \text{ or } \pi D$$

Let C = circumference of driver pulley
C' = circumference of driven pulley
D = diameter of driver pulley
D' = diameter of driven pulley
N = revolutions of driver pulley per minute
N' = revolutions of driven pulley per minute

(C' is read C prime.)

$C \times N$ = distance belt moves per minute on driver wheel
$C' \times N'$ = distance belt moves per minute on driven wheel

The distance represented by $C \times N$ is called rim speed of driver wheel.

The distance represented by $C' \times N'$ is called rim speed of driven wheel.

Since the surface speeds are equal,

$$(1)\quad C \times N = C' \times N', \qquad (2)\quad \frac{C}{C'} = \frac{N'}{N},$$

Since $C = \pi D$, $C' = \pi D'$ or $\dfrac{D}{D'} = \dfrac{N'}{N}$ $\qquad D = \dfrac{D'N'}{N}$

If we know any three of the above four quantities, the fourth can be found:

If $\qquad D = \dfrac{D'N'}{N} \qquad N = \dfrac{D'N'}{D}$

The proportion may be easily remembered by noting that the primes come together as middle terms:

$$D : D' :: N' : N$$

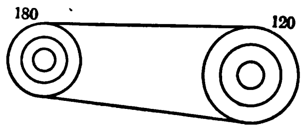
The above formula may be expressed in the form of rules: Whenever one pulley drives another we have four quantities to consider — the diameters of the two pulleys and the revo-

lutions per minute of the two pulleys. The above equation shows us that there is a definite relation between them. If we know any three of the quantities, we may find the fourth by transferring the factors of the equation.

NOTE. — It is often difficult to remember these rules. In that case draw a sketch and place the given information about each pulley. Let x represent the unknown quantity. Then multiply the two numbers known about one pulley and divide by the number, given in the other pulley. The quotient will represent x.

EXAMPLE. — Find the size of a pulley on a countershaft that runs 120 revolutions per minute, if the diameter of the pulley on line shaft is 18″ and runs 180 revolutions per minute.

$$D : D' :: N' : N$$
$$18 : D' :: 120 : 180$$
$$120\,D' = 18 \times 180$$
$$2\,D' = 54$$
$$D' = 27$$



EXAMPLES

1. The diameter of a pulley on the line shaft is 30″ and it runs 158 revolutions per minute; the countershaft runs 400 revolutions per minute. What is the size of the pulley on the countershaft?

2. What is the size of a pulley on a countershaft of an engine lathe if the diameter of the pulley on the line shaft is 15″ and it runs 150 revolutions per minute while the countershaft runs 140 revolutions per minute?

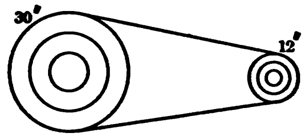
3. What is the size of a pulley on a countershaft of a planer if the diameter of the pulley on line shaft is 26″ and it runs 305 revolutions per minute while the countershaft runs 793 revolutions per minute?

4. What size pulley should be placed on the countershaft of a band saw if the diameter of the pulley on the main line shaft is 14″, if it runs 250 revolutions per minute, and the countershaft runs 225 revolutions per minute?

5. A pulley 30″ in diameter on a main shaft running 180 revolutions per minute is required to drive a countershaft 450 revolutions per minute. What will be the diameter of the pulley on the countershaft?

EXAMPLE. — Find the number of revolutions that a countershaft is running if the line shaft runs 140 revolutions per minute, and the diameter of the pulley on it is 30″ and the diameter of the pulley on the countershaft 12″.

$$D : D' :: N' : N$$
$$30 : 12 :: N' : 140$$
$$2N' = 700$$
$$N' = 350. \quad \textit{Ans.}$$



EXAMPLES

1. Find the number of revolutions per minute of a countershaft of an engine lathe if driven by an 18″ pulley on the line shaft which runs 168 revolutions per minute, if the diameter of the countershaft pulley of lathe is 12″.

2. What is the speed of a countershaft of a speed lathe, if the pulley on the main shaft is 10″ with 305 revolutions per minute and the diameter of the pulley on the countershaft is 4″?

3. Find the number of revolutions of a countershaft of a band saw if the pulley on the main shaft is 19″ with 215 revolutions per minute and the diameter of the pulley on the countershaft is 16″.

4. What is the speed of the countershaft of a cutting-off saw if the pulley on the main shaft is 15″ with 230 revolutions per minute and the diameter of the pulley on the countershaft is 12″?

5. A pulley 30″ in diameter making 180 revolutions per minute drives a countershaft with a 12″ pulley. What is the speed of the countershaft?

6. The main driving pulley of an engine is 12′ 6″ in diameter and makes 96 revolutions per minute; it is belted to a 48″ pulley on the main shaft. Find the speed of the latter.

EXAMPLE. — The line shaft of a machine shop runs 120 revolutions per minute, the diameter of the pulley on the countershaft is 15″, and the countershaft runs 240 revolutions per minute. Find the size of the pulley on the main line.

$$D : D' :: N' : N \qquad \frac{D}{D'} = \frac{N'}{N}$$

$$D : 15'' :: 240 : 120$$

$$120\,D = 15'' \times 240$$

$$D = 30''. \quad \textit{Ans.} \qquad N'D' = ND$$

EXAMPLES

1. The main line runs 160 revolutions per minute; the countershaft pulley is 9″ in diameter and runs 320 revolutions per minute. What is the diameter of the pulley on the main line?

2. A pulley 24″ in diameter running 144 revolutions per minute is to drive a shaft 192 revolutions per minute. What must the diameter of the pulley be on the driven shaft?

3. A driving shaft runs 140 revolutions per minute; the driven pulley is 10″ in diameter and is to run 350 revolutions per minute. What must the diameter of the driving pulley be?

4. A countershaft with a 12″ pulley runs 450 revolutions per minute; the revolutions of the main shaft are 180. What size pulley must be used on the main shaft?

5. A main line runs 189 revolutions per minute and the countershaft pulley is 10″ in diameter and runs 385 revolutions per minute. What is the size of the pulley on the main line?

6. A pulley 36″ in diameter running 168 revolutions per minute is to drive a shaft 212 revolutions per minute. What must be the diameter of the pulley on the driven shaft?

7. A driving shaft runs 184 revolutions per minute; the driven pulley is 12″ in diameter and is to run 350 revolutions per minute. What must be the diameter of the driving pulley?

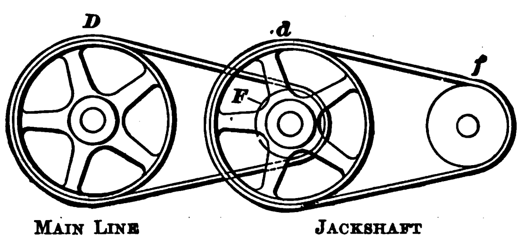
8. What is the speed of the main shaft if the pulley is 12″ and the revolutions per minute on the other shaft are 228 with a pulley 8″?

9. What is the speed of a countershaft if the pulley is 11″ and the main shaft runs 196 revolutions per minute with a 14″ pulley?

10. A 14″ flywheel running 98 revolutions per minute drives a 9″ pulley. What is the speed of the pulley?

Countershafts or Jackshafts

The first shaft belted off from a flywheel is often called a *jackshaft*. In the figure below the jackshaft carries the pulley *d*; on the main line is the large pulley *D* and the small pulley *F* on the jackshaft. On another main line is the pulley *f*. Pulley *D* is the first driver and by means of pulley *F* on the jackshaft drives pulley *d*. Pulley *d* is the second driver. Pulley *F* is the first driven and pulley *f* is the second driven pulley.



EXAMPLE. — The main shaft runs 160 R. P. M. (revolutions per minute); *D* is 60″ in diameter and drives *F* 140 R. P. M. What is the diameter of *F*?

The second main *f* is to run 186 R. P. M. What should be the diameter of *f* if the 72″ driver *d* is running 140 R. P. M.?

An examination of the data in connection with D and f in the figure above will show that the relative speed of f to D is

$$N \times D \times d = N' \times F \times f$$

where N = number of revolutions of driver

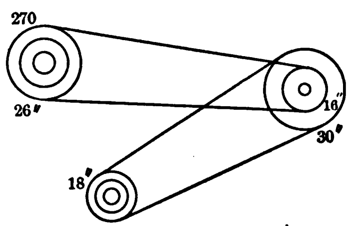
and N' = number of revolutions of driven

That is, the continued product of the speed of the first driver and the diameters of all the drivers is equal to the continued product of the speed of the last driven by the diameters of all the driven pulleys. In this combination of driving f by D there are six quantities, any of which can be found when we know the other five, by figuring from one shaft to the next step by step.

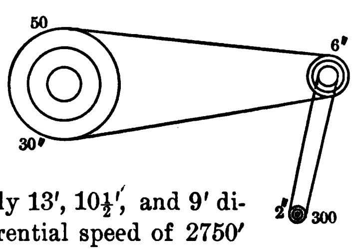
EXAMPLES

1. The R. P. M. of a 26″ driving pulley is 270. What are the revolutions per minute of an 18″ driven pulley?

2. If there is a shaft with a speed of 270 R. P. M. upon which there is a 26″ pulley driving a 16″ pulley, and on the shaft with the 16″ pulley is a 30″ pulley driving an 18″ pulley, what is the speed of the shaft which carries the 18″ pulley?



3. A flywheel which is 30 feet in diameter drives a countershaft by means of a pulley 6 feet in diameter; the flywheel makes 50 R. P. M. What size of pulley must be used on the countershaft to give 300 R. P. M. to a pulley 2 feet in diameter?



4. If 3 flywheels, respectively 13′, $10\frac{1}{2}'$, and 9′ diameter have the same circumferential speed of 2750′ per minute, how many revolutions per minute does each make?

5. A flywheel which is 40 feet in diameter drives a countershaft by means of a pulley $6\frac{1}{2}$ feet in diameter; the flywheel makes 54 R. P. M. What size of pulley must be used on the countershaft to give 341 R. P. M. to a pulley 3 feet in diameter?

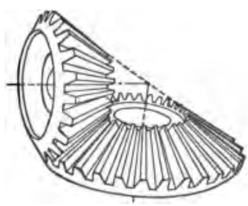
6. The size of a main driving pulley is 20 feet in diameter on a shaft with a speed of 70 R. P. M., driving a pulley 4 feet in diameter and two other pulleys 5.6 and 6.36 feet in diameter, respectively. What is the speed of each shaft?

CHAPTER X

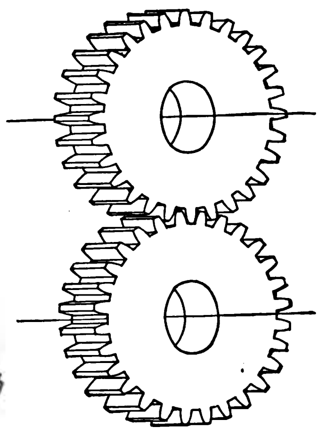
GEARING

In a machine shop power is transmitted from one part of a machine to another part by means of tooth-shaped interlocking wheels. The train of toothed wheels for transmitting motion is called *gearing*.

Gears are nothing but pulleys with teeth, and are made to drive one another by the teeth coming in contact with each other. If a small gear drives a larger gear, the larger gear will go more slowly than the smaller; that is, the larger gear will make fewer turns in a minute. Just the reverse is true if a larger gear drives a smaller one. The number of revolutions which a gear makes is always proportional to the number of its teeth.



Bevel Gear



Spur Gear

As in pulleys, there are the driver and the driven gears. The driver may be distinguished from the driven by examining the gears and noticing that it is the gear that is bright or worn on the front of the tooth—that is, the side of the wheel moving. The driven wheel is worn on the side away from the direction of the motion.

In order to express the relation between the driver and driven gears it is necessary to use abbreviations.

Let D = number of teeth in driver
D' = number of teeth in driven
N = number of revolutions of the driver
N' = number of revolutions of the driven

$$D : D' :: N' : N \text{ or } DN = D'N'$$

That is, the product of the teeth in the driver by its revolutions equals the tooth transits of the driver, which in turn equal the tooth transits of the driven or follower. If any three of these quantities are known, the fourth can be found.

EXAMPLE. — How many revolutions does a 12-tooth follower make to five revolutions of a 24-tooth driver?

$$12\,N = 24 \times 5$$
$$N = 10 \quad \textit{Ans.}$$

EXAMPLES

1. A driver has 98 teeth and its follower 42. How many revolutions will the follower make to one revolution of the driver?

2. In Example 1 how many revolutions of the driver will drive the follower one revolution?

3. How many teeth must a follower have in order to make three revolutions while a 96-tooth driver makes one?

4. How many teeth must a gear have to revolve 16 times while a 60-tooth mate revolves 12 times?

5. A 96-tooth gear drives a 48-tooth gear. What is the ratio of their speeds?

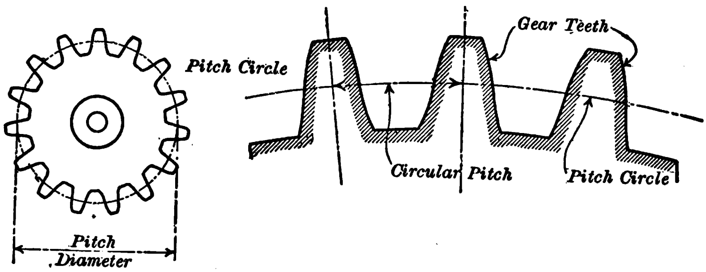
6. A 48-tooth gear drives a 120-tooth gear. What is the ratio of their speeds?

7. Two shafts are connected by gears, one of which turns 55 times a minute and the other 11 times a minute. If the small gear has 32 teeth, how many teeth has the larger gear?

Pitch

In order to solve problems connected with the use of gears it is necessary to know the different terms used in connection with gearing.

The *pitch circle* is shown in the illustration and is the circle which runs around the teeth. It is the same size as the fric-



tion rollers or cylinders would be if no teeth were there. When two spur gears roll together their pitch circles are considered to be always in contact. The *pitch diameter* is the diameter of the pitch circle. The word diameter when applied to gears always means the *pitch diameter.*

The *circular pitch* is the distance measured on the pitch circle from the center line of one tooth to the center of the next. This is illustrated in the diagram. In solving gearing problems the *circular pitch* is not nearly so important as the diametral pitch.

If the distance from the center of a tooth to the center of the next tooth is $\frac{1}{2}''$, the gear is $\frac{1}{2}''$ circular pitch.

The *diametral pitch* is the number of teeth for each inch of pitch diameter.

For example, if a gear has thirty teeth and the pitch diameter is three inches, then the *diametral pitch* is $30 \div 3 = 10$, or a 10 diametral pitch gear. Using P for the diametral pitch, D for the pitch diameter, and N for the number of teeth, we have a formula $P = N \div D$.

To find the *thickness of tooth* at the pitch line when the diametral pitch is given, divide 1.57 by the diametral pitch.

For example, if the diametral pitch is 3, divide 1.57 by 3, and the quotient, which is .523 inches, is the thickness of the tooth.

To find the *circular pitch* when the diametral pitch is given divide 3.1416 by the diametral pitch.

For example, if the diametral pitch is 4, divide 3.1416 by 4 and the quotient, which is .7854, is the *circular pitch*.

Diametral pitch is found when the circular pitch is given by dividing 3.1416 by the circular pitch. Since the circumference of any circle is equal to 3.1416 times its diameter, every inch of diameter of any circle is equal to 3.1416 inches of circumference, or in this instance, for every inch of pitch diameter we have 3.1416 inches of circumference measured on the pitch circle. But the diametral pitch is by definition *the number of teeth for each inch of pitch diameter*, and by the above reasoning we can also say that the diametral pitch is the number of teeth for each 3.1416 inches of circumference of pitch circle.

Since the circular pitch is the distance from the center of one tooth to the center of the next, it will also be equal to 3.1416 of circumference of pitch circle divided by the number of teeth in that 3.1416. But the number of teeth in 3.1416 is equal to the diametral pitch, and therefore the distance from the center of one tooth to the center of the next, or the circular pitch, is equal to 3.1416 divided by the diametral pitch. Therefore, using P' for circular pitch and P for diametral pitch, $P = 3.1416 \div P'$ or $P' = \dfrac{3.1416}{P}$.

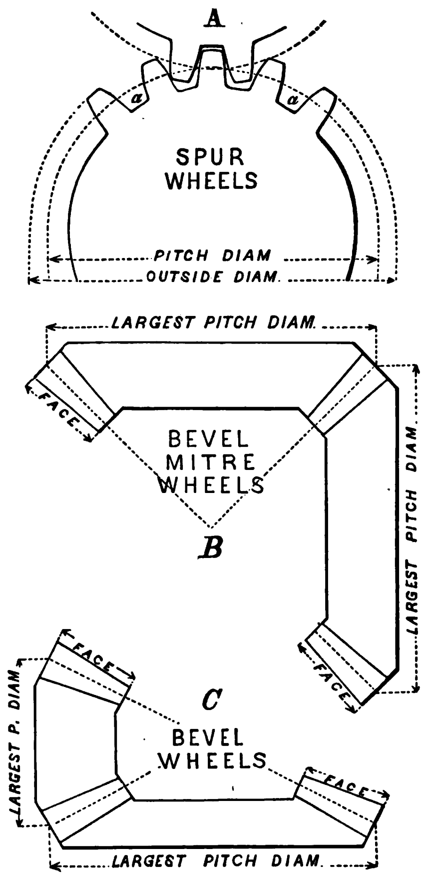
EXAMPLES

1. What is the diametral pitch of a gear having (*a*) 56 teeth and a pitch diameter of 8″? (*b*) 60 teeth and a pitch diameter of 12″?

2. What is the thickness of the tooth of a gear having a diametral pitch of (*a*) 8? (*b*) 4? (*c*) 14?

3. What is the circular pitch if the diametral pitch of a gear is (*a*) 8? (*b*) 10? (*c*) 5?

4. What is the diametral pitch if the circular pitch of a gear is (*a*) 1.5708? (*b*) .5236? (*c*) .2618? (*d*) .7854?



To find the *pitch diameter* when the number of teeth and the diametral pitch are given, divide the number of teeth by the diametral pitch. For example, if the number of teeth is 40 and the diametral pitch is 4, divide 40 by 4, and the quotient, which is 10″, is the pitch diameter.

To make a gear the outside diameter must first be known. If the diametral pitch and the number of teeth are given, the outside diameter may be found by the following rule:

$$D = \frac{N+2}{P}$$

where D = outside diameter, N = number of teeth, and P = diametral pitch.

For example, if a gear has 24 teeth and its diametral pitch is 2, the outside diameter will be 13″; $O = \frac{24+2}{2} = \frac{26}{2} = 13''$; and $P = \frac{N+2}{O}$.

To find the *diametral pitch* when the number of teeth and the diameter of the blank are given, add 2 to the number of teeth and divide by the diameter of the blank.

For example, if the number of teeth is 40 and the diameter of the blank is $10\frac{1}{2}''$, add 2 to the number of teeth, making 42, and divide by $10\frac{1}{2}$, and the quotient, which is 4, is the diametral pitch.

To find the *thickness of tooth* at the pitch line when the circular pitch is given, divide the circular pitch by 2.

For example, if the circular pitch is 1.047″, divide this by 2, and the quotient, which is .523, is the thickness of tooth.

To find the *number of teeth* when the pitch diameter and the diametral pitch are given, multiply the pitch diameter by the diametral pitch.

For example, if the pitch diameter is 10″ and the diametral pitch is 4, multiply 10 by 4, and the product 40 will be the number of teeth in the gear.

To find the *whole depth of tooth* divide 2.157 by the diametral pitch.

For example, if the diametral pitch is 6, divide 2.157 by 6, and the quotient, .3595″, is the whole depth of the tooth.

Formulas for Determining the Dimensions of Gears by Diametral Pitch

P = the *diametral pitch*, or the number of teeth to one inch of diameter of pitch circle.
D' = the diameter of pitch circle.
D = the whole (outside) diameter or the diameter of the blank.
N = the number of teeth.
V = the velocity.
t = the thickness of tooth or cutter on pitch circle.
D'' = the working depth of tooth.

$f =$ the amount added to depth of tooth for rounding the corners and for clearance.

$D'' + f =$ the whole depth of tooth.

π denotes the constant 3.1416.

P' denotes the circular pitch or the distance from the center of one tooth to the center of the next on the pitch circle.

Formulas		*Examples*[1]
$P = \frac{N+2}{D}$	$=$	$\frac{72+2}{6.166}$, or $\frac{72+2}{6\frac{2}{12}} = 12.$
$P = \frac{N}{D'}$	$=$	$\frac{72}{6} = 12.$
$D' = \frac{D \times N}{N+2}$	$=$	$\frac{6.166 \times 72}{72+2} = 6.$
$D' = \frac{N}{P}$	$=$	$\frac{72}{12} = 6.$
$N = PD'$	$=$	$12 \times 6 = 72.$
$N = PD - 2$	$=$	$12 \times 6.166 - 2$, or $12 \times 6\frac{2}{12} - 2 = 72.$
$D = \frac{N+2}{P}$	$=$	$\frac{72+2}{12} = 6.166$, or $6\frac{2}{12}$.
$D = D' + \frac{2}{P}$	$=$	$6 + \frac{2}{12}$, or $6 + .166 = 6.166.$
$t = \frac{1.57}{P}$	$=$	$\frac{1.57}{12} = .130.$
$D'' = \frac{2}{P}$	$=$	$\frac{2}{12} = .166$, or $\frac{2}{12}$.
$f = \frac{t}{10}$	$=$	$\frac{.130}{10} = .013.$
$D'' + f$	$=$	$.166 + .013 = .179.$
$P' = \frac{\pi}{P}$	$=$	$\frac{3.1416}{12} = .262.$
$P = \frac{\pi}{P'}$	$=$	$\frac{3.1416}{.262} = 12.$

[1] The examples placed opposite the formulas above are for a single gear of 12 pitch, 6.166″ or $6\frac{2}{12}$″ diameter.

EXAMPLES

Solve by using the above formulas and the rules on pages 161–165:

1. If the number of teeth of a gear is 46 and the diametral pitch is 8, what will the pitch diameter be?

2. If a gear is to have 54 teeth and the diametral pitch is 6, what will be the pitch diameter?

3. If a gear has 38 teeth and the diametral pitch is 6, what is the pitch diameter?

4. If the pitch diameter of a gear is 6″ and the diametral pitch is 8, what will be the number of teeth?

5. How many teeth has a gear if the pitch diameter is $7\frac{5}{6}$″ and the diametral pitch is 6?

6. How many teeth has a gear if the pitch diameter is $8\frac{1}{4}$″ and the diametral pitch is 4?

7. If the pitch diameter of a gear is 12″ and the diametral pitch 7, how many teeth has the gear?

8. If the diametral pitch is 16 and the pitch diameter of a gear is $6\frac{3}{4}$″, how many teeth has the gear?

9. What is the whole depth of the tooth if the diametral pitch of a gear is (*a*) 8? (*b*) 4? (*c*) 7? (*d*) 10?

10. If a gear has 48 teeth and the diameter of the blank is $6\frac{1}{4}$″, find the diametral pitch.

11. If a gear has 61 teeth and the diameter of the blank is $10\frac{1}{2}$″, find the diametral pitch.

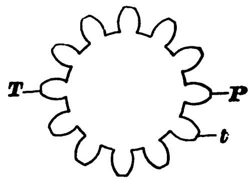
12. If a gear has 35 teeth and the diameter of the blank is $3\frac{1}{12}$″, find the diametral pitch.

13. If a gear has 78 teeth and the diameter of the blank is 5″, find the diametral pitch.

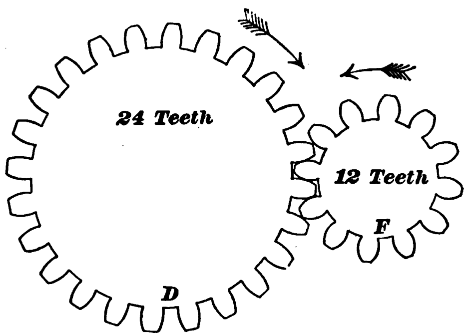
14. If a gear has 54 teeth and the diameter of the blank is $6\frac{2}{9}$″, find the diametral pitch.

15. What is the thickness of the tooth if the circular pitch of a gear is (*a*) .1848″? (*b*) .1428″? (*c*) 6.2832″? (*d*) 2″?

If one examines closely the movements of two gears on a machine, one will notice that all the teeth pass a given point at every revolution of the wheel. That is, the teeth marked *Tt*, on the gear shown in the illustration pass an outside point like *P* in making one complete revolution. So when a tooth and the space adjacent to it have passed a given point like *P*, one transit of a tooth has occurred. There are as many tooth transits at every complete revolution of a gear as there are teeth in the gear.



If we multiply the number of teeth by the number of revolutions, the product will be the number of transits. If this product (number of transits) be divided by the number of teeth, the quotient will be the revolutions of the gear. For example, if a 12-tooth gear makes 60 transits, then it has made ($\frac{60}{12} = 5$) five revolutions.



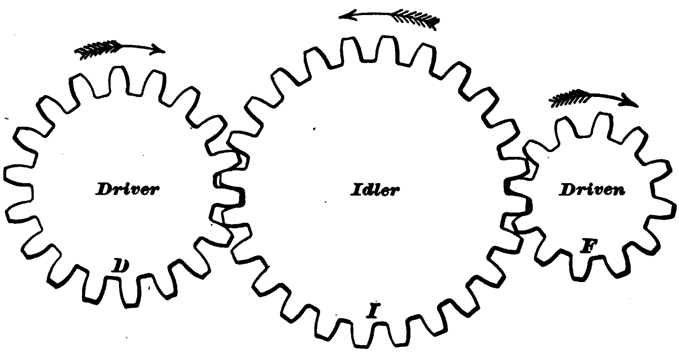
Two shafts, *D* and *F*, are to be connected by gears so that shaft *D* will make one revolution while shaft *F* makes two. To do this a gear must be put on shaft *D* having twice the number of teeth that the gear on shaft *F* has. If a gear having 24 teeth is put on shaft *D*, the gear on shaft *F* will have half as many. Each time the gear on *D* turns around once the

gear on F will turn twice; that is, the 24 teeth on gear D will have to turn gear F twice in order to mesh with the 24 teeth on F.

The relation or ratio of the speed of F to the speed of D is 2 to 1. This is called the ratio of the gearing. The ratios between the speeds and number of teeth can be written in the form of a proportion:

$$24 : 12 :: 2 : 1$$

And the number of teeth on gear D is to the number of teeth on gear F as the speed of F is to the speed of D.



Train of Gears

When two gears mesh, as in the illustration on page 167, one revolves in the opposite direction from the other. Three or more gears running together, as in the illustration above, are often called a *train of gears*. In a train of spur gears like these, one gear, I, which is called an intermediate gear, meshes with the other two gears, D and F, and causes the revolutions of both D and F to be made in one direction, while the intermediate revolves in the opposite direction. The intermediate does not change the relative speeds of D and F, so that they can be figured as explained on page 167. An intermediate gear is also called an *idler*.

We may calculate the number of revolutions of any follower for any number of revolutions of a driver, in a train of gears, step by step. To find the number of revolutions of the last follower when the number of revolutions of the first driver and the teeth in all the gears are known, take the continued product of the revolutions of the first driver and all the driving gears, and divide it by the continued product of all the followers; the quotient is the number of revolutions of the last follower. In other words, the product of the revolutions of the first driver and the teeth of all the driving gears is equal to the continued product of the revolutions of the last follower and the teeth of all the driven gears.

EXAMPLES

1. If a 60-tooth wheel is to mesh with one having 46 teeth and the 60-tooth gear makes 25 revolutions per minute, how many will the 46-tooth gear make?

2. A 168-tooth gear drives a 28-tooth gear. What is the ratio of the gearing?

3. What would be the ratio of the gearing in the example above if the 28-tooth gear were the driver?

4. If the 28-tooth gear is making 48 revolutions per minute, how many revolutions per minute is the 168-tooth gear making?

5. How could the gearing be changed to make the 28-tooth gear turn in the same direction as the 168-tooth gear?

6. Two gears running together have a speed ratio of 7 to 1. If the smaller one turns 14 times, how many times will the larger one turn?

7. If a 144-tooth gear makes one complete turn, how many turns will a 32-tooth gear make working with it?

8. In the above example if the 32-tooth gear turned once, how many turns will the 144-tooth gear make?

9. A train of 3 gears has 69, 30, and 74 teeth. If the 69-tooth gear makes 100 revolutions per minute, how many revolutions per minute will the 74-tooth gear make? Make a sketch showing the direction in which each gear turns.

10. A train of gears is made up of 6 gears having teeth as follows: 46, 60, 32, 72, 56, and 48; while the first gear in the train makes 10 turns, how many turns will the last gear make?

11. What two gears will give a ratio of speeds so that the driver will make $\frac{13}{14}$ as many turns as the follower; that is, while the driver makes 13 turns the follower will make 14?

12. If 24 gears work in a train, in what direction will the last one turn if the first turns right-handed?

REVIEW EXAMPLES

1. Two gears working together have pitch circles $14\frac{1}{2}''$ and $26\frac{7}{8}''$ in diameter. What is the distance between their centers?

The distance between centers may be obtained from the following formulas:

$$a = \frac{D' + d'}{2} : a = \frac{b}{2P}$$

where a = distance between centers
d = diam. of pitch circle of smaller gear
b = number of teeth on both wheels

2. Two gears in mesh have pitch circles 17.082″ and 31.3124″ in diameter. What is their center distance?

3. Two gears working together are four pitch, the larger has 36 teeth and the smaller has 14 teeth. What is their center distance?

4. The circular pitch of a planer table gear is 1″. What should be the total depth to cut the tooth space?

5. The circular pitch of a milling machine gear is $\frac{7}{8}''$ and it has 96 teeth. What is its outside diameter?

6. A flue rattler gear is to be renewed. Its outside diameter is 34″ and it has 100 teeth. What size shall we draw its pitch circle?

7. Two gears mesh together, one has 28 teeth and an outside diameter of $2\frac{1}{2}''$ and the other 68 teeth and an outside diameter of $5\frac{5}{6}''$. Find the center distance.

8. A gear has a circular pitch of $\frac{13}{16}''$. What is the thickness of the tooth at the pitch line?

9. A pitch circle is $18''$ in circumference. If the teeth are $\frac{1}{2}''$ thick, what is the circular pitch?

10. A gear is 10 pitch. What is the total depth of its teeth?

11. The diametral pitch of a gear is 4. What is the circular pitch?

12. The circular pitch of a gear is .157. What is the diametral pitch?

13. A gear is 12 pitch. What is its circular pitch?

14. A gear is 20 pitch. If it has 105 teeth, what is its outside diameter?

15. A gear of 10 pitch has 44 teeth. Find the pitch diameter.

16. A gear has 28 teeth and a pitch diameter of 8. What is the pitch?

17. A gear is set in a milling machine ready to cut the teeth, and if the pitch is to be one, what is the depth of the cut?

18. A gear is 14 pitch and has an outside diameter of $4\frac{1}{7}''$. Find the depth of cut for the teeth.

19. A gear has 75 teeth and is 18 pitch. What is the working depth of the tooth?

20. A pinion has 44 teeth and is 10 pitch. What is the outside diameter?

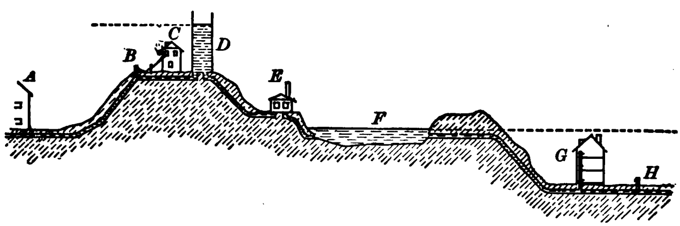
21. A lathe back gear has 108 teeth and is 5 pitch. What is its outside diameter?

PART VI—PLUMBING AND HYDRAULICS

CHAPTER XI

RECTANGULAR AND CYLINDRICAL TANKS

Water Supply.—The question of the water supply of a city or a town is very important. Water is usually obtained from lakes and rivers which drain the surrounding country. If a lake is located in a section of the surrounding country higher than the city (which is often located in a valley), the water may be obtained from the lake, and the pressure of the water in the lake may be sufficient to force it through the pipes



WATER SUPPLY

into the houses. But in most cases a reservoir is built at an elevation as high as the highest portion of the town or city, and the water is pumped into it. Since the reservoir is as high as the highest point of the town, the water will flow from it to any part of the town. If houses are built on the same hill with the reservoir, a stand-pipe, which is a steel tank, is erected on this hill and the water is pumped into it.

Water is conveyed from the reservoir to the house by means of iron pipes of various sizes. It is distributed to the different parts of the house by small lead, iron, or brass pipes. Since water exerts considerable pressure, it is necessary to know how to calculate the exact pressure in order to have pipes of proper size and strength.

EXAMPLES

1. Water is measured by means of a meter. If a water meter measures for five hours 760 cubic feet, how many gallons does it indicate?

NOTE. — 231 cubic inches = 1 gallon.

2. If a water meter registered 1845 cubic feet for 3 days, how many gallons were used?

3. A tank holds exactly 12,852 gallons; what is the capacity of the tank in cubic feet?

4. A tank holds 3841 gallons and measures 4 feet square on the bottom; how high is the tank?

Rectangular Tanks. — To find the contents in gallons of a square or rectangular tank, multiply together the length, breadth, and height in feet; multiply the result by 7.48.

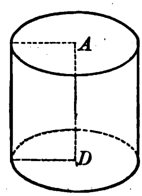
l = length of tank in feet
b = breadth of tank in feet
h = height of tank in feet

Contents = lbh cubic feet × 7.48 = 7.48 lbh gallons

(NOTE. — 1 cu. ft. = 7.48 gallons.)
If the dimensions of the tank are in inches, multiply the length, breadth, and height together, and the result by .004329.

5. Find the contents in gallons of a rectangular tank having inside dimensions (*a*) 12′ × 8′ × 8′; (*b*) 15″ × 11″ × 6″; (*c*) 3′ 4″ × 2′ 8″ × 8″; (*d*) 5′ 8″ × 4′ 3″ × 3′ 5″; (*e*) 3′ 8″ × 3′ 9″ × 2′ 5″.

Cylindrical Tank. — To find the contents of a cylindrical tank, square the diameter in inches, multiply this by the height in inches, and the result by .0034.



d = diameter of cylinder
h = height of cylinder

Contents = d^2h cubic inches × .0034 = d^2h .0034 gallons

6. Find the capacity in gallons of a cylindrical tank (*a*) 14″ in diameter and 8′ high; (*b*) 6″ in diameter and 5′ high;

(*c*) 15″ in diameter and 4′ high; (*d*) 1′ 8″ in diameter and 5′ 4″ high; (*e*) 2′ 2″ in diameter and 6′ 7″ high.

Inside Area of Tanks. — To find the area, for lining purposes, of a square or rectangular tank, add together the widths of the four sides of the tank, and multiply the result by the height. Then add to the above the area of the bottom. Since the top is usually open, we do not line it. In the following problems find the area of the sides and bottom.

7. Find the amount of zinc necessary to line a tank whose inside dimensions are 2′ 4″ × 10″ × 10″.

8. Find the amount of copper necessary to line a tank whose inside dimensions are 1′ 9″ × 11″ × 10″, no allowance made for overlapping.

9. Find the amount of copper necessary to line a tank whose inside dimensions are 3′ 4″ × 1′ 2″ × 11″, no allowance for overlapping.

10. Find the amount of zinc necessary to line a tank 2′ 11″ × 1′ 4″ × 10″.

Drainage Pipes

In order to have pipes of a proper size to carry away the water from the roof of a building, it is necessary to know the number of gallons of water that will be drained. To find the number of gallons that will drain from a roof in a month, multiply the number of square feet of the roof by the average number of inches of rainfall per month, and multiply this product by .623. When the roof is not flat, or very nearly so, its area should be considered as the area which it actually covers.

EXAMPLES

11. A roof is 48′ by 62′. How many gallons of water will drain from it in a month if the rainfall is 6″?

12. A roof is 29′ by 74′. How many gallons of water will drain from it if the rainfall is $4\frac{1}{4}$″?

NOTE. — The rainfall per month in Massachusetts varies from $\frac{1}{4}''$ to $12''$. In calculating the following problems consider $10''$ rainfall as the average amount.

13. Find the number of gallons that will drain from a flat roof (*a*) $112'$ by $64'$; (*b*) $88'$ by $49'$; (*c*) $120'$ by $80'$.

14. Find the number of gallons that will drain from a slanting roof each half of which is (*a*) $52'$ by $34'$; (*b*) $49'$ by $28'$; (*c*) $112'$ by $54'$; (*d*) $57'$ by $33'$; (*e*) $54'$ by $31'$.

Weight of Lead Pipe

Pipes, particularly those of lead, are sold by weight, and this depends upon the diameter and thickness of the metal in the pipe.

To find the weight of a length of pipe, subtract the square of the inner diameter in inches from the square of the outer diameter in inches, and multiply the remainder by the weight of 12 cylindrical inches. This product multiplied by the length in feet gives the required weight.

EXAMPLE. — What is the weight of 1450 feet of lead pipe, the outer diameter being $\frac{7}{8}''$ and the inner diameter being $\frac{7}{16}''$?

D = outer diameter
d = inner diameter
l = length of pipe

3.8697 lb. = weight of 12 cylindrical inches of lead pipe whose length is $1'$ and whose diameter is $1''$.

$$\text{Weight in lb.} = (D^2 - d^2) \times 3.8697 \times l$$

Then
$$\frac{(7)^2}{(8)^2} = \frac{7 \times 7}{8 \times 8} = \frac{49}{64} = .765625 \text{ (square of outer diameter)}$$

$$\frac{(7)^2}{(16)^2} = \frac{7 \times 7}{16 \times 16} = \frac{49}{256} = .191406 \text{ (square of inner diameter)}$$

$$.765625 - .191406 = .574219 \text{ (difference)}$$

$$.574219 \times 3.8697 \times 1450 = 3221.98 \text{ lb.}$$

EXAMPLES

1. What is the weight of $364'$ of lead pipe, outer diameter $\frac{3}{4}''$, inner diameter $\frac{5}{16}''$?

2. What is the weight of 1189′ of lead pipe, outer diameter $1\frac{3}{4}''$, inner diameter $1\frac{5}{16}''$?

3. What is the weight of 2189′ of lead pipe, outer diameter $2\frac{1}{2}''$, inner diameter $2\frac{1}{16}''$?

4. What is the weight of 112′ of lead pipe, outer diameter $3''$, inner diameter $2\frac{9}{16}''$?

5. What is the weight of 212′ of lead pipe, outer diameter $3\frac{1}{2}''$, inner diameter $3\frac{1}{16}''$?

Capacity of Pipe

Rules for Finding the Capacity in Gallons of a Foot of Pipe of any Diameter:

1. Find the cubical contents in inches, and divide by 231.

$$C = D^2 \times .7854 \times 12 \div 231$$

2. Multiply the square of the inside diameter by .0408.

$$C = D^2 \times .0408$$

Rules for Finding the Capacity of a Pipe of any Length and any Diameter:

1. Multiply the number of gallons in a foot of the pipe by the number of feet in the length of the pipe.

$$C = D^2 \times .0408 \times L$$

2. Find the cubical contents in inches, and divide by 231.

$$C = D^2 \times .7854 \times L \div 231$$

NOTE. — In the above formulas, let C = the capacity of the pipe in gallons, D = the diameter in inches, and L = the length in feet.

EXAMPLES

1. What is the capacity of a pipe having an inside diameter of (*a*) $\frac{3}{4}$ inch, 16 feet in length; (*b*) $1\frac{1}{4}$ inches, 20 feet in length; (*c*) $1\frac{1}{2}$ inches, 1 foot in length; (*d*) $3\frac{1}{2}$ inches, 1 foot in length; (*e*) 14 inches, 10 feet in length; (*f*) 5 inches, 22 feet in length?

Number of U. S. Gallons in Round Tanks

FOR ONE FOOT IN DEPTH

Dia. of Tanks Ft. In.	No. U. S. Gals.	Cubic Ft. and Area in Sq. Ft.	Dia. of Tanks Ft. In.	No. U. S. Gals.	Cubic Ft. and Area in Sq. Ft.	Dia. of Tanks Ft. In.	No. U. S. Gals.	Cubic Ft. and Area in Sq. Ft.
1	5.87	.785	3	52.88	7.069	5	146.88	19.63
1-1	6.89	.922	3-1	55.86	7.467	5-1	151.82	20.29
1-2	8.	1.069	3-2	58.92	7.876	5-2	156.83	20.97
1-3	9.18	1.227	3-3	62.06	8.296	5-3	161.93	21.65
1-4	10.44	1.396	3-4	65.28	8.727	5-4	167.12	22.34
1-5	11.79	1.576	3-5	68.58	9.168	5-5	172.38	23.04
1-6	13.22	1.767	3-6	71.97	9.621	5-6	177.72	23.76
1-7	14.73	1.969	3-7	75.44	10.085	5-7	183.15	24.48
1-8	16.32	2.182	3-8	78.99	10.559	5-8	188.66	25.22
1-9	17.99	2.405	3-9	82.62	11.045	5-9	194.25	25.97
1-10	19.75	2.640	3-10	86.33	11.541	5-10	199.92	26.73
1-11	21.58	2.885	3-11	90.13	12.048	5-11	205.67	27.49
2	23.50	3.142	4	94.	12.566	6	211.51	28.27
2-1	25.50	3.409	4-1	97.96	13.095	6-3	229.50	30.68
2-2	27.58	3.687	4-2	102.	13.635	6-6	248.23	33.18
2-3	29.74	3.976	4-3	106.12	14.186	6-9	267.69	35.78
2-4	31.99	4.276	4-4	110.32	14.748	7	287 88	38.48
2-5	34.31	4.587	4-5	114.61	15.321	7-3	308.81	41.28
2-6	36.72	4.909	4-6	118.97	15.90	7-6	330.48	44.18
2-7	39.21	5.241	4-7	123.42	16.50	7-9	352.88	47.17
2-8	41.78	5.585	4-8	127.95	17.10	8	376.01	50.27
2-9	44.43	5.940	4-9	132.56	17.72	8-3	399.88	53.46
2-10	47.16	6.305	4-10	137.25	18.35	8-6	424.48	56.75
2-11	49.98	6.681	4-11	142.02	18.99	8-9	449.82	60.13

31½ *gallons equal* 1 *barrel.*

To find the capacity of tanks greater than the largest given in the table, look in the table for a tank one half of the given size and multiply its capacity by 4, or one of one third its size and multiply its capacity by 9, etc.

EXAMPLES

Solve the following problems by use of tables:

1. Find the number of gallons contained in a round tank, 1′ 8″ in diameter and 1′ deep.

2. Find the number of gallons contained in a round tank, 4′ 11″ in diameter and 3′ deep.

3. Find the number of gallons contained in a round tank, 9′ 3″ in diameter and 10′ deep.

4. Find the number of barrels contained in a round tank, 2′ 4″ in diameter and 2′ deep.

5. Find the number of barrels contained in a round tank, 12′ 6″ in diameter and 4′ deep.

REVIEW PROBLEMS

1. A three-fourths-inch pipe has an actual internal diameter of .824 inch; what is the nearest rule measure?

2. A two-inch pipe is .154 inch in metal thickness; what is the nearest rule measure?

3. A one-inch pipe one foot long holds .373 pound of water; how long should it be to hold 5 pounds?

4. A five-inch pipe one foot long holds 1.038 gallons; how many gallons will a pipe 277 feet long hold?

5. If a five-inch steam pipe has an actual inside diameter of 5.045 inches and an actual outside diameter of 5.565 inches, what is the thickness of the pipe?

6. The inside diameter of a cylinder is 22.625 inches before boring and 22.875 inches after boring; how much larger is the opening after boring?

7. What must be the height of a rectangular tank, with base 12″ by 12″, to hold 60 gallons?

8. What is the height of a rectangular tank with a base 12″ × 14″, which will hold 189 gallons?

9. What size cylindrical tank, with a base of 40 square inches, will hold 89 gallons?

10. What size cylindrical tank, with a base of 60 square inches, will hold 42 gallons?

11. What size cylindrical tank, with a height of 8 feet, will hold 24 gallons?

12. What size cylindrical tank, with a height of 6 feet 4 inches, will hold 52 gallons?

13. What size cylindrical tank, with a base of 14 square inches, will hold 35 gallons?

14. What size rectangular tank, with a base 14 inches × 7 inches, will hold 14 gallons?

Capacity of Pipes

Law of Squares. — The areas of similar figures vary as the squares of their corresponding dimensions.

Pipes are cylindrical in shape and are, therefore, similar figures. The areas of any two pipes are to each other as the squares of the diameters.

EXAMPLE. — If one pipe is 4″ in diameter and another is 2″ in diameter, their ratio is $\frac{16}{4}$, and the area of the larger one is, therefore, 4 times the smaller one.

EXAMPLES

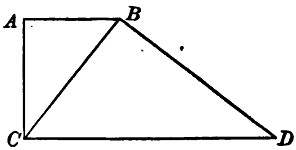
1. How much larger is a section of 5″ pipe than a section of 2″ pipe?

2. How much larger is a section of $2\frac{1}{2}$″ pipe than a section of 1″ pipe?

3. How much larger is a section of 5″ pipe than a section of 3″ pipe?

Rule of Thumb Method. — This method is used for finding the size of a pipe necessary to fill a number of smaller pipes, and is explained as follows:

To find the diameter of a pipe which will fill three pipes, having diameters of 2″, 2½″, and 4″, respectively. Draw a right angle, one arm (AB) being 2″ in length, the other arm (AC) being 2½″ in length. Connect the points B and C. The length of this line (BC) will give the size of a pipe necessary to fill the two smaller pipes (about 3¼″). From one end of this last line (BC) and at right angles to it draw another line (BD) 4″ in length. Connect the points C and D. The length of this line (CD) will represent the size of a pipe necessary to fill the three pipes, having diameters of 2″, 2½″, and 4″, respectively.



This process may be continued for as many pipes as desired.

The carrying capacity of circular pipes for conveying liquids depends upon the area of the opening of the pipe. Since the areas of pipes are proportional to the squares of the diameters squared, the principle of the right triangle may be used for determining an area equivalent to two or more areas, or a pipe equivalent to three or more pipes.

Geometric Proof. — (See above figure):

In the rt. $\triangle ABC$

$$\overline{CB}^2 = \overline{AB}^2 + \overline{AC}^2$$

In the rt. $\triangle CBD$

$$\overline{CD}^2 = \overline{CB}^2 + \overline{BD}^2$$

$$CD = \sqrt{\overline{CB}^2 + \overline{BD}^2}$$

Let

$$AB = 2''$$

$$AC = 2\tfrac{1}{2}''$$

$$BD = 4''$$

$$\overline{CB}^2 = 4'' + \tfrac{25}{4}'' = \tfrac{41}{4}''$$

$$CB = \sqrt{\tfrac{41}{4}}'' = \tfrac{6.4}{2}'' = 3.2''$$

Substituting the values of AB, AC, and BC in the above equations:

$$\overline{CB}^2 = (2)^2 + (2\tfrac{1}{2})^2 = \tfrac{41}{4}$$

$$\overline{CD}^2 = \tfrac{41}{4} + 4^2 = \tfrac{41}{4} + 16 = \tfrac{105}{4}''$$

$$CD = \tfrac{10.24}{2}'' = 5.12''$$

EXAMPLES

1. Find the size of pipe necessary to fill a $1\frac{1}{2}''$, $2''$, and $3''$ pipe.

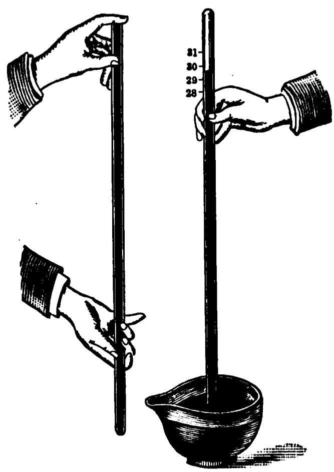
2. Find the size of pipe necessary to fill a $\frac{3}{4}''$, $1\frac{1}{2}''$, and $2\frac{1}{2}''$ pipe.

3. Find the size of pipe necessary to fill a $1\frac{1}{2}''$, $2\frac{1}{2}''$, and $3\frac{1}{2}''$ pipe.

Atmospheric Pressure

BAROMETER

Atmospheric pressure is often expressed as a certain number of "atmospheres." The pressure of one "atmosphere" is the weight of a column of air, one square inch in area.



BAROMETER TUBE

At sea level the average pressure of the atmosphere is approximately 15 pounds per square inch.

The pressure of the air is measured by an instrument called a barometer. The barometer consists of a glass tube, about $31\frac{1}{2}$ inches long, which has been entirely filled with mercury (thus removing all air from the tube) and inverted in a vessel of mercury.

The space at the top of the column of mercury varies as the air pressure on the surface of the mercury in the vessel increases or decreases. The pressure is read from a graduated scale which indi-

cates the distance from the surface of the mercury in the vessel to the top of the mercury column in the tube.

QUESTIONS

1. Four atmospheres would mean how many pounds?

2. Give in pounds the following pressures: 1 atmosphere; $\frac{1}{2}$ atmosphere; $\frac{3}{4}$ atmosphere.

3. If the air, on the average, will support a column of mercury 30 inches high with a base of 1 square inch, what is the pressure of the air? (One cubic foot of mercury weighs 849 pounds.)

Water Pressure

When water is stored in a tank, it exerts pressure against the sides, whether the sides are vertical, oblique, or horizontal. The force is exerted perpendicularly to the surface on which it acts. In other words, every pound of water in a tank, at a height above the point where the water is to be used, possesses a certain amount of energy due to its position.

It is often necessary to estimate the energy in the tank at the top of a house or in the reservoir of a town or city, so as to secure the needed water pressure for use in case of fire. In such problems one must know the perpendicular height from the water level in the reservoir to the point of discharge. This perpendicular height is called the *head*.

Pressure per Square Inch. — To find the pressure per square inch exerted by a column of water, multiply the head of water in feet by 0.434. The result will be the pressure in pounds.

The pressure per square inch is due to the weight of a column of water 1 square inch in area and the height of the column. Therefore, the pressure, or weight per square inch, is equal to the weight of a foot of water with a base of 1 square inch multiplied by the height in feet. Since the weight of a column of water 1 foot high and having a base of 1 square inch is 0.434 lb., we obtain the pressure per square inch by multiplying the height in feet by 0.434.

EXAMPLES

What is the pressure per square inch of a column of water (*a*) 8′ high? (*b*) 15′ 8″ high? (*c*) 30′ 4″ high? (*d*) 18′ 9″ high? (*e*) 41′ 3″ high?

Head. — To find the *head* of water in feet, if the pressure (weight) per square inch is known, multiply the pressure by 2.31.

Let p = pressure

h = height in feet

Then $p = h \times 0.434$ lb. per sq. in.

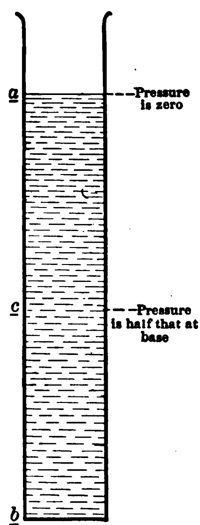
$$h = \frac{p}{0.434} = \frac{1}{0.434} \times p = 2.31\,p$$

EXAMPLES

Find the head of water, if the pressure is (*a*) 49 lb. per sq. in.; (*b*) 88 lb. per sq. in.; (*c*) 46 lb. per sq. in.; (*d*) 28 lb. per sq. in.; (*e*) 64 lb. per sq. in.

Lateral Pressure. — To find the lateral (sideways) pressure of water upon the sides of a tank, multiply the area of the submerged side, in inches, by the pressure due to one half the depth.

EXAMPLE. — A tank 18″ long and 12″ deep is full of water. What is the lateral pressure on one side?



LATERAL PRESSURE

length depth
18″ × 12″ = 216 square inches = area of side
depth
1′ × 0.434 = .434 lb. pressure at the bottom of the tank
0 = pressure at top

2).434 lb.
.217 lb. average pressure due to one half the depth of the tank

.217 × 216 = 46.872 pounds = pressure on one side of the tank

EXAMPLES

1. Find the lateral pressure on one end of a tank (*a*) 11″ long, 8″ wide, and 7″ deep; (*b*) 4′ long, 5′ wide, and 16″ deep.

2. Find the lateral pressure on one side of a tank (*a*) 8″ long, 4″ wide, and 6″ deep; (*b*) 2′ long, 3′ wide, and 11″ deep.

3. Find the lateral pressure on the bottom of a tank 7′ long, 3′ wide, and 11″ deep.

4. Find the force acting on the bottom of a box 9′ long, 6′ wide, and 2′ deep, filled with water.

Thickness of Pipe. — To find the thickness of a lead pipe necessary for a given head of water, multiply the head in feet by the size of the pipe required, expressed as a decimal, and divide this result by 750. The quotient will represent the thickness required in hundredths of an inch.

$$T = \frac{h \times .s}{750}$$

T = thickness of pipe
$.s$ = size of pipe expressed as a decimal
h = head of water

Example. — What thickness should a half-inch pipe have for a 50-foot head of water?

$$50 \times .5 = 25 \qquad 25 \div 750 = .33 \text{ inch}$$

EXAMPLES

1. What thickness of $\frac{1}{2}$″ pipe is necessary for a head of 28′?

2. What thickness of 1″ pipe is necessary for a head of 64′?

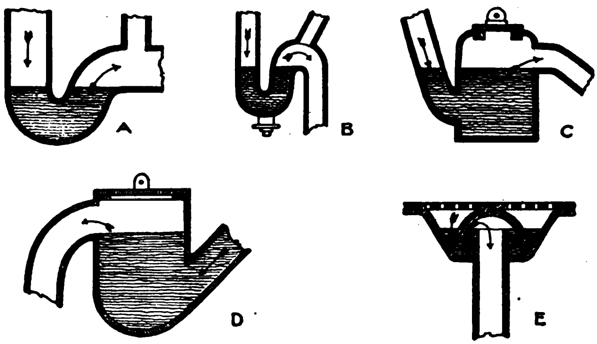
3. What thickness of $1\frac{1}{2}$″ pipe is necessary for a head of 112′?

4. What thickness of 2″ pipe is necessary for a head of 250′?

5. What thickness of $2\frac{1}{2}$″ pipe is necessary for a head of 275′?

Water Traps

The question of disposing of the waste water, called sewage, is of great importance. Various devices may be used to prevent odors from the sewage entering the house. In order to prevent the escape of gas



WATER TRAPS

from the outlet of the sewer in the basement of a house or building, a device called a trap is used. This trap consists of a vessel of water placed in the waste pipe of the plumbing fixtures. It allows the free passage of waste material, but prevents sewer gases or foul odors from entering the living rooms. The vessels holding the water have different forms; (see illustration). These traps may be emptied by back pressure or by siphon. It is a good plan to have sufficient water in the trap so that it will never be empty. All these problems belong to the plumber and involve more or less arithmetic.

To determine the pressure which the seal of a trap will resist:

EXAMPLE. — What pressure will a $1\frac{1}{2}$-inch trap resist?

If one arm of the trap has a seal of $1\frac{1}{2}$ inches, both arms will make a column twice as high, or 3 inches. Since a column of water 28 inches in height is equivalent to a pressure of 1 pound, or 16 ounces, a column of water 1 inch in height is equivalent to a pressure of $\frac{1}{28}$ of a pound, or $\frac{16}{28} = \frac{4}{7}$ ounces, and a column of water 3 inches in height is equivalent to $3 \times \frac{4}{7} = \frac{12}{7} = 1.7$ ounces.

Therefore, a $1\frac{1}{2}$-inch trap will resist 1.7 ounces of pressure.

EXAMPLES

1. What back pressure will a $\frac{3}{4}$-inch seal trap resist?
2. What back pressure will a 2-inch seal trap resist?
3. What back pressure will a $2\frac{1}{2}$-inch seal trap resist?
4. What back pressure will a $4\frac{1}{2}$-inch seal trap resist?
5. What back pressure will a 5-inch seal trap resist?

Velocity of Water

Velocity through Pipes. — To calculate the velocity of water flowing through a horizontal straight pipe of given length and diameter, the head of water above the center of the pipe being known, multiply the head of water in feet by 2500 and divide the result by the length of the pipe in feet multiplied by 13.9, divided by the inner diameter of the pipe in inches. The square root of the quotient gives the velocity in feet per second.

Let l = length of the pipe in feet
d = diameter of the pipe
H = head of water above the center of pipe
V = velocity of water in feet per second

Then $$V = \sqrt{\frac{H \times 2500}{l \times \frac{13.9}{d}}}$$

Example. — The head of water is 6 feet, the length of the pipe 1340 feet, and its diameter 5 inches. What is the velocity of water passing through the pipe?

Substituting in the above formula:

$$V = \sqrt{\frac{6 \times 2500}{1340 \times \frac{13.9}{5}}} = \sqrt{\frac{15000}{3725.2}} = \sqrt{4.02} = 2 \text{ ft. per sec.} \quad \textit{Ans.}$$

EXAMPLES

What is the velocity of water running through the following pipes:

1. Head 16′; length of pipe, 811′; diameter of pipe, 6″?
2. Head 10′; length of pipe, 647′; diameter of pipe, 4″?
3. Head 14′; length of pipe, 489′; diameter of pipe, 4″?
4. Head 26′; length of pipe, 264′; diameter of pipe, $2\frac{1}{2}$″?

Head and Velocity. — To find the head which will produce a given velocity of water through a pipe of a given diameter and length, multiply the square of the velocity, expressed in feet per second, by the length of pipe, multiplied by the quotient obtained by dividing 13.9 by the diameter of the pipe in inches. Divide this result by 2500, and the final quotient will give the head in feet.

$$h = \frac{V^2 \times L \times \frac{13.9}{d}}{2500}$$

V = velocity of water expressed in ft. per sec.
L = length of pipe expressed in feet
d = diameter of pipe expressed in inches

EXAMPLE. — The horizontal length of pipe is 1200 feet and the diameter is 4 inches. What head must be secured to produce a flow of 3 feet per second?

$$h = \frac{V^2 \times L \times \frac{13.9}{d}}{2500} = \frac{9 \times \overset{3}{\cancel{1200}} \times 13.9}{\underset{1}{\cancel{4}} \times \underset{25}{\cancel{2500}}} = \frac{27 \times 13.9}{25} = 15'. \quad \textit{Ans.}$$

EXAMPLES

1. The horizontal length of a pipe is 845 feet and the diameter is 3 feet. What head must be secured to produce a flow of $2\frac{1}{2}$ feet per second?

2. The horizontal length of a pipe is 980 feet and the diameter is $2\frac{1}{2}$ inches. What head must be secured to produce a flow of 4 feet per second?

3. The horizontal length of a pipe is 1500 feet and the diameter is 3 inches. What head must be secured to produce a flow of 4 feet per second?

4. The horizontal length of a pipe is 1280 feet and the diameter is $2\frac{1}{2}$ inches. What head must be secured to produce a flow of $3\frac{1}{2}$ feet per second?

5. The horizontal length of a pipe is 1890 feet and the diameter is 4 inches. What head of water must be secured to produce a flow of 5 feet per second?

Power

Power is the time rate of doing work. The unit of work is the horse power (H. P.), which represents 33,000 foot pounds a minute or 550 foot pounds a second. A foot pound is the amount of power necessary to lift a pound through the distance of one foot.

Raising Water. — To find the power necessary to raise water to any given height, multiply the number of feet through which it is to be lifted by 6.23 and divide this product by 33,000. This will give the nominal horse power required. If the amount of water required per minute is in gallons, the multiplier should be 83 instead of 6.23.

EXAMPLES

1. Find the power necessary to raise a bucket (weighing 2 lb. when empty) holding a quart of water 24′.

2. Find the power necessary to raise 3 cubic feet of water 15′.

3. Find the power necessary to raise 5 cubic feet of water 29′.

4. Find the power necessary to raise 1 gallon of water 31′.

5. Find the power necessary to raise 5 gallons of water 34′.

NOTE. — Change quarts to fractions of a gallon.

Water Power

When water flows from one level to another, it exerts a certain amount of energy, which is the capacity for doing work. Consequently, water may be utilized to create power by the use of such means as the water wheel, the turbine, and the hydraulic ram.

Friction, which must be considered when one speaks of water power, is the resistance which a substance encounters when moving through or over another substance. The amount of friction depends upon the pressure between the surfaces in contact.

When work is done a part of the energy which is put into it is naturally lost. In the case of water this is due to the friction. All the power which the water has cannot be used to advantage, and *efficiency* is the ratio of the useful work done by the water to the total work done by it.

Efficiency. — To find the work done by the water when a pump lifts or forces it to a height, multiply the weight of the water by the height through which it is raised.

Since friction must be taken into consideration, the useful work done by the water when the same power is exerted will equal the weight of the water multiplied by the height through which it is raised, multiplied by the efficiency of the pump.

EXAMPLE. — Find the power required to raise half a ton (long ton, or 2240 lb.) of water to a height of 40 feet, when the efficiency of the pump is 75 %.

$$\text{Total work done} = \textit{weight} \times \textit{height} \times \textit{efficiency counter}$$

$$1120 \times 40 \times \tfrac{100}{75} = 5973.3 \text{ ft. lb.}$$

$$\text{H. P. required} = \frac{5973.3}{33{,}000} = .18. \quad \textit{Ans.}$$

EXAMPLES

1. Find the power required to raise a cubic foot of water 28′, if the pump has 80 % efficiency.

2. Find the power required to raise a gallon of water 16′, if the pump has 85 % efficiency.

3. Find the power required to raise a quart of water 25′, if the pump has 70 % efficiency.

Cement and Solder

Cement. — In making joints on earthenware house sewers, plumbers use a mixture of one part Portland cement, and two parts clean sand, over a ring of oakum.

Portland cement is hydraulic lime and is made by burning a mixture of limestone, clay, and sand, and grinding the product to a very fine powder. Oakum is made from old ropes, which have been picked to pieces. After these ropes have been untwisted, loosened, and drawn out, the material, oakum, is heated with tar to make it flexible.

Solder. — In plumbing it is often desirable to join two metals or two pieces of the same metal. This may be done by means of an alloy called *solder*, which melts at a lower temperature than the metals that are to be united. This alloy melts at a comparatively low temperature (below 500° F.) and will hold the two pieces together unless they are subject to great heat or stress. The process of uniting metal surfaces in this way is called *soldering*.

Solder is made of tin and lead dross mixed with resin and charcoal, and heated in a furnace covered with a hood. After melting and stirring, the product is drawn off into a small pot furnace, dipped out with ladles, and run into molds.

There are different grades of solder: soft, hard, etc. Soft solder consists of two parts of tin and one part of lead, and melts at about 340° F. The addition of bismuth makes it more fusible. Hard solder, composed of equal parts of tin and lead, is used by the tinsmith.

A solder used by plumbers for wiping is composed of three parts of lead and two parts of tin.

Hard *spelter* is made of one part of copper and one part of zinc, while soft *spelter* is made from two parts of copper and three parts of zinc.

Solder is applied by means of a soldering iron made of copper and

pointed at the end. The handle is made of wood to prevent the heat from passing from the hot iron to the hand.

To solder: Brighten the copper with a file, moisten with acid, and then apply the solder until cooled.

For jointing, practical plumbers allow for each joint one inch in size one pound of calking lead.

EXAMPLE. — Estimate the pounds of calking required for 18 4″ joints and 25 2″ joints.

1 4″ joint requires 4 lb. lead (calking)	
1 2″ joint requires 2 lb. lead (calking)	
Therefore 18 4″ joints require	72 lb.
25 2″ joints require	50 lb.
	122 lb.

EXAMPLES

1. How much copper and zinc will be used (*a*) to make 29 lb. of soft spelter? (*b*) to make 13 lb. of hard spelter?

2. How much calking lead should be allowed for (*a*) 24 3″ joints and 18 2″ joints? (*b*) 17 $2\frac{1}{2}$″ joints and 21 4″ joints? (*c*) 41 2″ joints and 38 $2\frac{1}{2}$″ joints?

Density of Water

The mass of a unit volume of a subtance is called its *density*.

One cubic foot of pure water at 39.1° F. has a mass of 62.425 pounds; therefore, its density at this temperature is 62.425, or approximately 62.5. At this temperature water has its greatest density, and with a change of temperature the density is also changed.

With a rise of temperature, the density decreases until at 212° F., the boiling point of water, the weight of a cubic foot of fresh water is only 59.64 pounds.

When the temperature falls below 39.1° F., the density of water decreases until we find the weight of a cubic foot of ice to be but 57.5 pounds.

EXAMPLES

1. One cubic foot of fresh water at 62.5° F. weighs 62.355 lb., or approximately 62.5 lb. What is the weight of 1 cubic inch? What is the weight of 1 gallon (231 cubic inches)?

2. What is the weight of a gallon of water at 39.1° F.?

3. What is the weight of a gallon of water at 212° F.?

4. What is the weight of a volume of ice represented by a gallon of water?

5. What is the volume of a pound of water at ordinary temperature, 62.5° F.?

Specific Gravity

Some forms of matter are heavier than others, *i.e.* lead is heavier than wood. It is often desirable to compare the weights of different forms of matter and, in order to do this, a common unit of comparison must be selected. Water is taken as the standard.

Specific Gravity is the ratio of the mass of any volume of a substance to the mass of the same volume of pure water at 4° C. or 39.1° F. It is found by dividing the weight of a known volume of a substance in liquid by the weight of an equal volume of water.

EXAMPLE. — A cubic foot of wrought iron weighs about 480 pounds. Find its specific gravity.

NOTE. — 1 cu. ft. of water weighs 62.425 lb.

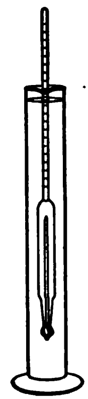
$$\frac{\text{Weight of 1 cu. ft. of iron}}{\text{Weight of 1 cu. ft. of water}} = \frac{480}{62.425} = 7.7. \quad \textit{Ans.}$$

To find Specific Gravity. — To find the specific gravity of a solid, weigh it in air and then in water. Find the difference between its weight in air and its weight in water, which will be the buoyant force on the body, or the weight of an equal volume of water. Divide the weight of the solid in air by its buoyant force, or the weight of an equal volume of water, and the quotient will be the specific gravity of the solid.

Tables have been compiled giving the specific gravity of different solids, so it is seldom necessary to compute it.

The specific gravity of liquids is very often used in the industrial world, as it means the "strength" of a liquid. In the carbonization of raw wool, the wool must be soaked in sulphuric acid of a certain strength. This acid cannot be bought except in its concentrated form (sp. gr. 1.84), and it must be diluted with water until it is of the required strength.

The simplest way to determine the specific gravity of a liquid is with a *hydrometer*. This instrument consists of a closed glass tube, with a bulb at the lower end filled with mercury. This bulb of mercury keeps the hydrometer upright when it is immersed in a liquid. The hydrometer has a scale on the tube which can be read when the instrument is placed in a graduate of the liquid whose specific gravity is to be determined.



But not all instruments have the specific gravity recorded on the stem. Those most commonly in use are graduated with an impartial scale.

In England, Twaddell's scale is commonly employed, and since most of the textile mill workers are English, we find the same scale in use in this country. The Twaddell scale bears a marked relation to specific gravity and can be easily converted into it.

Another scale of the hydrometer is the Beaume, but these readings cannot be converted into specific gravity without the use of a complicated formula or reference to a table.

Hydrometer Scale	Formula for Converting into S. G.
1. Specific gravity hydrometer	Gives direct reading
2. Twaddell	$\text{S. G.} = \dfrac{(.5 \times N) + 100}{100}$
3. Beaume	$\text{S. G.} = \dfrac{146.3}{146.3 - N}$

$N =$ the particular degree which is to be converted.

EXAMPLE. — Change 168 degrees (Tw.) into S. G.

$$\frac{(.5 \times 168) + 100}{100} = 1.84. \quad \textit{Ans.}$$

Another formula for changing degrees Twaddell scale into specific gravity is: $\frac{(5 \times N) + 1000}{1000} =$ specific gravity.

In Twaddell's scale, 1° specific gravity = 1.005
2° specific gravity = 1.010
3° specific gravity = 1.015

and so on by a regular increase of .005 for each degree.

To find the degrees Twaddell when the specific gravity is given, multiply the specific gravity by 1000, subtract 1000, and divide by 5. *Formula :*

$$\frac{(\text{S. G.} \times 1000) - 1000}{5} = \text{degrees Twaddell}$$

EXAMPLE. — Change 1.84 specific gravity into degrees Twaddell,

$$\frac{(1.84 \times 1000) - 1000}{5} = 169 \text{ degrees Twaddell}$$

EXAMPLES

1. What is the specific gravity of sulphuric acid of 116° Tw.?
2. What is the specific gravity of acetic acid of 86° Tw.?
3. What is the specific gravity of a liquid of 164° Be.?
4. What is the specific gravity of a liquid of 108° Be.?
5. What is the specific gravity of a liquid of 142° Tw.?
6. Change the following specific gravity readings into Tw: (*a*) 1.81; (*b*) 1.12; (*c*) 1.60; (*d*) 1.44; (*e*) 1.29.

PART VII — STEAM ENGINEERING

CHAPTER XII

HEAT

Heat Units. — The unit of heat used in the industries and shops of America and England is the British Thermal Unit (B. T. U.) and is defined as the quantity of heat required to raise one pound of water through a temperature of one degree Fahrenheit.

Thus the heat required to raise 5 lb. of water through 15 degrees F. equals

$5 \times 15 = 75$ British Thermal Units (B. T. U.)

Similarly, to raise 86 lb. of water through $\frac{1}{2}°$ F. requires

$86 \times \frac{1}{2} = 43$ B. T. U.

The unit that is used on the Continent and among scientific circles in America is the metric system unit, a calorie. This is the amount of heat necessary to raise 1 gram of water 1 degree Centigrade. (See Appendix for metric system.)

EXAMPLES

1. How many units (B. T. U.) will be required to raise 4823 lb. of water 62 degrees?

2. How many B. T. U. of heat are required to change 365 cubic feet of water from 66° F. to 208° F.?

3. How many units (B. T. U.) will be required to raise 785 lb. of water from 74° F. to 208° F.?

(Consider one cubic foot of water equal to $62\frac{1}{2}$ lb.)

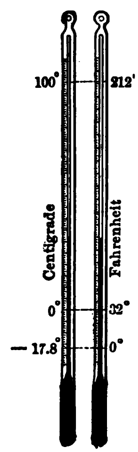
4. How many B. T. U. of heat are required to change 1825 cu. ft. of water from 118° to 211°?

5. How many heat units will it take to raise 484 gallons of water 12 degrees?

6. How many heat units will it take to raise 5116 gallons of water from 66° F. to 198° F.?

Temperature

The ordinary instruments used to measure temperature are called thermometers. There are two kinds — Fahrenheit and Centigrade. The Fahrenheit thermometer consists of a cylindrical tube filled with mercury with the position of the mercury at the boiling point of water marked 212, and the position of mercury at the freezing point of water 32. The intervening space is divided into 180 divisions. The Centigrade thermometer has the position of the boiling point of water 100 and the freezing point 0. The intervening space is divided into 100 spaces. It is often necessary to convert the Centigrade scale into the Fahrenheit scale, and Fahrenheit into Centigrade.



THERMOMETERS

To convert F. into C., subtract 32 from the F. degrees and multiply by $\frac{5}{9}$ or divide by 1.8, or C. = (F. − 32°) $\frac{5}{9}$, where C. = Centigrade reading and F. = Fahrenheit reading.

To convert C. to F., multiply C. degrees by $\frac{9}{5}$ or 1.8 and add 32.

$$F = \frac{9\,C}{5} + 32^\circ$$

EXAMPLE. — Convert 212 degrees F. to C. reading.

$$\frac{5(212^\circ - 32^\circ)}{9} = \frac{5(180^\circ)}{9} = \frac{900^\circ}{9} = 100^\circ \text{ C.}$$

EXAMPLE. — Convert 100 degrees C. to F. reading.

$$\frac{9 \times 100^\circ + 32^\circ}{5} = \frac{900^\circ}{5} + 32^\circ = 180^\circ + 32^\circ = 212^\circ \text{ F.}$$

If the temperature is below the freezing point, it is usually written with a minus sign before it: thus, 15 degrees below the freezing point is written $-15°$. In changing $-15°$ C. into F. we must bear in mind the minus sign.

Thus, $F = \frac{9\,C}{5} + 32° \quad F = \frac{-15° \times 9}{5} + 32° = -27° + 32° = 5°$

EXAMPLE. — Change $-22°$ F. to C.

$$C. = \tfrac{5}{9}(F. - 32°)$$
$$C. = \tfrac{5}{9}(-22° - 32°) = \tfrac{5}{9}(-54°) = -30°$$

EXAMPLES

1. Change 36° F. to C.
2. Change 89° F. to C.
3. Change 289° F. to C.
4. Change 350° F. to C.
5. Change 119° C. to F.
6. Change 225° C. to F.
7. Change 380° C. to F.
8. Change 415° C. to F.
9. Change 580° C. to F.

Value of Coal in Producing Heat

There are different kinds of coal on the market. Some grades of the same coal give off more heat than others in burning. The heating value of a coal may be found in three ways: (1) By chemical analysis to determine the amount of carbon, (2) by burning a definite amount in a calorimeter and noting the rise of temperature of the water, (3) by actual trial in a steam boiler. The first two methods give a theoretical value, the third gives the real result under the actual conditions of draft, heating surface, combustion, etc.

EXAMPLES

1. If 125 lb. of ashes are produced from one ton of coal, what is the percentage of ashes in that coal?

2. Twelve tons of coal are burned per day, and twenty-two baskets of ashes, each weighing 65 lb., are removed; what percentage of ashes does the coal contain?

3. If twelve tons of coal are burned per day, and 1450 lb. of ashes are produced, what percentage of ashes does the coal contain?

4. A quantity of coal is built into a rectangular stack 50 ft. long, 25 ft. broad, and 6 ft. high; what is the weight of the coal, allowing 45 cubic feet per ton?

5. If 92,400 lb. of coal are consumed in 60 hours and the engines regularly develop 480 I. H. P. (Indicated Horse Power), how much coal is consumed per H. P. per hour?

6. With the price of coal at $3.25 per ton of 2000 lb., and the power produced earning a profit of 25 % on the cost of production, what would be the amount of profit when running full power?

Latent Heat

By latent heat of water is meant that heat which water absorbs or discharges in passing from the liquid to the gaseous, or liquid to solid state, without affecting its own temperature. Thus, the temperature of boiling water at atmospheric pressure never rises above 212 degrees F., because the steam absorbs the excess of heat which is necessary for its gaseous state. Latent heat of steam is the quantity of heat necessary to convert a pound of water into steam of the same temperature as the steam in question.

To find the latent heat of steam, subtract ten times the square root of the gauge pressure from 977°.

Example. — Find the latent heat of steam at 169 lb. gauge pressure.

$$\sqrt{169} = 13$$

$$13 \times 10 = 130^\circ$$

$$977^\circ - 130^\circ = 847^\circ \text{ latent heat}$$

EXAMPLES

Find the latent heat of steam at the following gauge pressures:

1. 132 lb. gauge pressure.
2. 116 lb. gauge pressure.
3. 208 lb. gauge pressure.
4. 196 lb. gauge pressure.
5. 84 lb. gauge pressure.
6. 39 lb. gauge pressure.
7. 41 lb. gauge pressure.
8. 160 lb. gauge pressure.
9. 159 lb. gauge pressure.
10. $180\frac{1}{2}$ lb. gauge pressure.

Heat Units Required to Produce a Given Pressure

To find the number of units of heat required to raise the temperature corresponding to one gauge pressure to that of another, find the square roots of the gauge pressures, subtract these values, and multiply by $14\frac{1}{3}$.

Example.—Find the number of heat units required to raise the temperature of 64 pounds gauge pressure to 169 pounds gauge pressure.

$\sqrt{64} = 8$ $\quad 13 - 8 = 5$ $\quad 14\frac{1}{3} \times 5 = 71\frac{2}{3}$

$\sqrt{169} = 13$ $\quad$ Approximately 72 B. T. U. *Ans.*

EXAMPLES

Find the number of heat units required to raise the temperature between the following gauges:

1. From 64 to 128 lb. gauge.
2. From 26 to 131 lb. gauge.
3. From 39 to 149 lb. gauge.
4. From 49 to 165 lb. gauge.
5. From 42 to 121 lb. gauge.
6. From 28 to 132 lb. gauge.
7. From 33 to 144 lb. gauge.
8. From 55 to 164 lb. gauge.

Volume of Water and Steam

According to steam tables one cubic foot of steam at 100 pounds' pressure weighs 0.2307 lb., one cubic foot of water weighs $62\frac{1}{2}$ lb., and one gallon of water may be taken as $8\frac{3}{10}$ lb.

At atmospheric pressure one cubic foot of steam has nearly the weight of one cubic inch of water, and the weight increases very nearly as the pressure. Hence, to find the number of cubic inches of water required to make a certain amount of steam, multiply the number of cubic feet of steam by the absolute pressure in the atmosphere; the product is the number of cubic inches of water required. In all such calculations for practical purposes, a liberal allowance must be made for loss and leakage.

Absolute pressure is the total pressure, or the gauge pressure plus the atmospheric pressure (which at sea level is 14.7 lb. per sq. in.).

EXAMPLES

1. How much water will it take to make 800 cu. ft. of steam at 10 lb. pressure?

2. How much water will it take to make 3020 cu. ft. of steam at 65 lb. pressure?

3. How much water will it take to make 4812 cu. ft. of steam at 8 lb. pressure?

4. How much water will it take to make 512 cu. ft. of steam at 75 lb. pressure?

5. How much water will it take to make 1213 cu. ft. of steam at 80 lb. pressure?

Solve the following problems according to the table on the next page:

6. What is the total steam pressure if the steam gauge reads 55 lb.?

7. How many cubic feet of steam from 2 lb. of water at a steam gauge pressure of 65 lb.?

8. What is the latent heat of 1 lb. of water at a total pressure of 75 lb. at 307.5° F.?

9. What is the total heat required to generate 1 lb. of steam from water at 32° F. under total pressure of 90 lb.?

PROPERTIES OF SATURATED STEAM

Pressure		Temperature in Fahrenheit Degrees	Volume		Latent Heat in Fahrenheit Degrees	Total Heat required to generate 1 Lb. of Steam from Water at 32° under Constant Pressure
By Steam Gauge	Total		Compared with Water	Cubic Ft. of Steam from 1 Lb. of Water		Heat units
0	15	212.0	1642	26.36	965.2	1146.1
5	20	228.0	1229	19.72	952.8	1150.9
10	25	240.1	996	15.99	945.3	1154.6
15	30	250.4	838	13.46	937.9	1157.8
20	35	259.3	726	11.65	931.6	1160.5
25	40	267.3	640	10.27	926.0	1162.9
30	45	274.4	572	9.18	920.9	1165.1
35	50	281.0	518	8.31	916.3	1167.1
40	55	287.1	474	7.61	912.0	1169.0
45	60	292.7	437	7.01	908.0	1170.7
50	65	298.0	405	6.49	904.2	1172.3
55	70	302.9	378	6.07	900.8	1173.8
60	75	307.5	353	5.68	897.5	1175.2
65	80	312.0	333	5.35	894.3	1176.5
70	85	316.1	314	5.05	891.4	1177.9
75	90	320.2	298	4.79	888.5	1179.1
80	95	324.1	283	4.55	885.8	1180.3
85	100	327.9	270	4.33	883.1	1181.4
90	105	331.3	257	4.14	880.7	1182.4
95	110	334.6	247	3.97	878.3	1183.5
100	115	338.0	237	3.80	875.9	1184.5
110	125	344.2	219	3.50	871.5	1186.4
120	135	350.1	203	3.27	867.4	1188.2
130	145	355.6	190	3.06	863.5	1189.9
140	155	361.0	179	2.87	859.7	1191.5
150	165	366.0	169	2.71	856.2	1192.9

Steam Heating

A steam heating system with steam having a pressure of less than 15 lb. by the gauge is called a low pressure system. If the steam pressure is higher than 15 lb. it is called a high pressure system. When the water of condensation flows back to the boiler by gravity alone, the apparatus is known as a gravity circulating system. When the boiler is run at a high pressure and the heating system at a low pressure, the condensed steam must be returned to the boiler by a pump, steam return trap, or injector.

The quantity of heat given off by the radiators of steam pipes, in the ordinary methods of heating buildings by direct radiation, varies from $1\frac{3}{4}$ to 3 heat units per hour per square foot of radiating surface for each degree of difference in temperature; an average of from 2 to $2\frac{1}{4}$ is a fair estimate.

One pound of steam at about atmospheric pressure contains 1146 heat units, and if the temperature in the room is to be kept at 70° F., while the temperature of the pipes is 212 degrees, the difference in temperature is 142 degrees. Multiplying this by $2\frac{1}{4}$, the emission of heat will be $319\frac{1}{2}$ heat units per hour per square foot of radiating surface. A rule often given is to allow one square foot of heating surface in the boiler for every eight to ten square feet of radiating surface.

In steam heating the following rule is used: To find the amount of direct radiating surface required to heat a room, basing the calculation upon its cubic contents, allow one square foot of direct radiating surface for each cubic foot shown in the following table.

Proportion of Radiating Surface to Volume of Rooms

Bathrooms or living rooms, with 2 or 3 exposures	40 cu. ft.
Living rooms, with 1 or 2 exposures	50 cu. ft.
Sleeping rooms	55–70 cu. ft.
Halls	50–70 cu. ft.
Schoolrooms	60–80 cu. ft.
Large churches and auditoriums	65–100 cu. ft.
Lofts, workshops, and factories	75–150 cu. ft.

The above ratios will give reasonably good results on ordinary work if proper judgment is exercised.

EXAMPLES

1. How much radiating surface is necessary to heat a bathroom containing 485 cu. ft.?

2. How much radiating surface is necessary to heat a bathroom $10\frac{1}{2}' \times 5\frac{1}{2}' \times 9'$?

3. How much radiating surface is necessary to heat a living room with three windows, and containing 2798 cu. ft.?

4. How much radiating surface is necessary to heat a living room $18' \times 16\frac{1}{2}' \times 10'$, with three windows?

CHAPTER XIII

BOILERS

THERE are two classes of boilers — water tube and fire tube. The difference between the two is that water flows through water tube boilers and the fire to heat the water is outside, while in the fire tube boiler the conditions are reversed.

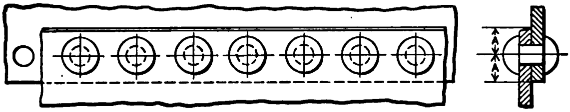
Return Tubular Boilers. — The boiler most widely used in America is the return tubular, which is a type of fire tube boiler. It is a closed tube, simple in construction, inexpensive to make, and easy to clean and repair. The first horizontal tubular boiler was an ordinary storage tank made of iron. Now horizontal tubular boilers, 16 to 20 feet long, and 4 to 8 feet in diameter, and even larger, are used and can withstand a pressure of 150 lb. per sq. in.

Boilers up to fourteen feet in length are constructed of two plates, each forming the entire circumference of the boiler. Above fourteen feet long the shell is constructed of three plates which make the required length of the boiler shell. These plates are $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, or $\frac{9}{16}$ in. thick, having from 45,000 to 85,000 lb. tensile strength.

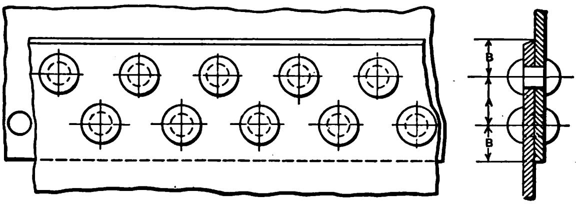
Tensile Strength. — The tensile strength is the pull applied in the direction of its length required to break a bar of boiler plate one square inch in area. Different pieces are taken from the various parts of the boiler plate, reduced to $\frac{1}{4}$ inch square, and subjected to pressure on a testing machine. The average strength of the samples is thus obtained and multiplied by 16 to determine the strength of one square inch. The tensile strength is usually stamped on the boiler steel. If it is not stamped on it, the tensile strength is 48,000 lb.

HORIZONTAL RETURN TUBULAR BOILER, 275 HORSE POWER, IN THREE COURSES
The flames pass through the tubes around which the water circulates.

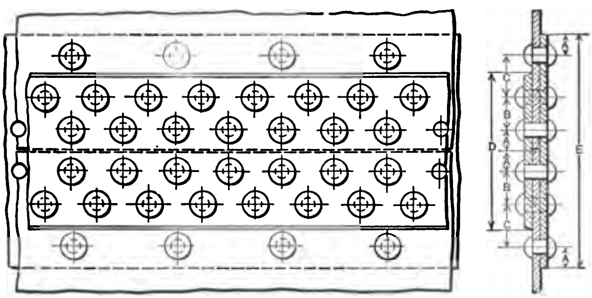
If four samples give tests of 3998 lb., 4001 lb., 4001 lb., and 4000 lb., then the average is 4000. Therefore, for one square inch the tensile strength is 4000 × 16 (the number of quarter sq. in. in one sq. in.) = 64,000 lb.



SINGLE RIVETED LAP JOINT;[1] EFFICIENCY ABOUT 56 %



DOUBLE RIVETED LAP JOINT; EFFICIENCY ABOUT 70 %



TRIPLE RIVETED BUTT-STRAP JOINT; EFFICIENCY ABOUT 85 %

Safe Working Pressure. — In order to know the safe working pressure of a single riveted boiler, it is necessary to multiply one sixth of the tensile strength by the thickness of the shell, and divide this product by the inside radius of the shell. If

[1] The efficiency of a riveted joint is the ratio of the strength of the joint to that of the solid plate.

a boiler is double riveted, add 20 per cent to the safe pressure of a single riveted boiler of the same dimensions.

Example. — What is the safe working pressure of a single riveted boiler having 72″ diameter, $\frac{1}{2}''$ shell, if the boiler plate has a tensile strength of 66,000 lb.?

$$\tfrac{1}{6} \times 66{,}000 = 11{,}000$$
$$11{,}000 \times .5'' = 5500$$
$$5500 \div 36'' \text{ (radius)} = 152\tfrac{7}{9} \text{ lb. approx.} \quad \textit{Ans.}$$

EXAMPLES

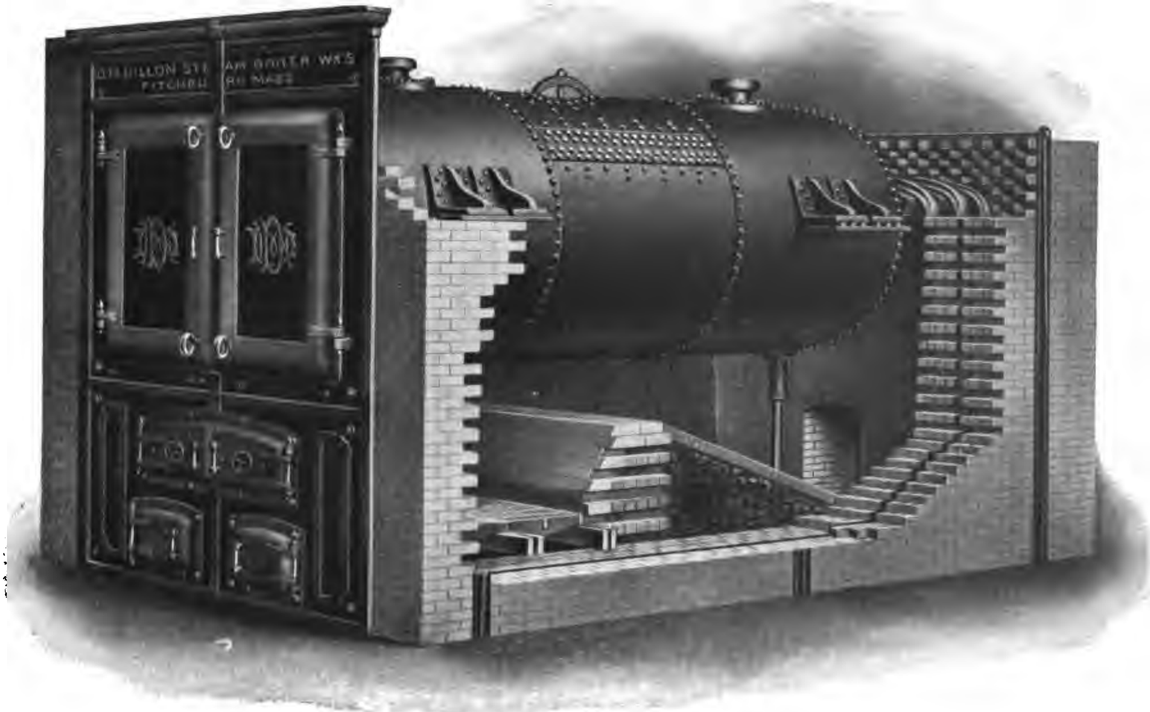
1. (*a*) What is the safe working pressure of a single riveted boiler of 60″ diameter, $\frac{6}{16}''$ shell, the T. S. being 60,000 lb. per sq. in.? (*b*) What is the safe working pressure of a double riveted boiler of the same dimensions?

2. (*a*) What is the safe working pressure of a single riveted boiler of 54″ diameter, $\frac{3}{8}''$ shell, the T. S. being 60,000 lb.? (*b*) What is the safe working pressure of a double riveted boiler of the same dimensions?

3. (*a*) What is the safe working pressure of a single riveted boiler of 48″ diameter, $\frac{3}{8}''$ shell, the T. S. being 60,000 lb.? (*b*) What is the safe working pressure of a double riveted boiler of the same dimensions?

4. (*a*) What is the safe working pressure of a single riveted boiler of 42″ diameter, $\frac{5}{16}''$ shell, the T. S. being 60,000 lb.? (*b*) Of a double riveted boiler of the above dimensions?

The Plate. — The diameter of the rivet used for boiler plate is generally double the thickness of the plate. The amount of space between the rivet holes of a boiler is found by dividing the area of the rivet hole by the thickness of the plate. The pitch or distance between the rivet hole centers in a boiler plate is found by dividing the area of the rivet by the thickness of the plate and adding the diameter of one hole. The thickness of plate is found, when the shearing strength is



HORIZONTAL RETURN TUBULAR BOILER, SHOWING BOILER SETTING

known, by multiplying the area of the rivet hole by the shearing strength, then multiplying the thickness of the plate by the tensile strength, dividing the first product by this product, and adding one rivet hole diameter to the quotient.

EXAMPLE. — What thickness of plate should be used on a 40-inch boiler to carry 125 lb. pressure, if the tensile strength of the plate is 60,000 lb.?

$$\begin{aligned} 125 &= \text{steam pressure} \\ 6 &= \text{factor of safety} \\ 40 &= \text{diameter of boiler} \\ 20 &= \tfrac{1}{2} \text{ diameter of boiler} \\ 60{,}000 &= \text{tensile strength of plate} \\ 125 \times 6 &= 750 \\ 750 \times 20 &= 15{,}000 \\ \frac{15{,}000}{60{,}000} &= .25 \text{ or } \tfrac{1}{4}'' \text{ thickness of plate.} \quad \textit{Ans.} \end{aligned}$$

The Boiler Inspection Department of Massachusetts recommends the following formula for determining the thickness of boiler plate:

$$T = \frac{P \times R \times F.S.}{T.S. \times \%}$$

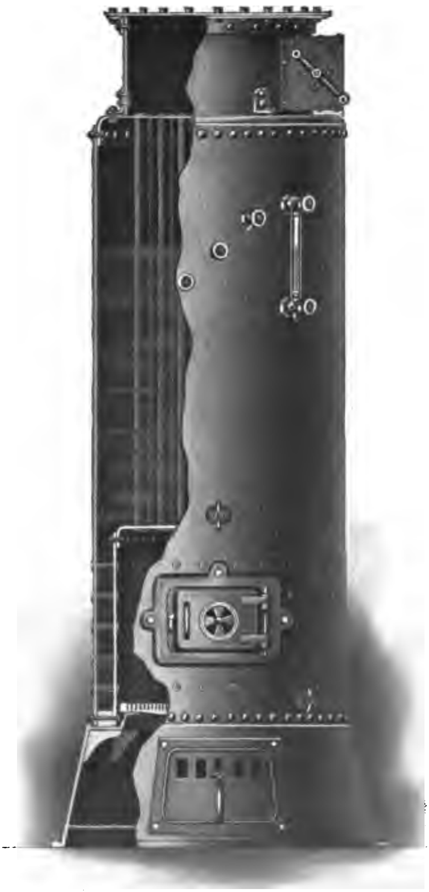
$$\begin{aligned} T &= \text{thickness of plate} \\ P &= \text{pressure} \\ R &= \text{radius } (\tfrac{1}{2} \text{ diameter of boiler}) \\ F.\ S. &= \text{safety factor} \\ T.\ S. &= \text{tensile strength} \\ \% &= \text{strength of joint} \end{aligned}$$

EXAMPLE. — What thickness of plate should be used on a 40-inch boiler to carry 125 lb. pressure, if the strength of the plate is 60,000 lb., using 50 % as the strength of the joint?

$$T = \frac{125 \times 20 \times 6}{60{,}000 \times .50} = \tfrac{1}{2}'' \text{ sheet.} \quad \textit{Ans.}$$

Find the safe working of the same boiler with the above figures:

$$\text{Safe working pressure} = \frac{60{,}000 \times .5 \times .50}{20 \times 6} = 125 \text{ lb.} \quad \textit{Ans.}$$



SMALL VERTICAL BOILER

Size.—The engineer often has to calculate the size of a boiler to carry a definite steam pressure.

The size of a single riveted boiler may be found by multi-

plying $\frac{1}{6}$ of the tensile strength by the thickness of the shell, and dividing this product by the steam pressure. The quotient is the radius of the boiler. Multiply this radius by the theoretical diameter (twice this radius equals the theoretical diameter); add one fifth of the diameter just found as a safety factor, and this sum gives the working diameter of a boiler that will safely carry the required pressure.

EXAMPLE. — What is the diameter of a boiler that will withstand 150 lb. pressure, if made of $\frac{3}{8}''$ steel, 60,000 lb. T. S.?

$$\frac{1}{6} \times 60{,}000 = 10{,}000$$
$$10{,}000 \times \frac{3}{8} = 3750$$
$$\frac{3750}{150} = 25'' \text{ theoretical radius}$$
$$50'' = \text{theoretical diameter}$$
$$50'' + 10'' = 60'' \text{ working diameter. } \textit{Ans.}$$

The Boiler Inspection Department of Massachusetts recommends the following formula for finding the diameter of a boiler when the pressure, thickness, tensile strength, and per cent are known.

$$D = \frac{T \times T.S. \times \%}{P.F.} \times 2$$

D = diameter of boiler
T = thickness of plate
$T.\ S.$ = tensile strength
$\%$ = strength of joint
P = pressure
F = safety factor

EXAMPLE. — What is the diameter of a boiler having $\frac{1}{2}''$ shell, allowing 50 per cent for the strength of the joint, with a tensile strength of 60,000 lb., when the factor of safety is 6 and the pressure of steam is 125 lb.?

$$D = \frac{5 \times 60{,}000 \times .50}{125 \times 6} \times 2 = 40'' \text{ diameter. } \textit{Ans.}$$

EXAMPLES

1. What is the diameter of the rivet for a plate $\frac{1}{4}''$ thick?

2. How much space is there between the rivet holes, $\frac{11}{16}''$ in diameter, on a $\frac{1}{4}''$ plate?

3. What is the pitch in a $\frac{1}{4}''$ boiler plate, if the rivet diameter is $\frac{5}{8}''$ and the whole diameter is $\frac{11}{16}''$?

4. What is the working diameter of a boiler made of $\frac{3}{8}''$ steel, 60,000 lb. T. S., that will withstand 90 lb. pressure?

5. What thickness of plate should be used on a 72″ boiler with a T. S. of 66,000 lb. to carry a pressure of 90 lb.?

6. How much space is there between rivet holes $\frac{7}{8}''$ in diameter on a $\frac{7}{16}''$ plate?

7. What is the working diameter of a boiler made of $\frac{5}{16}''$ steel, 60,000 lb. T. S., that will withstand 150 lb. pressure?

8. What is the pitch of a rivet hole of a $\frac{3}{8}''$ boiler plate with a diameter of $\frac{13}{16}''$, a shearing strength of 30,000 lb., and T. S. of 60,000 lb.?

9. What is the working diameter of a boiler made of $\frac{3}{8}''$ steel, 60,000 lb. T. S., that will withstand 125 lb. pressure?

10. What should be the diameter of a rivet to be used in a $\frac{7}{16}''$ plate?

11. What is the pitch in a $\frac{1}{2}''$ boiler plate with a rivet diameter of $\frac{7}{8}''$ and boiler diameter of $\frac{15}{16}''$?

12. What thickness of plate should be used on a 48″ boiler to carry 60 lb. pressure, with a T. S. of 60,000 lb.?

13. What is the working diameter of a boiler made of $\frac{3}{8}''$ steel, 60,000 lb. T. S., that will withstand 110 lb. pressure?

14. What is the pitch in a rivet hole $\frac{7}{8}''$ in diameter in a boiler with a shearing strength of 32,000 lb., if the plate has a tensile strength of 60,000 lb., and is $\frac{7}{16}''$ in thickness?

15. What is the working diameter of a boiler made of $\frac{7}{16}''$ steel, 60,000 lb. T. S., that will withstand 60 lb. pressure?

Boiler Tubes

A boiler tube is open at both ends. Therefore it is not necessary to consider the area of the bases in computing the heating surface of a tube. The area of the cylindrical surface is all that is necessary to find. Boiler tubes are often measured in terms of the heating surface per foot.

EXAMPLE.—What is the heating surface per foot for a 3″ tube, $\frac{1}{8}''$ thick?

$$3'' = \text{diam. of tube}$$
$$2 \times \tfrac{1}{8}'' = \tfrac{1}{4}'' \text{ or twice thickness of tube}$$
$$3 - \tfrac{1}{4}'' = 2\tfrac{3}{4}'' \text{ inside diam. of tube} = 2.75''$$
$$2.75'' \times 3.1416 = 8.6394'' \text{ inside circumference}$$
$$8.6394'' \times 12'' = 103.6728 \text{ sq. in.}$$
$$\frac{103.6728}{144} = .7199 \text{ sq. ft. heating surface}$$

To find the total tube heating surface of a boiler:

Multiply the heating surface per foot of length by the length of the tubes in feet and that product by the number of tubes in the boiler.

EXAMPLE.—If a boiler which has 110 3″ tubes $\frac{1}{8}''$ thick is 12′ between front and back heads, what is its tube heating surface?

$$.72 = \text{heating surface per foot length}$$
$$12 = \text{length of tubes in feet}$$
$$110 = \text{number of tubes}$$
$$.72 \times 12 = 8.64 \text{ sq. ft. per tube}$$
$$8.64 \times 110 = 950.4 \text{ sq. ft. tube heating surface.} \quad \textit{Ans.}$$

EXAMPLE.—In the above example what percentage of the total volume of the boiler do the tubes represent if the diameter is 60″?

$$60 = \text{diam. of shell}$$
$$60 \times 60 = 3600$$
$$3600 \times .7854 = 2827.44 \text{ sq. in. area}$$
$$\frac{2827.44}{144} = 19.635 \text{ sq. ft. area}$$
$$19.64 \times 12' = 235.68 \text{ cu. ft. volume of shell}$$

$$3'' = \text{diam. of tube } 3 \times 3 = 9$$
$$9 \times .7854 = 7.0686 \text{ sq. in. area}$$
$$\frac{7.0686}{144} = .049 \text{ sq. ft. area}$$
$$.049 \times 1 = \text{cu. ft. for 1 ft. of tube length}$$
$$.049 \times 12' = 0.588 \text{ cu. ft. (capacity of each tube)}$$
$$.588 \times 110 = 64.68 \text{ cu. ft. occupied by tubes}$$
$$\frac{64.7 \text{ tubes volume}}{235.7 \text{ shell volume}} = .27 = 27 \text{ per cent shell volume. } \textit{Ans.}$$

Fusible Plug. — A fusible plug is a brass plug with a tapering center of Banca tin. The large end is put next to the pressure to prevent the soft metal from blowing out. This plug is screwed into the rear head of a boiler not less than two inches above the top row of tubes and should extend one inch into the water to prevent it from becoming scaled. If the water falls below this plug the soft metal melts, allowing the steam to escape, thus giving warning.

Manhole. — A manhole, oval in shape, is put in the top or in the heads of the boiler, to allow a person to enter the inside to inspect the boiler. It is made tight by the use of a rubber gasket.

Hand Hole and Blow-off. — Hand-hole plugs are put into the bottom of the front and rear heads of a boiler to permit washing out. The blow-off is connected at the bottom of the shell at the rear end with a valve on the pipe outside the brickwork, called a blow-off valve. This is to empty the boiler, or to blow it down one gauge. It is necessary to blow down the boiler each morning in order to rid it of the sediment that has settled at the bottom each day. Boilers should be entirely emptied and washed out at least once a month; the necessity for this is determined by the quality of the feed water.

Water Gauge. — The water gauge registers the height of the water in a boiler. It consists of a small cast iron drum placed in an upright position in front of the boiler and provided with a glass gauge, cocks, water, and steam connections. The pipe connections are arranged so that dry steam enters the top and water the bottom, with a blow-off valve for the water column and gauges. The water should be kept up to the second gauge while the boiler is working, and up to the third gauge at night. The first duty of the engineer is to see that the water is at the proper level. To be sure that the glass is registering correctly, the gauge cocks have to be tried. One of these is below the water line and one above it. If the water in the boiler is right, steam will come out of the upper one and water out of the lower; if the water in the boiler is too low, steam will come out of both.

Safety Valves

The power of a boiler depends upon the amount of heating surface that the boiler contains. As the cylinder of the boiler is made to withstand a certain pressure, an excess may cause it to explode. So it is necessary that the engineer should know when the pressure is exceeded. Various devices to be attached to boilers have been invented to give warning. One of these is the safety valve.

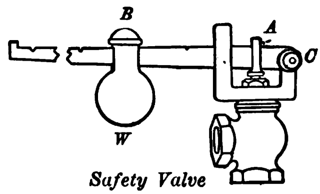
Steam Pressure Gauge

Every boiler should have at least two safety valves, a water gauge, and a pressure gauge. The function of a safety valve is to relieve the boiler of all pressure in excess of that at which the valve is set to blow off. It is placed at the top of the boiler and piped outside. The careful engineer tries the safety valve every day to see if it is in working order.

The size of the safety valve is very important. The area of the grate, the weight of the fuel burned, and the steam pressure have to be considered when calculating the size of the valve. The amount of steam generated in a given time and the pressure caused by the steam will depend upon the weight of coal burned. The velocity of the escape of the steam through the valve will depend upon the pressure of the steam. A low pressure safety valve is not higher than thirty pounds. The figure stamped on the lever of the safety valve shows the limit of pressure.

Lever Safety Valve. — The lever safety valve is placed on an opening in the top of the boiler. This valve consists of a disk,

a stem, and a lever which has a weight hung on the end. The weight keeps the valve in place until the pressure of the steam in the boiler overcomes that of the weight so that the valve is pushed up and gives warning that the pressure of the steam must be lessened to prevent an explosion.



Safety Valve

C = fulcrum
A = distance from fulcrum to center of valve
B = distance from fulcrum to center of weight
P = pressure
W = weight

$$BW = CP$$
$$B \times W \div P = A$$
$$A \times P \div W = B$$
$$P \times A \div B = W$$
$$W \times B \div A = P$$

Pop Safety Valve.—The pop safety valve has displaced the lever safety valve to a large extent.

The size of the safety valve that should be placed on a boiler may be determined by the following rule:

$$\text{Grate surface} \times 22.5 \qquad (1)$$
$$\text{Gauge pressure} + 8.62 \qquad (2)$$
$$(1) \div (2) = \text{area of valve}$$
$$A = \frac{22.5\ G}{P + 8.62}$$

Divide area of valve by .7854 and extract the square root of the quotient. The result is the diameter.

Example.—What shall be the diameter of a safety valve on a boiler with 36 sq. ft. of grate surface carrying 100 lb. pressure?

$$36 \times 22.5 = 810$$
$$100 + 8.62 = 108.62$$
$$\frac{810}{108.62} = 7.5'', \text{ area of valve}$$
$$\sqrt{\frac{7.5}{.7854}} = 3\tfrac{1}{8}'' \text{ approx. } \textit{Ans.}$$

The Boiler Inspection Department of Massachusetts recommends the following formula for calculating the size of safety valve.

$$\text{Diameter of valve} = \sqrt{\frac{W \times 70}{P} \times 11 \div .7854.}$$

D = diameter of valve
W = weight of water in pounds evaporated per square foot of grate surface per second
P = pressure absolute at which safety valve is set to blow

EXAMPLES

1. What must be the size of a safety valve on a boiler with 48 sq. ft. of grate surface, carrying 98 lb. pressure?

2. The diameter of a boiler tube is 4″ and the length 18′. Find the area of the surface of the tube in sq. ft.

3. What must be the size of a safety valve on a boiler with 42 sq. ft. of grate surface, carrying 84 lb. pressure?

4. The diameter of a boiler tube is $3\frac{1}{2}$″ and its length 16′. What is its area?

5. The diameter of a lever safety valve of a steam boiler is 3″, the length of the lever from its fulcrum to a point at which a 50 lb. weight is suspended is 24″, and the distance from the fulcrum to the point where the lever in a horizontal position presses upon the valve is 3″. At what steam pressure per sq. in. will the boiler blow off?

6. What is the total tube heating surface of a boiler having 112 3″ tubes, each $\frac{1}{8}$″ thick, if the distance between the front and back heads is 16′?

7. On a certain boiler the safety valve lever is 24″ long and weighs 3 lb. and carries at its extremity a weight of 30 lb. The length from the fulcrum of the lever to the valve spindle is 3″, and from the fulcrum to the center of gravity of the lever 16″. If the valve has an area of 10 sq. in. and weighs

with its spindle $1\frac{1}{2}$ lb., at what steam pressure per sq. in. will the boiler blow off?

8. In Example 7 if it is desired to reduce the maximum steam pressure by one half, at what point on the lever must the weight be hung?

9. What is the heating surface per foot of a $2\frac{1}{2}''$ boiler tube $\frac{3}{16}''$ thick?

10. What is the total tube heating surface of a boiler having 90 $3\frac{1}{2}''$ tubes, each $\frac{1}{8}''$ thick, if the boiler is 17′ long?

11. What is the total tube heating surface of a boiler having 80 3″ tubes, each $\frac{1}{8}''$ thick and 16′ long?

12. What is the percentage of the total tube volume to the total boiler volume of a boiler 72″ in diameter, with shell 18′ long, having 70 tubes, each 4″ in diameter?

13. What is the total tube heating surface of a 16′ boiler having 60 3″ tubes, each $\frac{1}{8}''$ thick?

14. What is the heating surface per foot of a $2\frac{1}{2}''$ boiler tube $\frac{1}{8}''$ thick?

15. The safety valve of a boiler is 4″ in diameter, the center of the valve is 5″ from the pin at the end of the lever, the lever is 51″ long from the pin and carries a weight of 112 lb. at the end; the weight of the valve is $7\frac{1}{2}$ lb., of the lever 42 lb.; the center of gravity of the lever is 16″ from the pin. At what pressure will the valve blow off?

Superheated Steam

The steam used in boilers should be as dry as possible. It may be made dry by heating it to a higher temperature by passing it through a vessel or coils of pipe separated from the boiler and called a superheater. Every passage conveying superheated steam must be well covered with non-conducting material.

The temperature of steam in contact with the water from which it is generated, as in the ordinary steam boiler, depends upon the pressure. But if the vessel is closed, as in the case of boilers, the pressure becomes greater and raises the boiling point of the water. Steam confined and under pressure has considerable energy stored up and is a powerful moving force when allowed to enter the piston chamber of an engine.

Boiler Pumps

The water inside a boiler is usually kept at the proper level by means of pumps or injectors. Most boilers have at least two means of feeding water. Steam pumps are most commonly used on stationary and marine boilers. There are several kinds of steam pumps: such as boiler feeders, general surface pumps, tank pumps, and municipal waterworks pumps. These are single or duplex.

The duplex pump is most commonly used because it is the simplest. The mechanism of it is much the same as that of the ordinary force pump, although it is a combination of two steam pumps, so arranged that the valve of one is operated by the piston of the other.

SECTIONAL VIEW OF INJECTOR

An *injector* is an apparatus for forcing water against pressure by the direct action of steam on the water. It is universally used on locomotives and occasionally on stationary boilers. Steam is led from the boiler through a pipe which terminates in a nozzle surrounded by a cone. This cone-shaped pipe is connected with the water tank or well where the water is stored.

When the steam passes into the injector, it rushes with great velocity from the nozzle and creates a partial vacuum in the cone. This causes atmospheric pressure to force water up to the cone, and there the kinetic action of the steam imparts velocity to it and overcomes the boiler pressure.

Size of Pump. — To find the size of a pump to supply a boiler: Multiply the volume in cu. in. of the water evaporated by each H. P. per hour and divide this product by the number of inches the plunger travels per minute. The quotient gives the area or size of the pump.

EXAMPLE. — Find the size of a pump that supplies a boiler furnishing steam for a 50 H. P. engine, the plunger making $47\frac{1}{2}$ ft. per minute, if 30 lb. of water are evaporated per hour for each H. P.

30 lb. water = $3\frac{1}{2}$ gal. approx.
$3\frac{1}{2} \times 231 = 808\frac{1}{2}$ cu. in.
$47\frac{1}{2} \times 12 = 570$
$808.5 \div 570 = 1.5$ sq. ft. in area. *Ans.*

The H. P. necessary to pump a given amount of water to a given height is found by multiplying the total weight of the water in pounds by the height in feet and dividing by 33,000 lb.; the quotient will be the required H. P.

EXAMPLE. — If the water end of a pump has a diameter of 6″ and the pump is running at a speed of 100′ per minute, if no leaks are accounted, how many gallons of water is it delivering?

Area of pump = $6'' \times .7854 = 28.27$ sq. in.
100 ft. or 1200 in. per minute
$28.27 \times 1200 = 33{,}924$ cu. in.
$\frac{33{,}924}{231} = 146.8$ gal. per minute. *Ans.*

Capacity of Pump. — To find the number of gallons of water that a pump of a given size is capable of raising per minute, multiply the volume of the water cylinder in cubic inches by

the number of feet the piston travels per minute, divide the result by 231, and the quotient will be the number of gallons per minute.

EXAMPLE. — How much water will a pump of 4 in. diameter deliver in one hour if the plunger makes $47\frac{1}{2}$ ft. per minute?

$$4 \times 4 \times .7854 = 12.56 \text{ sq. in.}$$
$$12.56 \times 47.5 \times 12 = 71592 \text{ cu. in.}$$
$$\frac{71592 \times 60}{231} = 18595.2 \text{ gallons per minute}$$

EXAMPLES

1. Find the size of a pump supplying a boiler which furnishes steam for 80 H. P. in 1 hour's time, if the plunger makes 55 ft. per minute and 30 lb. of water are evaporated per hour for each H. P.

2. How much water will a pump deliver in one hour, if the size is 3″ drain and the plunger makes 43′ per minute?

3. How many gallons of water will a 6″ pump with a piston speed of 60′ per minute raise in one hour?

4. What H. P. is necessary to pump 7583 gallons of water 134 feet?

5. Find the number of gallons of water a pump with an end diameter of 4″ and running at the rate of 84′ per minute will deliver.

6. Find the size of a pump that supplies a boiler furnishing steam at 98 H. P. in one hour's time, the plunger making 72 ft. per minute, if 30 lb. of water are evaporated per hour for each H. P.

7. How much water will a pump deliver in one hour, if the size of the drain is $3\frac{1}{2}$″ and the plunger makes $39\frac{1}{2}$′ per minute?

8. Find the number of gallons of water that a $5\frac{1}{2}$″ pump with a piston speed of 54′ per minute will raise in one hour.

9. What H. P. is necessary to pump 2981 gallons 47 feet high?

10. How much water will a pump deliver in 30 minutes if the drain is $4\frac{1}{2}''$ and the plunger makes $51\frac{1}{2}'$ per minute?

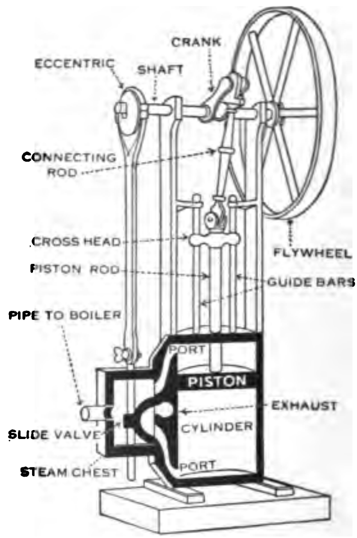
11. Find the number of gallons of water a pump with a diameter of $3''$ and running at rate of $109'$ per minute will deliver.

12. What H. P. is necessary to pump 21,809 gallons of water 60 feet?

CHAPTER XIV

ENGINES

The principal parts of a simple engine are the frame, cylinder, piston rods, eccentric, crank shaft, and governor. The cylinder is the long, round, iron band or tube in which the piston works. The piston is a device fitting into the cylinder and dividing it into compartments. Packing rings are provided to make it steam tight. The piston moves back and forth, forced by the steam which is alternately admitted on each side of it by means of valves. This back and forth movement thus imparted to the piston by the steam is transmitted to the crank and then to the large flywheel. The flywheel, by means of a belt or cable, transmits motion to smaller wheels or pulleys which drive machines.



VERTICAL STEAM ENGINE

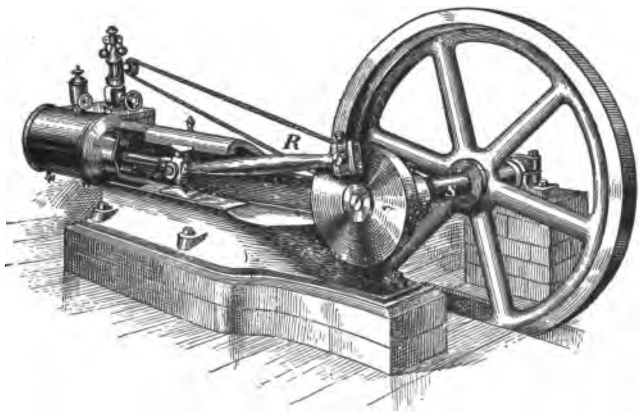
After the steam has moved the piston either way, it escapes into the air or passes into one or more cylinders, where it receives further expansion. An engine which allows steam to escape into one cylinder only is called a simple engine; if the steam is allowed to expand twice, it is a compound engine; and if three times, a triple expansion engine.

Maximum Pressure. — The maximum pressure on the guides between the crank pins and piston is found by multiplying the area of the piston by the average pressure. This product is the horizontal push on the crosshead. Multiply the horizontal push on the crosshead by the length of crank in inches and divide by the length of the connecting rod in inches. The quotient will represent the greatest (maximum) thrust at right angles.

EXAMPLE. — Find the maximum thrust at right angles of an engine 12″ × 24″ with a 40 lb. average pressure on the piston, the length of the rod being 60″ and of the crank 12″.

Area of piston = 12″ × 12″ × .7854 = 113.0976 sq. in. or 113 approx.
113.0976 sq. in. × 40 = 4523.9 lb. horizontal push on crosshead

$$\frac{4523.9 \times \cancel{12}}{\underset{5}{\cancel{60}}} = 905 \text{ lb. } \textit{Ans.}$$



HORIZONTAL ENGINE

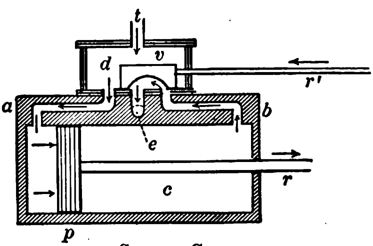
Weight of Flywheel. — The weight of the flywheel of an engine may be calculated by multiplying the area of the piston by the length of one stroke in feet, and then multiplying the product by the constant 12,000,000. Call this product No. 1.

Then multiply the square of the number of revolutions by the square of the diameter of the wheel in feet. Call this product No. 2. Divide product No. 1 by product No. 2, and the quotient will give the proper weight of the flywheel in pounds.

EXAMPLE. — Find the weight of a flywheel for an engine 12″ × 24″, if the diameter of the wheel is 6′, and makes 140 R. P. M.

$$12 \times 12 \times .7854 = 113.0976 \text{ sq. in., area of piston}$$
$$113.0976 \times 2 = 226.1952 \text{ sq. in.}$$
$$226.1952 \times 12{,}000{,}000 = 2{,}714{,}342{,}400 \quad (1)$$
$$140 \times 140 = 19{,}600$$
$$19{,}600 \times 6 \times 6 = 705{,}600 \quad (2)$$
$$2{,}714{,}342{,}400 \div 705{,}600 = 3847 \text{ lb. } \textit{Ans.}$$

Steam Lap. — The amount of steam lap that should be added to a common slide valve is obtained as follows: Divide the cut-off distance by the distance of the pull stroke and extract the square root of the quotient. Multiply this square root by one half of the valve travel and subtract one half of the lead from this product.



STEAM CHEST
v, slide valve p, piston
r', valve rod r, piston rod

EXAMPLE. — An engine has a 48″ stroke, with a valve travel of 6″ and a cut-off at half stroke and a valve of $\frac{1}{4}''$ lead. What is the necessary steam lap?

$$\tfrac{24}{48} = .5 \qquad \sqrt{.5} = .707$$
$$.707 \times 3 = 2.121 \quad 2.121 - \tfrac{1}{8} = 2.121 - .1250 = 1.996''. \quad \textit{Ans.}$$

Horse Power

The power of a steam engine is commonly designated as horse power. By one horse power is meant a force great enough to raise 33,000 lb. one foot high in one minute. There

are two measures of horse power in engines: indicated, and actual or net. Indicated horse power is obtained by multiplying the area of the piston in square inches, and the mean effective pressure in the cylinder in pounds per square inch, and the speed in feet per minute, and dividing the product by 33,000. The actual or net horse power is the difference between the indicated horse power and the amount expended in overcoming friction.

EXAMPLE. — What is the horse power of an engine which can pump in one minute 68 cu. ft. of water from a depth of 108 ft.?

$$68 \times 62\tfrac{1}{2} = 4250 \text{ lb.} \quad 4250 \times 108 = 459{,}000 \text{ ft. lb.}$$

$$\frac{459{,}000}{33{,}000} = \frac{153}{11} = 13\tfrac{10}{11} \text{ H. P.} \quad \text{Approx. 14 H. P.} \quad \textit{Ans.}$$

The horse power of an engine is expressed by the following formula:

A = area of piston in square inches
P = mean effective pressure of steam on piston per sq. in.
V = velocity of piston per minute in feet

$$\text{H. P.} = \frac{A \times P \times V}{33{,}000}$$

NOTE. — A quick method to find the H. P. of an engine is to square the diameter of the cylinder in inches and divide the product by 2.5. The quotient is approximately the H. P.

$$\text{H. P.} = \frac{D^2}{2.5}$$

$$D = \sqrt{2.5 \times \text{H. P.}} = 1.58\sqrt{\text{H. P.}} \quad \text{approx.}$$

Diameter of Cylinder. — To find the diameter of a cylinder of an engine of a required nominal horse power:

$$\frac{33{,}000}{PV} \times \text{H. P.} = A \qquad \frac{A}{.7854} = D^2$$

$$D = \sqrt{\frac{A}{.7854}}$$

Diameter of Supply Pipe. — The diameter of the steam supply pipe for a given engine may be calculated from the H. P. of the engine by dividing the H. P. by 6, and extracting the square root of the quotient.

$$D = \sqrt{\frac{\text{H. P.}}{6}}$$

Example. — What is the diameter of a steam supply pipe of a 216 H. P. engine?

$$216 \div 6 = 36$$

$\sqrt{36} = 6''$, diameter of supply pipe. *Ans.*

EXAMPLES

1. Find the diameter of the steam supply pipe of a 180 H. P. engine.

2. What is the H. P. of an engine whose area of piston is 114 sq. in., mean effective pressure 80, and velocity of piston 112 ft. per minute?

3. What is the approximate diameter of a cylinder of an engine of 50 H. P.?

4. What is the approximate H. P. of an engine the cylinder diameter of which is 28″?

5. What is the effective area, for power calculation, of the piston of a steam engine, the bore of the cylinder being 28″, and the diameter of the piston rod which passes through both ends of the cylinder 2″?

6. What is the H. P. of an engine that can raise 3 tons of coal (1 ton = 2240 lb.) from a mine 289 ft. deep?

7. What is the approximate H. P. of an engine the cylinder diameter of which is 16″?

8. Find the diameter of the steam supply pipe of a 24 H. P. engine.

9. What is the approximate diameter of a cylinder of an engine of 80 H. P.?

10. What is the approximate H. P. of an engine the cylinder diameter of which is 20″?

11. What is the H. P. of an engine whose diameter of piston is 15″, mean effective pressure 110, velocity of piston per minute 189′?

12. How many pounds of water per half minute can an 8 H. P. fire pump raise to a height of 86 ft.?

13. What is the effective area in square inches of the piston of a steam engine if the diameter of the cylinder is 20″ and the diameter of the piston rod is 3″?

14. What is the horse power of an engine that is required to pump out a basement 51′ × 22′ × 10′ deep, full of water, in 20 minutes?

15. (*a*) Find the diameter of the steam supply pipe of a 98 H. P. engine.

(*b*) Find the approximate diameter of a cylinder of an engine of 48 H.P.

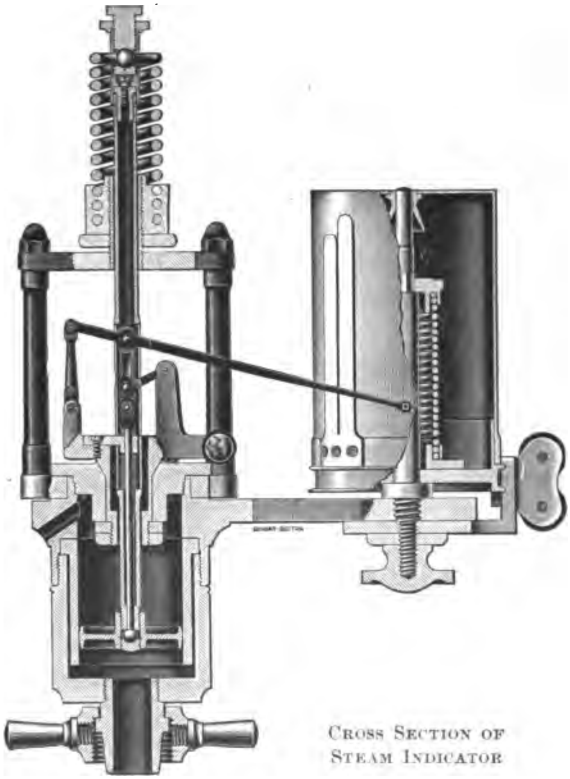
(*c*) What is the approximate H.P. of an engine the cylinder diameter of which is 28″?

Steam Indicator

In order to know the condition of the steam within the cylinder of an engine an indicator is used. This consists of a small cylinder containing a piston, the rod of which is enclosed in a spiral spring which opposes the motion of the piston. The piston rod, after passing through the top of the cylinder cover, is connected with a long light lever, on the end of which is a pencil. This pencil moves in a vertical straight line whenever the piston moves.

Another cylinder with an axis parallel to the first carries a paper drum, and this drum is connected to the crosshead of the engine by means of a cord and a reducing motion, so that

the movement of the drum is proportional to that of the cross-head. When the pipe between the indicator and the engine is closed by means of a cock, the pencil, when held against the drum, makes a horizontal line called an atmospheric line. If



CROSS SECTION OF STEAM INDICATOR

the cock is opened, admitting to the small cylinder of the indicator the pressure that exists in the engine cylinder, the pencil will trace a figure, every point of which is at a height from the atmospheric line proportional to the number of pounds' pressure in the engine cylinder at every point in the stroke.

Operating Power. — To find the power required to drive a certain machine when driven direct from the shaft or engine: Indicate the engine with the machinery running and calculate from the card. Then indicate the engine (from formula on p. 225) without the machinery running and from this obtain the H. P. The difference will give the power for operating.

To indicate an engine means to place a steam engine indicator on the steam engine and record on the diagrams the pressure in the steam cylinder at every part of the stroke. The diagrams are measured by means of a planimeter or by means of ordinates. The latter way may be done by dividing the diagram into 10 or 20 equal spaces by vertical lines. On these verticals measure the length between the back pressure line and mark each length on a long strip of paper. Dividing the sum of all these lengths by 10 or 20 will give the average length of ordinate which multiplied by the scale of the spring will give the mean effective pressure (M.E.P.).

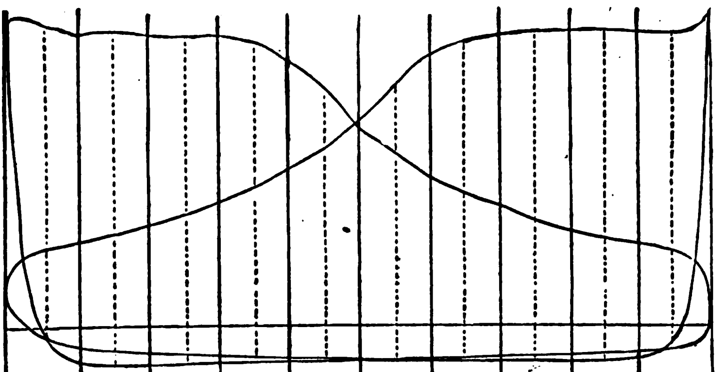
A  B

DIAGRAM FROM HARTFORD ENGINE

Cylinder, 16 × 24 inches. Boiler pressure, 87 pounds. Vacuum by gauge, 23¼ inches. 130 revolutions per minute. Scale, 50. The vertical lines from *A* are called ordinates.

EXAMPLE. — If the total length is 9″ and the spring used is 40 lb., how is the mean effective pressure (M. E. P.) found?

$$\frac{9}{10}' = .9''$$

$$40 \times .9 = 36 \text{ lb. M. E. P.} \quad \textit{Ans.}$$

EXAMPLES[1]

1. If the total length of the ordinates is 11″ and the spring used is 56 lb., what is the M. E. P.?

2. A 9 × 10 engine has a 48 lb. average pressure on the piston, the length of rod is 35″, and the crank is 5″. Find the maximum thrust at right angles.

3. Find the weight of a flywheel of an engine 12″ × 24″, diameter of wheel 7′, with 168 R. P. M.

4. What is the necessary steam lap of an engine with a 45″ stroke, valve travel 5″, and cut-off at half stroke, and valve $\frac{1}{4}$″ lead?

5. If the total length of ordinates is 10″ and the spring used is 49 lb., what is the M. E. P.?

6. A 12″ × 24″ engine has a 50 lb. average pressure on the piston, the length of the rod is 59″, and the crank is 12″; what is the maximum thrust at right angles?

7. What is the necessary steam lap of an engine with a 64″ stroke, valve travel 7, cut-off at half stroke, and valve $\frac{1}{4}$″ lead?

8. If the total length of ordinates is 8″ and the spring used is 38 lb., what is the M. E. P.?

9. Find the weight of a flywheel of an engine 24″ × 60″, the diameter of wheel being 24′ and having 75 R. P. M.

10. If the total length of ordinates is 12″ and the spring used is 61 lb., what is the M. E. P.?

11. Find the weight of a flywheel of an engine 30″ × 60″, the diameter of the wheel 30′, with 63 R. P. M.

12. An engine 10″ × 12″ has a 35 lb. average pressure on the piston, the length of the rod is 42″, and the crank is 6″. Find the maximum thrust at right angles.

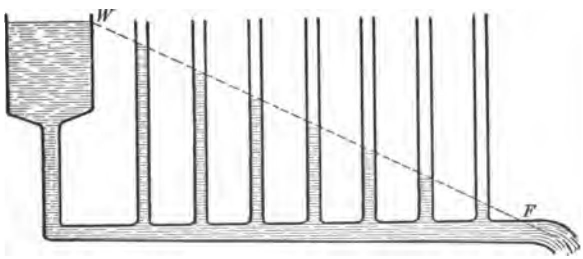
[1] Use 10 ordinates in solving problems.

PART VIII — MATHEMATICS FOR ELECTRICAL WORK

CHAPTER XV

COMMERCIAL ELECTRICITY

Amperes. — What electricity is no one knows. Its action, however, is so like that of flowing water that the comparison is helpful. A current of water in a pipe is measured by the amount which flows through the pipe in a second of time, as one gallon per second. So a current of electricity is measured



WATER ANALOGY OF FALL OF POTENTIAL

by the amount which flows along a wire in a second, as one *coulomb* per second, — a coulomb being a unit of measurement of electricity, just as a gallon is a unit of measurement of water. The rate of flow of one coulomb per second is called one *ampere*. The rate of flow of five coulombs per second is five amperes.

Volts. — The quantity of water which flows through a pipe depends to a large extent upon the pressure under which it flows. The number of amperes of electricity which flow along

a wire depends in the same way upon the pressure behind it. The electrical unit of pressure is the *volt.* In a stream of water there is a difference in pressure between a point on the surface of the stream and a point near the bottom. This is called the *difference* or *drop* in level between the two points. It is also spoken of as the pressure head, "head" meaning the difference in intensity of pressure between two points in a body of water, as well as the intensity of pressure at any point. Similarly the pressure (or voltage) between two points in an electric circuit is called the *difference* or *drop* in pressure or the *potential.* The *amperes* represent the amount of electricity flowing through a circuit, and the *volts* the pressure causing the flow.

Ohms. — Besides the pressure the resistance of the wire helps to determine the amount of the current: — the greater the resistance, the less the current flowing under the same pressure. To the electrical unit of resistance the name *ohm* is given. A wire has a resistance of one ohm when a pressure of one volt can force no more than a current of one ampere through it.

Ohm's Law. — The relation between current (amperes), pressure (volts), and resistance (ohms) is expressed by a law known as *Ohm's Law.* This is the fundamental law of the study of electricity and may be stated as follows:

An electric current flowing along a conductor is equal to the pressure divided by the resistance.

$$\textit{Current}\text{ (amperes)} = \frac{\textit{Pressure}\text{ (volts)}}{\textit{Resistance}\text{ (ohms)}}$$

Letting $I =$ amperes, $E =$ volts, $R =$ ohms,

$$I = E \div R \text{ or } I = \frac{E}{R}$$

$$E = IR$$

$$R = \frac{E}{I}$$

EXAMPLE. — If a pressure of 110 volts is applied to a resistance of 220 ohms, what current will flow?

$$I = \frac{E}{R} = \frac{110}{220} = \frac{1}{2} = .5 \text{ ampere.} \quad \textit{Ans.}$$

EXAMPLE. — A current of 2 amperes flows in a circuit the resistance of which is 300 ohms. What is the voltage of the circuit?

$$IR = E$$
$$2 \times 300 = 600 \text{ volts.} \quad \textit{Ans.}$$

EXAMPLE. — If a current of 12 amperes flows in a circuit and the voltage applied to the circuit is 240 volts, find the resistance of the circuit.

$$\frac{E}{I} = R \qquad \frac{240}{12} = 20 \text{ ohms.} \quad \textit{Ans.}$$

Ammeter and Voltmeter. — Ohm's Law may be applied to a circuit as a whole or to any part of it. It is often desirable to

AMMETER

VOLTMETER

know how much current is flowing in a circuit without calculating it by Ohm's Law. An instrument called an *ammeter* is used to measure the current. This instrument has a low resistance so that it will not cause a drop in pressure. A *voltmeter* is used to measure the voltage. This instrument has high resistance so that a very small current will flow through

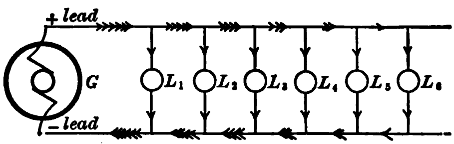
it, and is always placed *in shunt,* or parallel (see p. 235) with that part of the circuit the voltage of which is to be found.

EXAMPLE. — What is the resistance of wires that are carrying 100 amperes from a generator to a motor, if the drop or loss of potential equals 12 volts?

Drop in voltage $= IR$ $I = 100$ amperes
Drop in volts $= 12$ $R =$? ohms

$$R = \frac{E}{I} \qquad R = \frac{12}{100} = 0.12 \text{ ohm. } \textit{Ans.}$$

EXAMPLE. — A circuit made up of incandescent lamps and conducting wires is supplied under a pressure of 115 volts. The lamps require a pressure of 110 volts at their terminals



WIRING OF INCANDESCENT LAMP CIRCUIT

and take a current of 10 amperes. What should be the resistance of the conducting wires in order that the necessary current may flow?

Drop in conducting wires $= 115 - 110 = 5$ volts
Current through wires $= 10$ amperes

$$R = \frac{E}{I} = \frac{5}{10} = 0.5 \text{ ohm resistance. } \textit{Ans.}$$

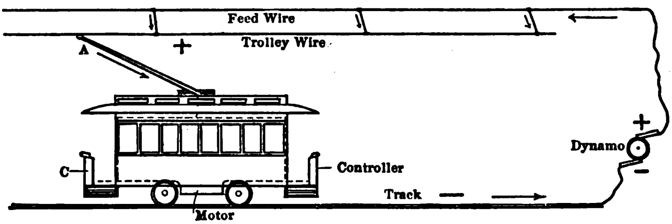
EXAMPLES

1. How much current will flow through an electromagnet of 140 ohms' resistance when placed across a 100-volt circuit?

2. How many amperes will flow through a 110-volt lamp which has a resistance of 120 ohms?

3. What will be the resistance of an arc lamp burning upon a 110-volt circuit, if the current is 5 amperes?

4. If the lamp in Example 3 were to be put upon a 150-volt circuit, how much additional resistance would have to be put into it in order that it might not take more than 5 amperes?



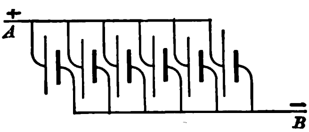
ELECTRIC ROAD SYSTEM

5. In a series motor used to drive a street car the resistance of the field equals 1.06 ohms; the current going through equals 30 amperes. What would a voltmeter indicate if placed across the field terminals?

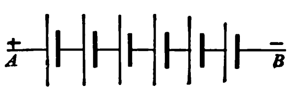
6. If the load upon the motor in Example 5 were increased so that 45 amperes were flowing through the field coils, what would the voltmeter then indicate?

Series and Parallel Circuits

Pieces of electrical apparatus may be connected in two ways. When the pieces are connected so that the current passes through them in a single path, they are said to be *in series.*



CELLS CONNECTED IN SERIES



CELLS CONNECTED IN PARALLEL

When the pieces are connected so that the current is divided between them, they are said to be *in parallel* with one another.

The total *resistance* of a *series circuit* is equal to the sum of the resistances of the separate parts of the circuit. The total

voltage of a series circuit is equal to the sum of the voltages across the separate resistances.

A —————— - - - - R_1 - - - - ——— - - - - - R_2 - - - - —————— *B*

Total resistance A to $B = R_1 + R_2$.

EXAMPLE. — If there is a circuit of 240 volts and the lamps are of 240 ohms' resistance but made to carry only ½ an ampere, two lamps would have to be put *in series* in order to use them on the 240-volt circuit.

The resistance of the two lamps *in series* would then be 480 ohms, the voltage of the circuit 240 volts, and the current by Ohm's Law

$$I = \frac{E}{R} = \frac{240}{480} = \frac{1}{2} \text{ ampere.}$$

A —————— - - - - R_1 - - - - - - - - - - - - - - - - - - R_2 - - - - —————— *B*

In any closed circuit, the algebraic sum of the products found by multiplying the resistance of each part by the current passing through it, is equal to the voltage of the circuit.

EXAMPLE. — If three lamps of 110 ohms' resistance are connected in series and take ½ ampere, the voltage of the circuit is:

$$I_1R_1 + I_2R_2 + I_2R_2 =$$
$$(\tfrac{1}{2} \times 110) + (\tfrac{1}{2} \times 110) + (\tfrac{1}{2} \times 110) =$$
$$55 + 55 + 55 = 165 \text{ volts.} \quad \textit{Ans.}$$

EXAMPLE. — A current of 50 amperes flowed through a circuit when the voltage was 550. What resistance should be added in series with the circuit to reduce the current to 11 amperes?

$$\text{Resistance} = \tfrac{550}{50} = 11 \text{ ohms}$$
$$\text{Resistance} = \tfrac{550}{11} = 50 \text{ ohms}$$
$$\text{Additional resistance} = 50 - 11 = 39 \text{ ohms.} \quad \textit{Ans.}$$

EXAMPLE. — The voltage required by 15 arc lamps connected in series is 900 and the current is 6 amperes. If the resistance of the connecting wires is 5 ohms, how much additional voltage will be necessary so that the lamp voltage may not drop below 900?

Drop in voltage in connecting wires $= E = IR$

$6 \times 5 = 30$ volts = additional voltage necessary. *Ans.*

Example. — The field coil of a motor having 4 poles is measured for voltage across the terminals and the following readings are take n:

Voltage across line $= 220$
Voltage across coil No. 1 $= 73.33$
Voltage across coil No. 2 $= 00.00$
Voltage across coil No. 3 $= 73.33$
Voltage across coil No. 4 $= 73.33$

Current flowing $= 1.5$ amperes.

What is the total resistance and what is the trouble at coil No. 2?

Total resistance $= R = \frac{E}{I} = \frac{220}{1.5} = 146.6$ ohms. *Ans.*

As there is no drop across the terminal of coil No. 2, there is practically no resistance and the current is not going around the coil but through a path of extremely low resistance.

EXAMPLES

1. If three electromagnets are connected in series and the resistances are 3, 5, and 17 ohms, respectively, what is the total resistance of this set?

2. The field coils of a series motor have a resistance of 10 ohms and the armature has a resistance of 7 ohms; what is the total resistance of the motor?

3. (*a*) What would be the total resistance of two 110-volt incandescent lamps placed in series across a 110-volt line if each lamp has a resistance of 220 ohms? (*b*) Would these lamps light on this voltage in this position? (*c*) Why?

4. Three coils are connected in series and have a resistance of 3, 5, and 8 ohms, respectively. What current will flow if the voltage of the circuit is 64?

5. Five arc lamps, each having a resistance of 4 ohms, are connected in series. The resistance of the connecting wires and the other apparatus is 5 ohms. What must be the voltage of the circuit so that a current of 10 amperes may flow?

6. A current of 10 amperes was passing through a circuit under a pressure of 550 volts. The circuit was made up of three sections connected in series, and the resistance of two sections was 8 and 12 ohms, respectively. What was the resistance of the third section?

Example. — If 5 electromagnets are arranged in series and marked A, B, C, D, and E, the resistance of the circuit is 45 ohms and the resistance of each coil is: A, 5 ohms; B, 10 ohms; C, 7 ohms; D, 8 ohms; E, 15 ohms. How much E. M. F. would be required to cause 10 amperes to flow through the coils, and what would be the E. M. F. across the terminals of each coil?

$$\begin{aligned}
\text{Voltage across } A &= 50\\
\text{Voltage across } B &= 100\\
\text{Voltage across } C &= 70\\
\text{Voltage across } D &= 80\\
\text{Voltage across } E &= \underline{150}\\
&\ 450
\end{aligned}$$

$10 \times 45 = 450$, total voltage. *Ans.*

Example. — What E. M. F. is necessary to send a current through 10 field coils connected in series, if each has a resistance of 10 ohms and 3 amperes are required to produce the necessary magnetization?

10 ohms = resistance of each coil
10 coils are in series
$10 \times 10 = 100$ ohms, total resistance
$R = 100$ ohms
$I = 3$ amperes
$E = IR = 3 \times 100 = 300$ volts. *Ans.*

In any closed circuit the algebraic sum of the products found by multiplying the resistance of each part by the current passing through it, is equal to the voltage of the circuit. This is practically an inverse statement of the law of series circuits.

Example. — If we have five lamps of 110 ohms' resistance, connected in series and taking $\frac{1}{2}$ ampere, the voltage of the circuit is:

$E = I_1R_1 + I_2R_2 + I_3R_3 + I_4R_4 + I_5R_5 = \frac{1}{2} \times 110 + \frac{1}{2} \times 110 + \frac{1}{2} \times 110 + \frac{1}{2} \times 110 + \frac{1}{2} \times 110 = 55 + 55 + 55 + 55 + 55 = 275$ volts. *Ans.*

In a parallel circuit the voltage across each branch is the same as the voltage across the combination. The current is equal to the sum of the currents in the separate parts.

The resistance is equal to the reciprocal of the sum of the conductances of the separate parts.

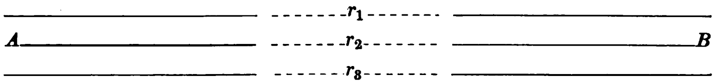
Conductance is the reciprocal of resistance and is equal to $\frac{1}{R}$. The unit of conductance is mho (ohm spelled backwards).

$$\text{mho} = \frac{1}{\text{ohm}} \text{ or ohm} = \frac{1}{\text{mho}}$$

Series and parallel circuits may be combined and may exist in the same circuit. In parallel circuits the reciprocal of the total resistance is equal to the sum of the reciprocals of the paralleled resistance.

If R_0 = resistance of circuit and

r_1, r_2, and r_3 = parallel resistances,



resistance between A and $B = R_0$.

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

EXAMPLE.—Suppose that r_1, r_2, and r_3 are lamps of 300 ohms' resistance each, then

$$\frac{1}{R_0} = \frac{1}{300} + \frac{1}{300} + \frac{1}{300} = \frac{3}{300} = \frac{1}{100}$$

$R_0 = 100$ ohms. *Ans.*

Or, $r_1 = 100$ ohms, $r_2 = 50$ ohms, $r_3 = 300$ ohms

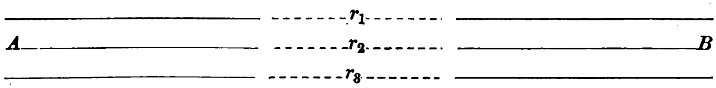
$$\frac{1}{R_0} = \frac{1}{100} + \frac{1}{50} + \frac{1}{300} = \frac{10}{300} = \frac{1}{30}$$

$R_0 = 30$ ohms. *Ans.*

When it is necessary to get the resistance of parallel circuits, it is often more convenient to use the sum of the conductances as the total conductance of the circuit.

EXAMPLE. — Three resistances in parallel:

$$r_1 = 2 \text{ ohms}$$
$$r_2 = 5 \text{ ohms}$$
$$r_3 = 1 \text{ ohm}$$



Conductance

$$(1)\ \frac{1}{r_1} = \frac{1}{2} = \ .5 \text{ mho}$$
$$(2)\ \frac{1}{r_2} = \frac{1}{5} = \ .2 \text{ mho}$$
$$(3)\ \frac{1}{r_3} = \frac{1}{1} = 1 \quad \text{mho}$$
$$\overline{1.7} \text{ mho conductance of the circuit}$$

Resistance of circuit $AB = \frac{1}{1.7} = .588$ ohm. *Ans.*

EXAMPLES

1. Ten arc lamps of 200 ohms' resistance are connected in series and the voltage of the circuit is 300. How much current does each lamp take?

2. What will be the E. M. F. necessary to supply 60 Thompson-Houston arc lamps arranged in series, the resistance of each lamp being 5 ohms when burning, making a total resistance of 300 ohms in the circuit, if the current required is 10 amperes?

3. Four parallel circuits of 2, 4, 5, and 10 ohms' resistance, respectively, have 40 volts impressed upon their terminals. (*a*) What is the total current flowing? (*b*) How much current flows in each branch?

4. Three incandescent lamps of different sizes are placed in parallel on a circuit. These have respective resistances of 100, 150, and 300 ohms. What is the total current flowing through these lamps when the pressure applied is 100 volts?

5. What is the total resistance of four incandescent lamps placed in parallel, if each lamp has a resistance of 220 ohms?

6. What is the resistance of a shunt-wound generator, if the field and the armature are respectively 25 and 10 ohms?

7. Two coils on a large electromagnet for lifting iron ore are connected in parallel and the resistance of each coil is 40 ohms. What is the whole resistance?

8. The resistance of a line from a power house to a mill is 6 ohms; there are 50 lamps in the mill, each lamp having a resistance of 220 ohms. What is the total resistance from the power house?

9. In an electric street car 4 heaters are all connected in series and each has a resistance of 20 ohms with a voltage of 500 across the circuit. (*a*) What is the total resistance of these? (*b*) How many amperes will go through them?

Power Measurement

The flow of an electric current has been compared to the flow of water through a pipe. The water current is measured by the number of gallons or pounds flowing per minute. A current of electricity is measured by the number of amperes or coulombs per second. When a gallon of water is raised a foot by means of a pump a certain amount of work is done. So when a coulomb of electricity is passed through a wire under the pressure of one volt, a certain amount of work is done. In the case of water the work done is measured in foot pounds. A foot pound is the work done in raising a weight of one pound through a distance of one foot.

$$Work = Force \times Distance$$

When one coulomb of electricity is passed through a wire under a pressure of one volt, the amount of work done is called one *joule*.

The power required to keep a current of water flowing is the product of the current in pounds per minute by distance in feet.

This gives the power in foot pounds per minute. Mechanical power is usually expressed in horse power (H. P.). The power required to keep a current of electricity flowing is the product of the current in amperes by the pressure in volts and is expressed in *watts*.

1 H. P. = 746 watts
1000 watts = 1 kilowatt
1 kilowatt (K. W.) = 1.34 or $1\frac{1}{3}$ H. P.

$$\frac{\textit{Volts} \times \textit{Amperes}}{1000} = \textit{Kilowatts}$$

Let $P =$ power in watts
$I =$ current in amperes
$E =$ pressure in volts

then (1) $P = IE$ (equation for power)
but $E = IR$ (Ohm's Law)
therefore $P = I\ (IR)$ or I^2R (by substitution)
(2) $P = I^2R$
but $I = \frac{E}{R}$ (Ohm's Law)
therefore $P = \frac{E}{R}\ (E)$ or $\frac{E^2}{R}$ (by substitution)
(3) $P = \frac{E^2}{R}$
thus $P = IE = I^2R = \frac{E^2}{R}.$

To measure power a *wattmeter* is used, which is a combination of a voltmeter and an ammeter.

In order to find the amount of work done by a certain engine, it is necessary to know the time it has been running and the power it has been supplying, *i.e.* its rate of doing work. If the power is measured in horse power and the time in hours, the work is done in horse power hours. Similarly, if the power is measured in kilowatts and the time in hours, the work done is measured in kilowatt hours (K. W. H.).

1 H. P. H. = 0.746 K.W. H.
1 K. W. H. = 1.34 H. P. H.

These units are too large to be used conveniently in all problems, so a smaller electrical unit called the *watt second*, or *joule*, is used.

EXAMPLES

1. If the resistance of a circuit is 1 ohm and the current 30 amperes, what energy is expended in one half hour?

2. With a potential difference of 95 volts and a current of 15 amperes, what energy is expended in 20 minutes?

3. If a current of 100 amperes flows for 2 minutes under a pressure of 500 volts, what is the work done in joules?

4. If 12 incandescent lamps burn for 10 hours under a pressure of 110 volts, each lamp consuming $\frac{1}{2}$ an ampere, how many kilowatt hours are used?

5. Fifty horse power expended continuously for one hour will produce how many kilowatt hours?

6. If 4000 watts are expended in a circuit, how much horse power is being developed?

7. If 20 horse power of mechanical energy were converted into electrical energy, how many watts would be developed?

8. If a current of 50 amperes flows through a circuit under a pressure of 220 volts, what is the power?

9. If 200 watts are expended in a circuit by a current of 4 amperes, what is the voltage required to drive the current through the wire?

10. If an incandescent lamp requires $\frac{1}{2}$ an ampere of current and the resistance of its filament is 220 ohms, how many watts are required for it?

Measurement of Resistance

The amount of current in a circuit depends upon the voltage and upon the resistance. To control the current it is necessary to change one of these two factors. The resistance to the flow of water through a pipe depends upon the shape of the pipe and

its length. The electrical resistance of a conductor depends upon the nature of the metal from which the conductor is made, its size, its length, and the temperature. The greater the size of the conductor, the greater is its power for conducting electricity, and therefore the less its resistance. The longer the wire, the less its conducting power, and therefore the greater the resistance. As the resistance of a large pipe is less than the resistance of a small one, so the resistance of a large wire is less than the resistance of a small one.

Copper is the material generally used for wires, and its conductivity, or capacity for conducting current, is taken as the standard. The conductivity of pure copper is expressed as 100 %. Commercial copper usually has from 98 to 99 % conductivity. Other materials used in electric wires are iron, aluminum, brass, etc. Iron has a conductivity of about 16 % and brass of about 25 %. It would require an iron wire with over 6 times the cross section of a copper wire to give the same conductivity, and brass wire would have to be about 4 times as large in its cross section for the same conductivity. Generally it is assumed that all electric wires are copper.

In measuring the length of wires the unit used is feet, while the cross section area is measured in circular *mils;* $\frac{1}{1000}$ of an inch is called a *mil,* and a round wire one mil in diameter is said to have a cross-section area of one *circular mil.* A wire 1 foot long, with a cross-section area of 1 circular mil, is called a mil-foot wire. The area of any wire in circular mils may be found by squaring the number of thousandths of an inch in the diameter.

If R = resistance of wire in ohms
L = resistance of wire
D = diameter of wire in mils
d^2 = area of wire in circular mils
K = resistance of 1 mil foot in ohms called resistivity or specific resistance of material

then $$R = \frac{KL}{d^2}$$

NOTE. — K varies with the material and the temperature. The resistance of 1 mil foot of soft copper wire at 50° F. is 10.4 ohms.

EXAMPLE. — What is the area of a wire 0.1 inch in diameter?

0.1 inch = 100 thousandths of an inch
$100 \times 100 = 10{,}000$, number of circular mils. *Ans.*

EXAMPLE. — What is the resistance at ordinary temperature of a copper wire 2500 ft. long with a cross-section area of 10,000 circular mils? ($K = 10$.)

$$R = \frac{KL}{d^2} = \frac{10 \times 2500}{10000} = 2.5 \text{ ohms.} \quad \textit{Ans.}$$

EXAMPLES

1. What must be the diameter of a wire in mils in order that it may have a cross-section area of 200 circular mils?

2. How many circular mils are there in a wire 50 mils in diameter?

3. How many circular mils are there in a wire 150 mils in diameter?

4. What is the diameter in inches of a copper wire which has a cross-section area of 20,000 circular mils?

5. If it is desired to have a copper wire of $\frac{1}{2}$ ohm resistance and 2000 ft. long, what must its cross section be?

6. What pressure is required to force a current of 50 amperes over a copper wire 1600 ft. long which has a cross-section area of 20,000 circular mils?

7. A current is forced through a copper wire 2000 ft. long under a pressure of 50 volts. If the wire has an area of 5000 circular mils, what is the value of the current flowing?

8. If a wire 5000 ft. long carries a current of 5 amperes under a pressure of 100 volts, what is the cross-section area of the wire?

9. A wire 100 ft. long is in series with another wire 500 ft. long and the first wire is $\frac{1}{2}''$ in diameter. If the second wire has a cross-section area of 20,000 circular mils, what is the resistance of the circuit?

10. How many amperes will a wire $\frac{1}{8}''$ in diameter carry if a wire 1000 mils in diameter will carry 650 amperes?

Size of Wire

If an electrician wishes to know the size of a wire to carry a certain current a certain distance with a certain drop of voltage, he may ascertain it by substituting values in the following formula, which is called a two-wire formula,

$$CM = \frac{21.6 \times DI}{e}$$

CM = size of wire in circular mils
D = distance from distribution
I = amperage
e = drop or volts lost

Example. — What size of wire will be required for a motor situated 85 ft. from the center of distribution, if the motor is 5 H. P., operating at difference of potential of 110 volts, allowing 3 % drop?

$P = EI$
1 H. P. = 746 watts
$P = 746 \times 5 = 3730$
$E = 110$

$$I = \frac{P}{E} \qquad I = \frac{3730}{110} \qquad e = 110 \times .03 = 3.30 \qquad D = 85 \text{ ft.}$$

$$CM = \frac{21.6 \times 85 \times \frac{3730}{110}}{3.30} = \frac{21.6 \times 85 \times 33.90}{3.30}$$

$$= 1886.07\tfrac{8}{11} \text{ wire.} \quad \textit{Ans.}$$

REVIEW EXAMPLES

1. What size of wire will be required for a 15 H. P. motor operating at 550 volts and situated 35 ft. from the center of distribution, allowing a 2 % drop?

2. How many coulombs are delivered in a minute when the current is $17\frac{1}{2}$ amperes?

3. What is the current when 480 coulombs are delivered per minute?

4. In what time will 72,000 coulombs be delivered when the current is 80 amperes?

5. What size of wire will be required for a $7\frac{1}{2}$ H. P. motor operating at 220 volts and situated 65 ft. from the center of distribution and allowing a 5 % drop?

6. If a current of 20 amperes flows through a circuit for 21.2 hours, what quantity of electricity is delivered?

7. How many ampere hours pass in a circuit in $2\frac{1}{2}$ hours when the current is 16 amperes?

8. What size of wire will be required for a 10 H. P. motor operating at 110 volts and situated 105 ft. from the center of distribution, allowing a drop of 3 volts?

9. If 200 coulombs of electricity are passed through an electrolytic vat each second under a pressure of 10 volts, how many joules of work are expended in an hour?

10. What quantity of electricity must flow under a pressure of 5 volts to do 125 joules of work?

11. If 10 coulombs do 10 joules of work flowing through a wire, what is the pressure?

12. What must be the diameter of a wire in mils in order that it may have a cross section of 200 circular mils?

13. A wire 5000 ft. long carries a current of 5 amperes under a pressure of 100 volts. What is the cross-section area of the wire?

14. A copper wire $\frac{1}{2}$ inch in diameter and 100 ft. long is in series with another copper wire 500 ft. long with a cross section of 20,000 circular mils. What is the resistance of the circuit?

15. What is the resistance at ordinary temperature of a copper wire 2500 ft. long having a cross-section area of 10,000 circular mils?

16. If it is desired to have a copper wire of $\frac{1}{2}$ ohm resistance and 2000 ft. long, what must be its cross-section area?

17. A voltmeter which measures the pressure on a circuit registers 500 volts and the ammeter on the same circuit shows 25 amperes. What is the resistance of the circuit?

18. An electromagnet has a resistance of 25 ohms, and there must be $4\frac{1}{2}$ amperes passing through it in order that the magnetism may be strong enough. What must be the voltage?

19. What current is needed to light a 16 C. P. lamp, if the hot resistance of the lamp is 220 ohms and the voltage is 110?

20. A storage battery gives 2.3 volts and it is connected with a coil having a resistance of 25 ohms. What current will flow through the circuit if the internal resistance of the cell is zero?

21. What is the power necessary to drive a current of 500 amperes through a resistance of 5 ohms?

22. How many watts of power are going to an electric motor if the voltage of the line is 500 and there are 7 amperes entering the motor?

23. How many H. P. are needed to run a dynamo that is lighting 258 lamps in parallel, if each lamp takes $\frac{1}{2}$ an ampere at 110 volts?

24. How many 40-watt electric glow lamps can be run on a 110-volt circuit with an expenditure of 48 amperes of current?

Brown and Sharpe Wire Table

The unit chosen for this table is a copper wire .1 inch in diameter. This is called No. 10 wire and has the following characteristics: No. 10, B & S wire, diameter, .1 inch; area in circular mils, 10,000 c.m.; resistance, per 1000 ft., 1 ohm;

weight per 1000 ft., 31.5 lb. Since this table was made, the standard has changed slightly, so that at present No. 10 wire is .1019 inch in diameter and the other values are changed proportionately, but for all commercial work the values as originally given are sufficiently accurate.

The table is so arranged that the area of the wire is doubled every three gauges down and halved every three numbers up.

EXAMPLE. — Find the area of No. 7 wire.

No. 7 is three numbers below No. 10, whose area is 10,000 c.m., so that its area is $2 \times 10,000 = 20,000$ c.m.

Area of No. 4 wire $= 2 \times 20,000 = 40,000$ c.m.
Area of No. 13 wire $= \frac{1}{2} \times 10,000 = 5000$ c.m.

The resistance is reduced to one half every three numbers down and doubled every three numbers up. The weight doubles every three numbers down and halves every three numbers up.

EXAMPLE. —

R per 1000 ft. of No. 10 B & S wire equals 1 ohm
R per 1000 ft. of No. 7 B & S wire equals .5 ohm
R per 1000 ft. of No. 4 B & S wire equals .25 ohm
R per 1000 ft. of No. 13 B & S wire equals 2 ohms
R per 1000 ft. of No. 16 B & S wire equals 4 ohms

EXAMPLE. —

W per 1000 ft. of No. 10 B & S wire equals 31.5 lb.
W per 1000 ft. of No. 7 B & S wire equals 63 lb.
W per 1000 ft. of No. 4 B & S wire equals 126 lb.
W per 1000 ft. of No. 13 B & S wire equals 15.8 lb.
W per 1000 ft. of No. 16 B & S wire equals 7.9 lb.

If the gauge number is not three or a multiple of three below or above No. 10, get the area, resistance, or weight desired which is less than the value for the wire required, and if it is one below the required number, multiply by 1.26, and if two below, by 1.59.

In computing the resistance and weight of cables the following formula is used:

$$Resistance = R = \frac{10000}{\text{c.m.}} \text{ in ohms per 1000 ft.}$$

$$Weight = .00305 \times \text{c.m. in lb. per 1000 ft.}$$

EXAMPLES

Find the resistance, area, and weight of the following wires:

1. No. 16.
2. No. 13.
3. No. 7.
4. No. 3.
5. No. 1.
6. No. 14.
7. No. 15.
8. No. 10.
9. No. 2.
10. No. 11.

PART IX — MATHEMATICS FOR MACHINISTS

CHAPTER XVI

MATERIALS

EVERY machinist should be familiar with the strength and other properties of the materials that he uses — such metals as cast iron, wrought iron, steel, copper, bronze, and brass.

Cast Iron. — Since much of the machinist's work is on cast iron, he should know something of its nature and manufacture. Iron ore as found in the earth generally contains many impurities, such as silicon, sulphur, phosphorus, manganese, combined carbon, and graphitic carbon. To free the iron from the grosser impurities, the ore is crushed and mixed with coke and limestone and intense heat applied in a blast furnace. The melted iron, being heavier than the other materials, falls to the bottom of the furnace. When a sufficient quantity has accumulated, it is allowed to flow out of a tap hole into molds of sand. After it has cooled it is broken into lengths suitable to be remelted in foundries and made into iron castings. Such iron is called pig iron.

During the process of smelting in the blast furnace, the liquid iron combines with a considerable quantity of carbon, sulphur, silicon, phosphorus, and manganese from the impurities in the ore and coke. Some of the carbon combines with the iron chemically and forms iron carbide, while the remainder exists in the iron as a mixture of carbon and is known as graphite. The amount of carbon may weaken the iron by making it soft, and it may also make the iron too brittle to work. So the man in charge of the foundry must use his judgment in mixing different grades and quantities of pig iron to obtain a casting of the desired strength, hardness, toughness, and clearness of grain.

Castings. — Machines are made of iron castings, forgings, steel parts, etc. Castings and forgings can be distinguished from each other by the appearance of the fractures in them. After the machines are designed and the wooden patterns made in the pattern shop the patterns are sent to

the foundry, where an impression of the machine is made in sand. During this operation of molding, the sand is confined in an iron or wooden device of two or more parts called a *flask*. The lower or bottom part of the flask is the *drag* or *nowel*, while the top or upper part is the *cope;* and other parts are the *checks*.

Sometimes pig iron and old scrap iron are melted together in a furnace called a *cupola*. The liquid iron is taken from the furnace in ladles and poured into different molds. As the hot iron flows into the mold and cools, it becomes solid and takes the shape of the mold.

When the castings are removed from the mold they present a rough surface and have to be cleaned and smoothed or "machined" before they can be put to the use intended for them. They are cleaned in various ways — by means of emery wheels and revolving wire brushes, by being rotated in "tumblers," by chipping with pneumatic chisels, or by means of a sand blast. The scales on the castings are removed by wetting them with diluted sulphuric acid. This process is called *pickling*. After this the casting is attached to the plate or table of the machine tool that is to perform the necessary work upon it. Special devices are made to hold castings when they are being machined.

Wrought Iron. — One of the valuable qualities of wrought iron is the comparative ease with which it can be united with another piece by welding. When two pieces of wrought iron are heated to a white heat, they assume a viscous condition, and when hammered together become united. Wrought iron differs from cast iron in that it is capable of assuming any shape under the hammer. It is readily made from cast iron by heating in a *puddling* furnace. In this furnace the cast iron is subjected to great heat and constant stirring, which allows the carbon to pass off as a gas and the other impurities to rise to the surface, where they can be removed. When the impurities are removed the iron is hammered to remove particles of slag and then rolled in order to make it more compact. After this it is heated again and rolled into bars for different purposes.

Wrought iron is sometimes *case-hardened* when it is used in machine parts that need to be harder than the common iron. After the piece has been finished and properly sized it is heated a bright red and the surface rubbed with prussiate of potash. When it has cooled to a dull red, it is immersed in water. Three parts of prussiate of potash and one of sal ammoniac is a good case-hardening mixture. The temperature (Fahrenheit) for cherry red is 1832°. If there are holes in such iron work, the hardening by this process reduces them slightly.

Steel. — Steel is a form of iron which contains, as a rule, more carbon and other elements than wrought iron and less than cast iron.

There are many grades of steel, and each one is made by a special process. Steel may be recognized by the appearance of a dark spot when nitric acid is placed on its surface. The darker the spot, the harder the steel. Iron, on the contrary, shows no sign when touched with nitric acid. Good steel will not stand a high heat, but will crumble under the hammer blow at a bright red heat, while at a moderate heat — a full dull red or cherry red — it may be drawn out to a fine edge tool. Steel that has once been overheated or burned cannot be restored.

Steel for cutting-tools on lathes and planers should be drawn to a straw color, or 430°, while for wood tools, taps, and dies, dark straw color, or 470°; for chisels for chipping, brown yellow or 500°; for springs, dark purple, or 550°.

Steel may be softened or *annealed* if heated to a low red and placed in a box of slaked lime and well covered, or in a box of fine bonedust, care being taken in either case to cover the piece all around and on top to a depth of not less than one and one half inches.

Copper. — Copper is used to a great extent because it may be easily forged when cold. It may also be pressed into different shapes by means of molds. Its strength is greatly increased by hammering and rolling. It is used principally in wires and plates.

Brass. — Brass is an alloy or mixture of copper and zinc. Its tensile strength is nearly equal to that of copper.

Weight of Bars of Steel. — The weight of bars of steel, as they are usually made, is found by multiplying the area of the cross section or end in inches by the length in inches, and multiplying the resulting number of cubic inches in the bar by 0.3. This will give practically an accurate result, since all bars, unless otherwise ordered, will be rolled or hammered slightly "full" to the dimensions given, or a bar that is "full" to size may be trimmed to exact dimensions if necessary. Whereas, if the bar is slightly under size, it cannot easily be made larger.

To find the weight of a triangular bar of steel, multiply the area of the base in square inches by the height in inches and then by 0.3. The base of the triangular bar is found by multiplying the length of one of the bars or sections by one half the perpendicular height; that is, by the distance to the opposite vertex of the cross section.

EXAMPLES

1. What is the weight of a triangular bar of steel when the base contains 16 sq. in. and the height is 6 ft.?

2. What is the weight of a triangular bar of steel when the base contains 13 sq. in. and the height is 4 ft.?

3. What is the weight of a triangular bar of steel when the base contains 21 sq. in. and the height is 15 feet?

4. What is the weight of a triangular bar of steel when the base contains 23 sq. inches and the height is 17 feet?

5. What is the area of the base or section of a triangular bar one side of which is 8 inches and the altitude 6 inches?

6. What is the area of the base or section of a triangular bar one side of which is 11 in. and the altitude 9 in.?

7. What is the weight of a triangular bar of steel one side of which and altitude of whose base or section are respectively 13 in. and 11 in., and the length of the bar 23 ft.?

8. What is the weight of a triangular bar whose section is 24 sq. in. and whose length is 22 ft.?

9. The weight of a cast-iron wheel is approximately sixteen times as heavy as the white pine pattern from which it is cast. What is the probable weight of a casting if the pattern for it weighs $2\frac{3}{4}$ pounds?

10. A white pine pattern weighs 12.5 pounds. What will be the weight of an iron casting from it? (Use data in Ex. 9.)

11. If the weight of a brass casting is approximately fifteen and a half times that of its white pine pattern, what will be the weight of a casting if the pattern weighs 15 oz.?

12. A white pine pattern weighs 1.75 pounds. What will be the weight of 50 brass castings made from it? (Use data in Ex. 11.)

13. Since the shrinkage of brass castings is about $\frac{1}{8}$ inch in 10 inches, what length would you make the pattern for a brass collar which is required to be 6 inches long?

CHAPTER XVII

LATHES

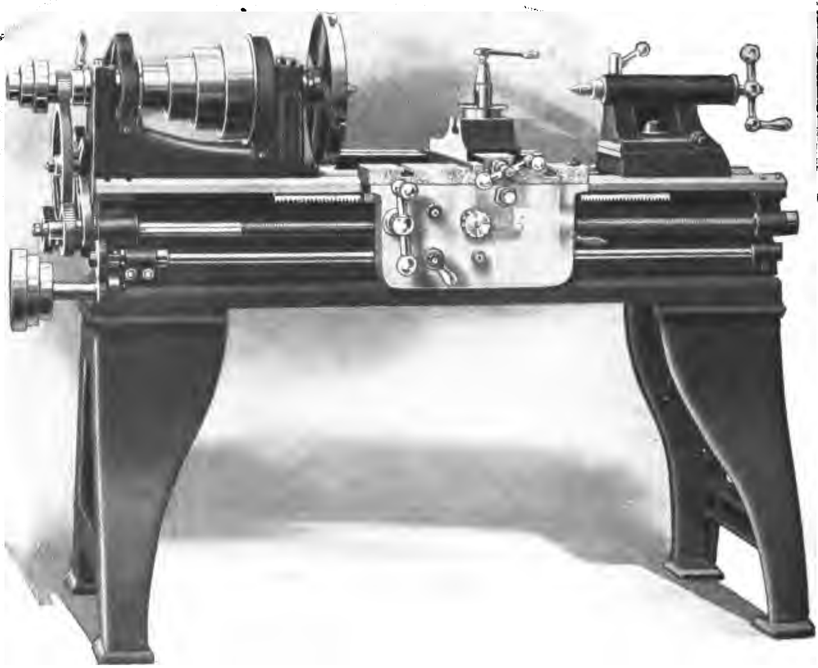
The Engine Lathe. — The lathe is one of the most important machines in the machine shop. There are different kinds of lathes, each adapted to certain kinds of work, the engine lathe being the most important. The motion of the *tool* is controlled by power speed; that is, the tool is moved automatically parallel with and at right angles to the center line of the lathe spindle. Most lathes are furnished with a series of clutch gears and lead screws by means of which threads of different pitch may be cut. All lathes have a series of stepped cones in order to obtain a variety of speeds which are necessary in order to work on hard and soft metals and to obtain constant surface speeds for different diameters. The slower speed makes the deeper cut.

Size. — The size of the lathe is expressed by stating the length of the bed and the largest diameter it will swing on centers. The *swing* is found by measuring from the point of the headstock center to the *ways* on the bed and then multiplying by 2. The English measurement is from the center to the way. The next important measurement is the length of the bed, which is the entire amount of distance the tailstock will move backward. If, however, accuracy is desired in this measurement, the figure given should be the distance between the two centers when the tailstock is in its extreme backward position, as the lathe will turn no longer piece than will go between centers.

Gear and Pitch. — Lathes that will cut a thread the same pitch as the lead screw, with gears having the same number of teeth on the stud and screw, are called *geared even.* If a

lathe will not do this, find what thread will be cut with even gears on both stud and lead screw and consider that as the pitch of the lead screw.

Adjusting Gears. — A simple or single-geared lathe is one having a straight train of gearing from its spindle to its feed



ENGINE LATHE

screw, excepting intermediate gears, which only serve as idlers to take up the distance between the driver and driven gears or spindle and screw gears. Index plates are usually found on lathes giving the change gear used for different threads,

but when threads are called for that are not indexed, or when those ending in fractions are to be cut, the machinist must make his own figures.

Refer to the screw cutting table and see what number of threads to an inch are cut with equal gears. This number is the number of turns to an inch that we assume the lead screw has, no matter what its real number of turns to an inch is. Write above the line the number of turns to an inch of the lead screw and below the line the number of turns to an inch of the screw to be threaded, thus expressing the ratio in the form of a fraction, the lead screw being the numerator and the screw to be threaded the denominator. Now find an equal fraction in terms that represent numbers of teeth in available gears. The numerator of this new fraction will be the spindle or stud gear and the denominator the lead screw gear. The new fraction is usually found by multiplying the numerator and denominator of the first fraction by the same number.

EXAMPLE. — It is required to cut a screw having $11\frac{1}{2}$ threads per inch.

The index gives 48 to 48 cuts 4 threads per inch.

$$\frac{4}{11\frac{1}{2}} \times \frac{6^*}{6} = \frac{24}{69}$$

Put the 24-tooth gear on the stud and the 69-tooth gear on the lead screw to cut $11\frac{1}{2}$ threads per inch.

EXAMPLES

1. What gears should be used to cut a screw having 16 threads per inch, if a 40-gear on the stud and an 80-gear on the screw will cut 8 threads to the inch?

2. What gears should be used to cut a screw having 3 threads per inch, if a 48-gear on the stud and a 56-gear on the screw will cut 14 threads to the inch?

* If multiplying by 6 will not give the gears available, use any other number.

3. What gears should be used to cut a screw 32 threads per inch, if the pitch of the lead screw is 12?

4. What gears should be used to cut the following threads per inch, if the pitch of the lead screw is 12?

a. 36 threads
b. 42 threads
c. 56 threads
d. 64 threads
e. $3\frac{1}{4}$ threads
f. $3\frac{1}{2}$ threads
g. 12 threads

Compound Lathes. — The term *compound* applied to a lathe means that in its train of gearing from its spindle to lead screws there is a stud or spindle having two different sized gears, both connected in such a way as to change the link of revolution between the spindle and the lead screw to a different number of revolutions from that which would take place if the straight line of gears were used.

First Method. — Write the number of turns to an inch of the lead screw as the numerator of a fraction and the turns of the screw to be threaded as the denominator. Factor this fraction into an equal compound fraction. Change the terms of this compound fraction either by multiplying or dividing into another equal compound fraction whose terms represent numbers of teeth in available gears. Then the two terms in the numerator represent the number of teeth in the gears to be used as drivers and those in the denominator the gears to be used as driven gears.

Example. — It is required to cut a screw having $3\frac{1}{4}$ inches lead or $\frac{4}{13}$ turns to an inch. The lead screw is $1\frac{1}{2}$ inches lead or $\frac{2}{3}$ turns to an inch.

$$\frac{2}{3} \div \frac{4}{13} = \frac{2 \times 13}{3 \times 4}$$

Multiply numerator and denominator by 5, $\frac{2 \times (13 \times 5)}{1 \times (12 \times 5)} = \frac{2 \times 65}{1 \times 60}$

Multiply numerator and denominator by 24, $\frac{(24 \times 2) \times 65}{(24 \times 1) \times 60} = \frac{48 \times 65}{24 \times 60}$

The 48-tooth and the 65-tooth gears will be the drivers and the 24-tooth and 60-tooth gears the driven.

NOTE. — Any multiplier may be used to obtain the gear that is available.

Second Method. — Another satisfactory method of working out the change gears is by proportion. If it is desirable to cut a screw having 10 threads per inch and the lead screw has 6 threads per inch, the first two terms of the proportion would be 10 : 6. As a rule, the smallest gear in the gear box is used on the spindle, if it will serve the purpose, and as the number of teeth on this gear is generally a multiple of the number of threads per inch on the lead screw, in the present case it would probably be 24. As the number of teeth on the lead screw is to be the third term of the proportion, and as this is unknown, x is used to represent it, and then the proportion is $10 : 6 : : x : 24$. By multiplying the first and fourth terms together and the second and third terms together, the result is $6x = 240$. Then $x = 40$, the number of teeth on the screw gear.

If the lathe is compound geared, it is necessary to find the proportional speed of spindle and stud. If the stud makes three quarters of a revolution while the spindle makes a complete revolution, it is necessary to use a gear on the screw with but three quarters the number of teeth represented by x in the proportion.

EXAMPLE. — What gear should be used on the screw of a compound geared lathe with the stud turning only three quarters as fast as the spindle, in order to cut a screw having 13 threads per inch, if the lead screw has 6 threads per inch and the stud gear 48 teeth?

$$13 : 6 : : x : 48$$

Cancelling, $13 : \cancel{6} : : x : \overset{8}{\cancel{48}} = 104$

Three quarters of $104 = 78$

With a 78-tooth gear on the screw, a 48-tooth gear on the stud of the compound geared lathe will cut a screw having 13 threads per inch.

Note.—If the stud turned but one half as fast as the spindle, then a gear should be used on the screw with one half as many teeth as shown under the method for simple geared lathes.

QUESTIONS AND EXAMPLES

1. Is the lathe in the classroom simple or compound geared?

2. What gears should be used to cut a screw having 18 threads per inch, if a 40-gear on the stud and an 80-gear on the screw will cut 8 threads to the inch?

3. How many threads per inch has the lead screw? Is it a square, V, or acme thread? Is it right or left hand?

4. What gears should be used to cut a screw having 6 threads per inch, if a 48-gear on the stud and a 56-gear on the screw will cut 14 threads to the inch?

5. How many gears are there between the spindle gear and the lead screw gear?

6. If the pitch of the lead screw is 16, what gears should be used to cut a screw with 38 threads per inch?

7. Has this lathe reversing gears? If not, state briefly how the reversing is accomplished.

8. If the pitch of the lead screw is 18, what gears should be used to cut a screw with 44 threads per inch?

9. Put even gears on the first change gear stud and lead screw and put a smooth round piece of wood on the lathe centers. Clamp a pencil on the tool post so that it will mark on the wood, then turn the lathe spindle until the carriage has moved 1 inch. How many threads did the pencil draw on the wood?

10. What gears should be used to cut the following threads per inch, if the pitch of the lead screw is 18?

a. 44 threads	*c.* 16 threads
b. 8 threads	*d.* 58 threads

11. What is the true pitch of the lead screw? Is it the same as the actual pitch?

12. A 1″ bolt is to have 8 threads per inch cut in it. If a 56-tooth gear is on the lead screw, what gear must be put on the stud?

13. A lathe has a feed rod turning at the same rate as the lead screw, while the carriage travels one quarter as fast as it would when screw cutting. If geared for 12 threads per inch and the feed shaft is used, what will be the feed in fractions of an inch per revolution?

14. At 64 revolutions per minute (R. P. M.) how long will it take to make a roughing cut with $\frac{3}{16}''$ feed and a finishing cut with $\frac{1}{16}''$ feed, if both cuts are 21″ long and 1 minute is allowed for changing tools?

To Cut Double or Multiple Threads

In modern machine construction there are many studs, screws, and feed rods having threads for rapid travel, and instead of a single spiral thread, there are two and sometimes three spirals. If a machinist is called upon to replace or duplicate such a thread, the method would not be to cut the multiple threads at one time but to cut one thread at a time. If the pitch of the double thread is measured, the pitch of every second thread will be measured and the lathe set for 4 threads per inch. The cut of 4 threads is chased out to size and the lathe left geared and the tool unchanged, and by turning the gears on the lathe spindle one half revolution, the tool position for beginning the second thread is gained.

Before fixing the gear wheel position, the carriage should be reversed in the position of the cut, thus taking up all lost motion in screws, lathe nuts, and gears. Then make the exact position of the mesh on the teeth of the spindle and on the lead screw and count the number of teeth on the spindle gear, which equals one half the entire number, and make this

tooth. Now take off the spindle and turn the lathe one half a revolution, bringing the second marked tooth to the position of the first, and the lathe is then ready for cutting the second thread. It is necessary in cutting multiple threads to select a driving gear wheel having a number of teeth exactly divisible by the number of threads cut.

Machine Speeds

Every casting is cut into a definite shape by removing a certain portion of it, called a *cut* or *turning*, by one of the machine shop tools. The casting must be cut in the most economical way and in the shortest possible time. The tool that does the cutting must attain the highest possible speed, but there is a limit to the speed of a tool on account of the heat generated as it moves against the casting. If too great speed is used, the heat generated takes the temper out of the tool, renders it useless, and causes the casting to expand. The effect of the expansion on the casting is to make it no longer *true*. The machine that is doing the cutting should be accurate to .005 of an inch.

The cutting capacity of a machine depends on (1) the speed of the cut, (2) the distance traversed by the tool in passing from one cutting portion to the next, (3) the depth of cut, *i.e.*, the thickness of the strip removed from the casting.

The volume of metal removed from a casting may be calculated as follows:

The volume of metal removed from good steel in one minute equals: Cutting speed (length) *times* the feed (width) *times* the depth of cut (thickness).

EXAMPLE. — If the speed of a cut is 18 ft. per minute, the feed .06 in., and the depth of cut is $\frac{1}{4}$ in., what is the volume of metal removed in one hour?

$18' \times 12'' \times .06 \times .25 = 3.24$ cu. in.
weight of 1 cu. in. of good steel $= .277$ lb.
$3.24 \times .277 = 0.897$ lb.
$.897 \times 60 = 53.8$ lb. in 1 hr. *Ans.*

Speeds for Different Metals. — Various materials, such as iron, copper, and wrought iron, possess different standards of hardness. A harder metal will wear away the tool and strain the machine more than a softer metal. Therefore, there are different speeds for each metal. With carbon steel cutting tools, the surface speed varies from 30 to 40 feet per minute for cast iron, wrought iron, and soft steel; 15 to 25 feet for well annealed tool steel, and from 60 to 80 feet per minute for brass, while the speed for dies and taps varies from 12 to 18 feet per minute on steel, and from 30 to 50 feet per minute on brass, depending upon the quality of the metal and the shape of the piece. With cutting tools of high-speed steel these speeds, except for dies and taps, can be nearly doubled.

Cast iron should be worked at a speed which is $\frac{1}{10}$ to $\frac{1}{20}$ of that for copper, or $\frac{1}{5}$ to $\frac{1}{10}$ of that for wrought iron.

Net Power for Cutting Iron or Steel

To find the net power for cutting cast iron and steel, multiply the section of cut or chip in square inches by 230,000 pounds for steel, or 168,000 pounds for cast iron, to get the pressure on the tool; and multiply this product by the cutting speed in feet per minute, and divide the result by 33,000 to obtain the horse power required.

EXAMPLE. — Steel is being cut with $\frac{1}{4}''$ cut, 1–64″ feed at a speed of 20′ per minute. What is the H. P.?

$\frac{1}{4}'' \times 1\text{–}64'' = .0039$ sq. in.

$.0039 \times 230{,}000$ lb. $= 897$ lb., pressure on tool

$897 \times 20 = 17{,}940$

$17{,}940 \div 33{,}000 = .54$ horse power. *Ans.*

EXAMPLE. — If the cutting speed at the rim of iron stock should be 40 feet per minute, at what speed should the lathe (spindle) be driven for a piece of stock 3″ in diameter?

3″ diameter

$3 \times 3.1416 = 9.4248''$, nearly $9.5''$, or $\frac{3}{4}'$

$40' \div \frac{3}{4}'$

$40 \times \frac{4}{3} = \frac{160}{3} = 53$ R. P. M. *Ans.*

EXAMPLES

1. What is the amount of metal removed in one hour from a casting with a cutting speed of 69′ per minute and a feed of $\frac{1}{16}''$ and a depth of $\frac{3}{8}''$?

2. As a tool will stand a cutting speed of 35 feet per minute when turning cast iron, how many revolutions per minute should the lathe spindle make when a piece of cast iron 8″ in diameter is being turned?

3. Write the formula for the feed of a lathe tool, making $N =$ number of revolutions, $D =$ distance the tool moves, and $F =$ the feed.

4. A lathe tool moves $2\frac{1}{2}''$ along the work in one minute, and the speed of the lathe is 400 R. P. M. What is the feed? (Use the formula.)

5. In turning up in the lathe a gun metal valve, 4 inches in diameter, it is desirable that the surface speed shall not exceed 45 ft. per minute. How many revolutions per minute may the wheel make?

6. Find the time required and the speed of the lathe in turning one 20-foot length of 3-inch wrought iron shafting, one cut, traverse 28 per inch, cutting speed 20 ft. per minute, no allowance being made for grinding or breaking of tools or for setting stays.

7. How many revolutions per minute may be made in turning up a steel shaft 6 inches in diameter, if the surface speed must not exceed 12 ft. per minute?

8. A piece of tool steel $1\frac{3}{4}''$ in diameter is turned in a lathe at 74 R. P. M. What is the cutting speed?

9. What is the amount of metal removed in one hour from a casting with a cutting speed of 64 ft. per minute and a feed of $\frac{1}{16}''$ and a depth of $\frac{1}{2}''$?

10. If a tool will stand a cutting speed of 37 R. P. M. when

turning cast iron, how many R. P. M. should the lathe (spindle) make when a piece of cast iron 6″ in diameter is being turned?

11. A valve yoke stem 2″ in diameter is being turned in a lathe. If the lathe spindle makes 50 R.P.M., what is the cutting speed in F. P. M.?

12. What number of revolutions must a lathe spindle make to cut 15 ft. per minute, in turning up an iron shaft $7\frac{1}{2}$ inches in diameter?

13. (*a*) With a tool steel that can stand a cutting speed of 30′ per minute, how many revolutions per minute may a lathe be run in turning one cut off a piece of shafting 15″ in diameter?

(*b*) To hold the same cutting speed, how many revolutions per minute would be required if the shaft were but $7\frac{1}{2}$″ in diameter?

14. In turning a cast iron piston head 15 inches in diameter in the lathe, it is desired that the surface speed shall not exceed 15 ft. per minute. How many revolutions per minute may the work make?

15. How many revolutions per minute should the lathe spindle make in turning up a cast iron pulley $33\frac{3}{4}$ inches in diameter, at a cutting speed of 15 feet per minute?

Table of Surface Speeds

The table on page 266 has been computed to facilitate the figuring of speeds for machines and is used as follows:

Having first determined the proper surface speed, refer in the table to the column of *revolutions per minute* corresponding to this surface speed, and in this column opposite the diameter corresponding to that of the work under consideration will be found the required revolutions per minute of spindle or work. Other surface speeds than those for which the table is computed can be readily obtained from the table by multiplying or dividing, as the case may require.

TABLE OF SURFACE SPEEDS

FT. PER MIN.	30	32½	35	37½	40	42½	45	47½	50
DIAM.	*Revolutions per Minute*								
$\frac{1}{32}$	3667.8	3973.5	4279.1	4584.8	4890.4	5196.1	5501.8	5807.4	6113.0
$\frac{1}{16}$	1833.9	1986.7	2139.5	2292.4	2445.2	2598.0	2750.9	2903.7	3056.5
$\frac{3}{32}$	1222.6	1324.5	1426.3	1528.2	1630.1	1732.0	1833.9	1935.8	2037.7
$\frac{1}{8}$	916.9	993.3	1069.8	1146.2	1222.6	1299.0	1375.4	1451.8	1528.3
$\frac{5}{32}$	733.6	794.7	855.8	916.9	978.0	1039.2	1100.3	1161.4	1226.6
$\frac{3}{16}$	611.3	662.2	713.2	764.1	815.7	866.0	916.9	967.9	1018.8
$\frac{7}{32}$	523.9	567.6	611.3	654.9	698.6	742.3	785.9	829.6	873.3
$\frac{1}{4}$	458.4	496.6	534.9	573.1	611.3	649.5	687.7	725.9	764.1
$\frac{9}{32}$	407.5	441.5	475.4	509.4	543.3	577.3	611.3	645.2	679.2
$\frac{5}{16}$	366.7	397.3	427.9	458.4	489.0	519.6	550.1	580.7	611.3
$\frac{11}{32}$	333.4	361.2	389.0	416.9	444.5	472.3	500.0	527.9	555.7
$\frac{3}{8}$	305.6	331.1	356.6	382.0	407.5	433.0	458.5	483.9	509.4
$\frac{13}{32}$	282.1	305.6	329.1	352.3	376.1	399.7	423.2	446.7	470.2
$\frac{7}{16}$	261.9	283.8	305.6	327.4	349.3	371.1	392.9	414.8	436.6
$\frac{15}{32}$	244.5	264.9	285.1	305.6	326.0	346.4	366.7	387.8	407.5
$\frac{1}{2}$	229.2	248.3	267.4	286.5	305.6	324.7	343.8	362.9	382.0
$\frac{9}{16}$	203.7	220.7	237.7	254.7	271.6	288.6	305.6	322.6	340.0
$\frac{5}{8}$	183.4	198.6	213.9	229.2	244.5	259.8	275.0	290.3	306.2
$\frac{11}{16}$	166.7	180.6	194.5	208.4	222.3	236.1	250.0	263.9	277.8
$\frac{3}{4}$	152.8	165.5	178.3	191.0	203.7	216.5	229.2	241.9	254.2
$\frac{13}{16}$	141.0	152.8	164.5	176.2	188.0	199.8	211.6	223.3	234.8
$\frac{7}{8}$	130.9	141.9	152.8	163.7	174.6	185.5	196.4	207.4	218.2
$\frac{15}{16}$	122.2	132.4	142.6	152.8	163.0	173.2	183.4	193.9	203.8
1	114.6	124.2	133.7	143.2	152.8	162.3	171.9	181.4	191.0
$1\frac{1}{16}$	108.2	117.2	126.2	135.2	143.2	153.2	162.2	170.8	180.4
$1\frac{1}{8}$	101.9	110.3	118.8	127.3	135.8	144.3	152.8	161.3	170.0
$1\frac{3}{16}$	96.8	104.8	113.0	121.0	129.2	137.0	145.0	152.8	161.2
$1\frac{1}{4}$	91.7	99.3	106.9	114.6	122.2	129.9	137.5	145.1	153.1
$1\frac{5}{16}$	87.5	94.7	102.1	109.4	116.8	124.0	131.2	138.2	146.0

EXAMPLES

1. If the cutting speed of soft steel is 25, find the speed (spindle) for $1\frac{1}{4}''$ stock.

2. If the cutting speed of soft steel is 25, find the speed (spindle) for $\frac{11}{16}''$ stock.

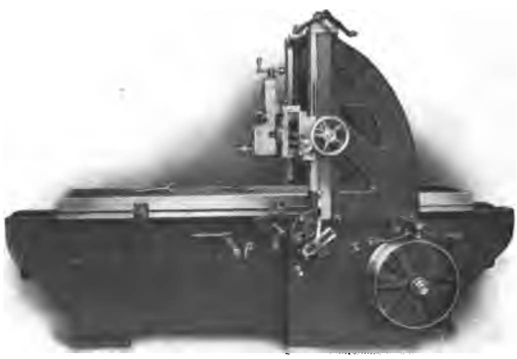
3. If the cutting speed of tool steel is 75, find the speed (spindle) for $\frac{15}{32}''$ stock.

4. According to the table find the speed of (*a*) soft steel $1\frac{1}{2}''$ in diameter; (*b*) soft steel $\frac{7}{32}''$ in diameter; (*c*) tool steel $2\frac{5}{8}''$ stock; (*d*) tool steel $2\frac{1}{2}''$ stock; (*e*) soft steel $\frac{5}{16}''$ in diameter; (*f*) soft steel $\frac{15}{16}''$ in diameter.

CHAPTER XVIII

PLANERS, SHAPERS, AND DRILLING MACHINES

Planers. — The planer is usually one of the heaviest machines in the machine shop. Planing is rough and heavy work and stiffness is needed in order to make the heavy cuts the planer usually has on castings and heavy forgings. In many shops much of this rough surface cutting is saved, however, by the use of a pickling solution of one part of sulphuric acid to eight parts of water. This is painted over the casting and, after four or five hours, washed off with clear water, removing the sand and hard grit from the surface.



MACHINE SHOP PLANER

Care must be taken, during the operation of a planer, that chips and dirt from the platen are not swept or allowed to blow into the V-ways, as this will cause injury to the machine. Great care must be taken, also, that oil is freely supplied to the machinery under the planer platen, as these parts are hidden and likely to be neglected.

A planer 24 inches by 24 inches by 6 feet will consume an average of 0.035 horse power for every pound of cast iron removed per hour, and a consumption of 0.065 horse power for every pound of machinery steel removed per hour under good operation with sharp tools. The operator

of a planer may, by poor grinding and not setting his tools, waste much power in driving the machine.

The cutting speed for planers is about the same as for lathe tools, the advantage of the planer being that heavier cuts can be produced on account of the rigid support of the platen.

The principal objection to planing machines is that they perform useful work in only one half of the motion. When the work is drawn back, no cutting takes place. Then again, since the forces are completely changed when the motion is changed, the slides upon which the table moves, suffer. In this way accurate work is difficult on account of the backlash. The return on capital invested in such machines is smaller than in the case of machines with continuous motion.

Planer and Shaper. — The planer and shaper, and such modified machines as the slotting machine and key seater, are in a distinct class. Their use is to machine and plane irregular surfaces that can be machined by a straight line cut. The cutting tools of the planer and shaper are practically the same as those on the lathe. In the planer the work moves, and vertical and lateral feeds are given to the tools. In the shaper the tools move over the work, lateral to the work and vertical to the tool. The shaping machine is designed for small pieces and short travels. It is nicely adapted for cutting grooves, slots, and dovetails.

Shaper tools should be kept in the best condition as to shear, clearance, and cutting edge, as they are called upon to do accurate shaping of metal parts of machinery not possible on other machines. *Shear* is a certain amount of angle given the face of a tool, which throws its cutting edge forward into the metal to be cut. Tools without *clearance* drag and pull heavily through the metal.

For work on the shaper, the student should have a good knowledge of the small try-square and the surface block. These tools are constantly used, the square showing when finished pieces are square with the shaping machine vise or when one cut is square with another. A universal bevel or a bevel protractor should also be used, enabling angles to be laid out for planing. A *scriber* and a four-inch outside caliper will enable the beginner to grasp the first operations of shaping and planing. The ram should be adjusted to proper length of stroke to save time. If a piece of work which measures two and three fourths inches is to be placed on the machine, one fourth inch is sufficient for the tool to enter and one eighth inch is even more than enough to allow the head to overreach. These additions then give us a total stroke of three and one eighth inches, and any amount over this loses time and causes unnecessary wear on the machine.

EXAMPLES

1. If a planer has a cutting speed of 30 ft. per minute and a return speed of 147 ft. per minute, what is the ratio of the cutting to the return speed?

2. On a 36″ planer the ratio of the cutting speed to the return speed of the table is 1 to 2.94, with a cutting speed of 58 ft. per minute. What is the return speed?

3. It is necessary to plane a bench block on the top and bottom surfaces. An equal amount of stock is to be removed from each side, and the thickness of the casting should be reduced from $1\frac{9}{16}$ in. to $1\frac{3}{8}$ in. What thickness of stock should be cut from the top surface?

4. A piece of work on the planer is 10 in. thick; it is reduced to a certain size in 5 cuts; at the first cut the tool takes off $\frac{5}{32}$ in. from the thickness; then $\frac{1}{8}$ in.; $\frac{3}{64}$ in., $\frac{1}{32}$ in., and the fifth cut is $\frac{1}{64}$ in. What is the thickness of the finished piece?

5. On a cast iron block 6 in. square[1] a groove $\frac{3}{16}$″ wide and $\frac{7}{8}$″ deep must be planed. The planer makes 12 strokes per minute and the down feed is set at $\frac{1}{32}$″ for every stroke. How long will it take to cut the groove?

Drilling Machines. — Machines for drilling holes in the different pieces made in a machine shop are divided into two general classes — vertical and horizontal. The vertical drilling machines include those with a number of drill spindles called multiple spindles. Besides, there are special machines of both classes, as portable drills, hand drills, etc.

The most common form of drill is the vertical drilling machine or drill-press. The machine consists of a frame supporting the drill spindle and the drilling table, and an arrangement for feeding the tool into the work by hand or power. On this machine the work to be drilled is

[1] The information 6″ square is not necessary for the solution of the problem, but is given to add interest to it. The same thing applies to some of the problems in drilling on the following pages, where the size of hole is given but not required.

SLIDING HEAD DRILLING MACHINE

placed on the drilling table, and is held stationary by means of a clamp or vise, while the revolving drill is fed through the work by hand or by power. The feeding mechanism is similar to that of the lathe. It is more convenient, usually, to drill small work in a speed lathe and to use the drill-press on heavy work.

There are two classes of drills — straight and twist; the twist drill being the latest and most approved in the leading shops. Whenever a manufacturer is making standardized parts, that is, uniform parts of the same machine, day after day, he designs and builds special drilling machines, also special milling and planing machines, gauges called *jigs*, and gauges to produce the standard or duplicate parts.

Power is transmitted from the pulley on the shaft to the countershaft at the bottom of the machine. Here are tight and loose pulleys so that the machine may be stopped by shifting the belt from the tight to the loose pulley. Different materials have qualities which make it necessary to use different cutting speeds in order to remove the metal quickly and efficiently. In order to provide for this a cone pulley giving a number of different speeds is used. The cone pulley on the countershaft is the same as the cone pulley on the headstock. The power is transmitted from the headstock to the drill spindle by bevel gears. The power is transmitted from the headstock shaft to the feeding spindle — which in turn lets the drill spindle down.

Reamers. — Drills cannot be relied upon to make holes as round, straight, smooth, or uniform in diameter as are required in the construction of accurate machinery. To make these holes accurately a tool called a *reamer* is used. It has two or more teeth, either parallel or at an angle with each other. The periphery of a drill or reamer is the distance around the outside, and the peripheral speed is the distance a point in the circumference travels in a minute. It is equal to the number of turns that the tool makes in a minute multiplied by the circumference. The proper speed at which to run a drill depends upon the kind of drill, its size, and the material to be drilled. A large drill must run more slowly than a smaller one, the turns per minute becoming less the larger the size of the drill. For instance, a drill $\frac{1}{2}''$ in diameter should make twice as many revolutions per minute as a $1''$ drill and four times as many as one $2''$ in diameter, used on the same material. The peripheral speeds usually recommended for carbon steel drills are as follows:

Wrought iron or steel, 30 ft. per min.	High speed drills, 50 to 70 ft. per min.
Cast iron, 35 ft. per min.	High speed drills, 60 to 80 ft. per min.
Brass, 60 ft. per min.	High speed drills, 100 to 140 ft. per min.

The feed of a drill is the amount the drill enters the hole for each turn or revolution, and in good practice the feed may be set to give 1″ depth of the hole for every 95 to 125 revolutions, according to the size of the drill, the material of which the drill is made and the kind of metal drilled.

EXAMPLE. — How long will it take a one-inch drill, making 134 R. P. M., with a feed of .012″ per revolution, to drill a hole $1\frac{1}{2}''$ deep in cast iron?

$$\frac{1.5}{.012} = 125 \text{ rev.} \qquad 1 \text{ rev.} = \frac{1}{134} \text{ of a minute}$$

$$125 \text{ rev.} = 125 \times \frac{1}{134} = \frac{125}{134} = .933 \text{ of a minute. } \textit{Ans.}$$

EXAMPLE. — A piece of wrought iron 2.69″ thick is to have two $\frac{13}{16}''$ holes drilled through it. If the drill makes 112 R. P. M., what must the feed be to drill each hole in two minutes?

$$112 \times 2 = 224 \text{ revs. in 2 min.}$$

$$\tfrac{1}{224} \text{ of } 2.69'' = .012'' \text{ feed. } \textit{Ans.}$$

EXAMPLE. — A 2″ drill is used in drilling an electrical generator bed plate. If the drill is making 67 R. M. P., and is being fed to the work at thé rate of .015″ per revolution, how deep will the hole be when the drill has worked $4\frac{1}{2}$ minutes?

$$67 \times .015'' = 1.005'' \text{ depth per min.}$$

$$1.005 \times 4.5 = 4.5225''. \ \textit{Ans.}$$

EXAMPLE. — What will be the R. P. M. of a drill $1\frac{3}{4}''$ in diameter used for drilling out a lathe spindle 30.24″ long, the feed being .015″ per revolution, and the time given the machinist to do the job being 42 minutes?

$$\frac{30.24}{.015} = 2016 \qquad \frac{2016}{42} = 48 \text{ R. P. M. } \textit{Ans.}$$

EXAMPLE. — Conditions being the same as in the above example, what length of a spindle could be drilled in 50 minutes?

$$48 \times .015 = .720'' \text{ per min.}$$

$$.720'' \times 50 = 36'' \text{ in 50 min. } \textit{Ans.}$$

EXAMPLES

1. How long will it take a carbon steel drill to drill a $\frac{7}{8}''$ hole through a piece of wrought iron $1\frac{13}{16}''$ thick, if the drill makes 105 R. P. M., with the feed at .011″ per revolution?

2. With the feed at .015″ per revolution and the speed at 115 R. P. M., how long will it take to drill a hole with a 2″ carbon steel drill through a brass bushing $4\frac{5}{8}''$ long?

3. A machinist using a high speed drill $1\frac{3}{4}''$ in diameter, is to drill 4 bolt holes in the base of an electrical generator, the holes to be $2\frac{1}{2}''$ long, the drill to make 164 R. P. M., with the feed at .018″ per revolution. How long will it take him to do the job if 3 minutes are allowed for the setting for each hole?

4. A $\frac{3}{4}''$ drill working on brass is running at 306 R. P. M. and advancing at the rate of .015″ per revolution. How long will it take to drill through a piece $1\frac{7}{16}''$ thick?

5. The R. P. M. of a carbon steel drill $1\frac{3}{16}''$ in diameter is 113, and the feed is .013″ per revolution. How much time will a machinist require to drill 18 holes in a cylinder head $1\frac{1}{4}''$ thick, allowing one minute for the setting of each hole?

6. A high speed drill $1\frac{5}{8}''$ in diameter is advancing at the rate of .018″ per revolution, making 229 R. P. M. while drilling an angle iron $1\frac{3}{4}''$ thick. How long will it take the drill to go through it?

7. In Example 6, what is the periphery speed in feet per minute of the drill?

8. A cast iron casing for a steam turbine is to have a series of holes $1\frac{1}{8}''$ in diameter drilled in it $2\frac{1}{4}''$ deep with a high speed drill making 250 R. P. M. What must the feed be per revolution to drill each hole in $\frac{4}{5}$ of a minute?

9. The holes in a face plate 4″ thick are to be drilled with a 1-in. carbon steel drill which makes 134 R. P. M. (*a*) What feed will be required to drill through the plate in

$2\frac{1}{2}$ minutes? (*b*) What will be the periphery speed in feet per minute of the drill?

10. What must be the R. P. M. of a $1\frac{1}{2}''$ drill, feeding at the rate of .013" per revolution, to drill a hole $3\frac{1}{2}''$ deep in an engine bed in 3 minutes?

11. A 2" drill is used in drilling an electrical generator bed platen. If the drill is making 87 R. P. M., and is being fed to the work at the rate of .015" per revolution, how deep will the hole be when the drill has worked $4\frac{1}{2}$ minutes?

12. What will be the R. P. M. of a $1\frac{1}{8}''$ carbon drill used in drilling out an engine lathe spindle 24.05" in 24.66 minutes? What will be the total number of revolutions?

PART X — TEXTILE CALCULATIONS

CHAPTER XIX

YARNS

Worsted Yarns. — All kinds of yarns used in the manufacture of cloth are graded into sizes according to the weight per definite length. To illustrate: Worsted yarns are made from combed wools, and the size, technically called the *counts*, is

Roving or Yarn Scales

These scales will weigh one pound by tenths of grains or one seventy-thousandth part of one pound avoirdupois, rendering them well adapted for use in connection with yarn reels, for the numbering of yarn from the weight of hank, giving the weight in tenths of grains to compare with tables.

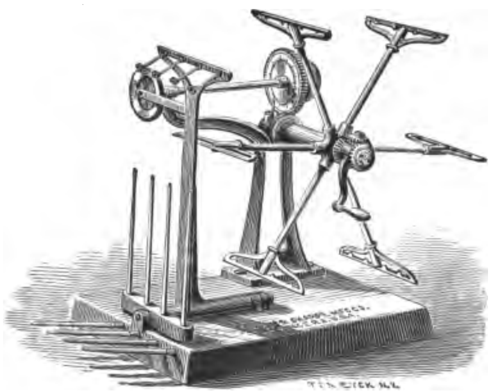
based upon the number of lengths (called *hanks*) of 560 yards required to weigh one pound. Thus, if one hank weighs one pound, the yarn will be number one counts, while if 20 hanks

are required for one pound, the yarn is the 20's, etc. The greater the number of hanks necessary to weigh one pound, the higher the counts and the finer the yarn. The hank, or 560 yards, is the unit of measurement for all worsted yarns.

LENGTH FOR WORSTED YARNS

No.	Yards per Lb.	No.	Yards per Lb.	No.	Yards per Lb.	No.	Yards per Lb.
1	560	5	2800	9	5040	13	7280
2	1120	6	3360	10	5600	14	7840
3	1680	7	3920	11	6160	15	8400
4	2240	8	4480	12	6720	16	8960

Woolen Yarns.—In worsted yarns the fibers lie parallel to each other, while in woolen yarns the fibers are entangled. This difference is due entirely to the different methods used



YARN REEL

For reeling and measuring lengths of cotton, woolen, and worsted yarns.

in working up the raw stock. In woolen yarns there is a great diversity of systems of grading, varying according to the districts in which the grading is done. Among the many systems

are the English *skein*, which differs in various parts of England; the Scotch *spyndle;* the American *run;* the Philadelphia *cut;* and others. In these lessons the run system will be used unless otherwise stated. This is the system used in New England. The run is based upon 100 yards per ounce, or 1600 yards to the pound. Thus, if 100 yards of woolen yarn weigh one ounce, or if 1600 yards weigh one pound, it is technically termed a No. 1 run; and if 300 yards weigh one ounce, or 4800 yards weigh one pound, the size will be No. 3 run. The finer the yarn, or the greater the number of yards necessary to weigh one pound, the higher the run.

LENGTH FOR WOOLEN YARNS (RUN SYSTEM)

No.	Yards per Lb.	No.	Yards per Lb.	No.	Yards per Lb.	No.	Yards per Lb.
$\frac{1}{8}$	200	1	1600	2	3200	3	4800
$\frac{1}{4}$	400	$1\frac{1}{4}$	2000	$2\frac{1}{4}$	3600	$3\frac{1}{4}$	5200
$\frac{1}{2}$	800	$1\frac{1}{2}$	2400	$2\frac{1}{2}$	4000	$3\frac{1}{2}$	5600
$\frac{3}{4}$	1200	$1\frac{3}{4}$	2800	$2\frac{3}{4}$	4400		

Raw Silk Yarns. — For raw silk yarns the table of weights is:

16 drams = 1 ounce
16 ounces = 1 pound
256 drams = 1 pound

The unit of measure for raw silk is 256,000 yards per pound. Thus, if 1000 yards — one skein — of raw silk weigh one dram, or if 256,000 yards weigh one pound, it is known as 1-dram silk, and if 1000 yards weigh two drams the yarn is called 2-dram silk, hence the following table is made:

1-dram silk = 1000 yards per dram, or 256,000 yards per lb.
2-dram silk = 1000 yards per 2 drams, or 128,000 yards per lb.
4-dram silk = 1000 yards per 4 drams, or 64,000 yards per lb.

Drams per 1000 Yards	Yards per Pound	Yards per Ounce
1	256,000	16,000
1¼	204,800	12,800
1½	?	?
1¾	146,286	9143
2	128,000	8000
2¼	113,777	7111
2½	102,400	6400
2¾	93,091	5818
3	?	?
3¼	78,769	4923
3½	73,143	4571

Linen Yarns. — The sizes of linen yarns are based on the *lea* or cuts per pound and the pounds per spindle. A cut is 300 yards and a spindle 14,000 yards. A continuous thread of several cuts is a hank, as a 10-cut hank, which is $10 \times 300 = 3000$ yards per hank. The number of cuts per pound, or the leas, is the number of the yarn, as 30's, indicating $30 \times 300 = 9000$ yards per pound. Eight-pound yarn means that a spindle weighs 8 pounds or that the yarn is 6-lea $(14{,}400 \div 8) \div 300 = 6$.

Cotton Yarns. — The sizes of cotton yarns are based upon the system of 840 yards to 1 hank. That is, 840 yards of cotton yarn weighing 1 pound is called No. 1 counts.

Spun Silk. — Spun silk yarns are graded on the same basis as that used for cotton (840 yards per pound), and the same rules and calculations that apply to cotton apply also to spun silk yarns.

Two or More Ply Yarns. — Yarns are frequently produced in two or more ply; that is, two or more individual threads are twisted together, making a double twist yarn. In this case the size is given as follows:

2/30's means 2 threads of 30's counts twisted together, and 3/30's would mean 3 threads, each a 30's counts, twisted together.

(The figure before the line denotes the number of threads twisted together, and the figure following the line the size of each thread.)

Thus when two threads are twisted together, the resultant yarn is heavier, and a smaller number of yards are required to weigh one pound.

For example: 30's worsted yarn equals 16,800 yd. per lb., but a two-ply thread of 30's, expressed 2/30's, requires only 8400 yards to the pound, or is equal to a 15's; and a three-ply thread of 30's would be equal to a 10's.

When a yarn is a two-ply, or more than a two-ply, and made up of several threads of equal counts, divide the number of the single yarn in the required counts by the number of the ply, and the result will be the equivalent in a single thread.

To find the Weight in Grains of a Given Number of Yards of Worsted Yarn of a Known Count

EXAMPLE. — Find the weight in grains of 125 yards of 20's worsted yarns.

No. 1's worsted yarn = 560 yards
No. 20's worsted yarn = 11,200 yards to 1 lb.
1 lb. worsted yarn = 7000 grains

If 11,200 yards of 20's worsted yarn weigh 7000 grains, then $\frac{125}{11{,}200}$ of $7000 = \frac{125}{11{,}200} \times 7000 = \frac{625}{8} = 78.125$ grains.

NOTE. — Another method: Multiply the given number of yards by 7000, and divide the result by the number of yards per pound of the given count.

$$125 \times 7000 = 875{,}000$$
$$1 \text{ pound } 20\text{'s} = 11{,}200$$
$$875{,}000 \div 11{,}200 = 78.125 \text{ grains. } \textit{Ans.}$$

To find the Weight in Grains of a Given Number of Yards of Cotton Yarn of a Known Count

EXAMPLE. — Find the weight in grains of 80 yards of 20's cotton yarn.

$$\text{No. 1's cotton} = 840 \text{ yards to a lb.}$$
$$\text{No. 20's cotton} = 16{,}800 \text{ yards to a lb.}$$
$$1 \text{ lb.} = 7000 \text{ grains}$$

$$1 \text{ yd. 20's cotton} = \frac{7000}{16{,}800} \text{ grains}$$

$$80 \text{ yd. 20's cotton} = \frac{7000}{16{,}800} \times 80 = \frac{700}{21} = 33.33 \text{ grains. } \textit{Ans.}$$

It is customary to solve examples that occur in daily practice by rule.

The rule for the preceding example is as follows:

Multiply the given number of yards by 7000 and divide the result by the number of yards per pound of the given count.

$$80 \times 7000 = 560{,}000$$
$$560{,}000 \div (20 \times 840) = 33.33 \text{ grains. } \textit{Ans.}$$

NOTE. — 7000 is always a multiplier and 840 a divisor.

To find the weight in ounces of a given number of yards of cotton yarn of a known count, multiply the given number of yards by 16, and divide the result by the yards per pound of the known count.

To find the weight in pounds of a given number of yards of cotton yarn of a known count, divide the given number of yards by the yards per pound of the known count.

To find the weight in ounces of a given number of yards of woolen yarn (run system), divide the given number of yards by the number of runs, and multiply the quotient by 100.

NOTE. — Calculations on the run basis are much simplified, owing to the fact that the standard number (1600) is exactly 100 times the number of ounces contained in 1 pound.

EXAMPLE. — Find the weight in ounces of 6400 yards of 5-run woolen yarn.

$$6400 \div (5 \times 100) = 12.8 \text{ oz. } \textit{Ans.}$$

To find the weight in pounds of a given number of yards of woolen yarn (run system) the above calculation may be used, and the result divided by 16 will give the weight in pounds.

To find the weight in grains of a given number of yards of woolen yarn (run system), multiply the given number of yards by 7000 (the number of grains in a pound) and divide the result by the number of yards per pound in the given run, and the quotient will be the weight in grains.

EXAMPLES

1. How many ounces are there (*a*) in 6324 grains? (*b*) in $34\frac{1}{2}$ pounds?

2. How many pounds are there in 9332 grains?

3. How many grains are there (*a*) in $168\frac{1}{2}$ pounds? (*b*) in 2112 ounces?

4. Give the lengths per pound of the following worsted yarns: (*a*) 41's; (*b*) 55's; (*c*) 105's; (*d*) 115's; (*e*) 93's.

5. Give the lengths per pound of the following woolen yarns (run system): (*a*) $9\frac{1}{4}$'s; (*b*) 6's; (*c*) 19's; (*d*) 17's; (*e*) $1\frac{1}{2}$'s.

6. Give the lengths per pound of the following raw silk yarns: (*a*) $1\frac{1}{2}$'s; (*b*) 3's; (*c*) $3\frac{3}{4}$'s; (*d*) 20's; (*e*) 28's.

7. Give the lengths per ounce of the following raw silk yarns: (*a*) $4\frac{1}{2}$'s; (*b*) $6\frac{1}{2}$'s; (*c*) 8's; (*d*) 9's; (*e*) 14's.

8. What are the lengths of linen yarns per pound: (*a*) 8's; (*b*) 25's; (*c*) 32's; (*d*) 28's; (*e*) 45's?

9. What are the lengths per pound of the following cotton yarns: (*a*) 10's; (*b*) 32's; (*c*) 54's; (*d*) 80's; (*e*) 160's?

10. What are the lengths per pound of the following spun silk yarns: (*a*) 30's; (*b*) 45's; (*c*) 38's; (*d*) 29's; (*e*) 42's?

11. Make a table of lengths per ounce of spun silk yarns from 1's to 20's.

12. Find the weight in grains of 144 inches of 2/20's worsted yarn.

13. Find the weight in grains of 25 yards of 3/30's worsted yarn.

14. Find the weight in ounces of 24,000 yards of 2/40's cotton yarn.

15. Find the weight in pounds of 2,840,000 yards of 2/60's cotton yarn.

16. Find the weight in ounces of 650 yards of $1\frac{1}{2}$-run woolen yarn.

17. Find the weight in grains of 80 yards of $\frac{1}{2}$-run woolen yarn.

18. Find the weight in pounds of 64,000 yards of 5-run woolen yarn.

Solve the following examples, first by analysis and then by rule:

19. Find the weight in grains of 165 yards of 35's worsted.

20. Find the weight in grains of 212 yards of 40's worsted.

21. Find the weight in grains of 118 yards of 25's cotton.

22. Find the weight in grains of 920 yards of 18's cotton.

23. Find the weight in pounds of 616 yards of $16\frac{1}{2}$'s woolen.

24. Find the weight in grains of 318 yards of 184's cotton.

25. Find the weight in grains of 25 yards of 30's linen.

26. Find the weight in pounds of 601 yards of 60's spun silk.

27. Find the weight in grains of 119 yards of 118's cotton.

28. Find the weight in grains of 38 yards of 64's cotton.

29. Find the weight in grains of 69 yards of 39's worsted.

30. Find the weight in grains of 74 yards of 40's worsted.

31. Find the weight in grains of 113 yards of $1\frac{1}{4}$'s woolen.

32. Find the weight in grains of 147 yards of $1\frac{3}{4}$'s woolen.

33. Find the weight in grains of 293 yards of 8's woolen.

34. Find the weight in grains of 184 yards of $16\frac{1}{2}$'s worsted.

35. Find the weight in grains of 91 yards of 44's worsted.

36. Find the weight in grains of 194 yards of 68's cotton.

37. Find the weight in pounds of 394 yards of 180's cotton.

38. Find the weight in pounds of 612 yards of 60's cotton.
39. Find the weight in grains of 118 yards of 44's linen.
40. Find the weight in pounds of 315 yards of 32's linen.
41. Find the weight in grains of 84 yards of 25's worsted.
42. Find the weight in grains of 112 yards of 20's woolen.
43. Find the weight in grains of 197 yards of 16's woolen.
44. Find the weight in grains of 183 yards of 18's cotton.
45. Find the weight in grains of 134 yards of 28's worsted.
46. Find the weight in grains of 225 yards of 34's linen.
47. Find the weight in pounds of 369 yards of 16's spun silk.
48. Find the weight in pounds of 484 yards of 18's spun silk.

To find the Size or the Counts of Cotton Yarn of Known Weight and Length

EXAMPLE. — Find the size or counts of 84 yards of cotton yarn weighing 40 grains.

Since the counts is the number of hanks to the pound,

$$\frac{7000}{40} \times 84 = 14{,}700 \text{ yd. in 1 lb.}$$
$$14{,}700 \div 840 = 17.5 \text{ counts. } \textit{Ans.}$$

RULE. — Divide 840 by the given number of yards; divide 7000 by the quotient obtained; then divide this result by the weight in grains of the given number of yards, and the quotient will be the counts.

$$840 \div 84 = 10$$
$$7000 \div 10 = 700$$
$$700 \div 40 = 17.5 \text{ counts. } \textit{Ans.}$$

To find the Run of a Woolen Thread of Known Length and Weight

EXAMPLE. — If 50 yards of woolen yarn weigh 77.77 grains, what is the run?

$$1600 \div 50 = 32$$
$$7000 \div 32 = 218.75$$
$$218.75 \div 77.77 = 2.812\text{-run yarn. } \textit{Ans.}$$

RULE. — Divide 1600 (the number of yards per pound of 1-run woolen yarn) by the given number of yards; then divide 7000 (the grains per pound) by the quotient; divide this quotient by the given weight in grains and the result will be the run.

To find the Weight in Ounces for a Given Number of Yards of Worsted Yarn of a Known Count

EXAMPLE. — What is the weight in ounces of 12,650 yards of 30's worsted yarn?

$$12{,}650 \times 16 = 202{,}400$$
$$202{,}400 \div 16{,}800 = 12.047 \text{ oz. } \textit{Ans.}$$

RULE. — Multiply the given number of yards by 16, and divide the result by the yards per pound of the given count, and the quotient will be the weight in ounces.

To find the Weight in Pounds for a Given Number of Yards of Worsted Yarn of a Known Count

EXAMPLE. — Find the weight in pounds of 1,500,800 yards of 40's worsted yarn.

$$1{,}500{,}800 \div 22{,}400 = 67 \text{ lb. } \textit{Ans.}$$

RULE. — Divide the given number of yards by the number of yards per pound of the known count, and the quotient will be the desired weight.

EXAMPLES

1. If 108 inches of cotton yarn weigh 1.5 grains, find the counts.

2. Find the size of a woolen thread 72 inches long which weighs 2.5 grains.

3. Find the weight in ounces of 12,650 yards of 2/30's worsted yarn.

4. Find the weight in ounces of 12,650 yards of 40's worsted yarn.

5. Find the weight in pounds of 1,500,800 yards of 40's worsted yarn.

6. Find the weight in pounds of 789,600 yards of 2/30's worsted yarn.

7. What is the weight in pounds of 851,200 yards of 3/60's worsted yarn?

8. If 33,600 yards of cotton yarn weigh 5 pounds, find the counts of the cotton.

Buying Yarn, Cotton, Wool, and Rags

Every fabric is made of yarn of definite quality and quantity. Therefore, it is necessary for every mill man to buy yarn or fiber of different kinds and grades. Many small mills buy cotton, wool, yarn, and rags from brokers who deal in these commodities. The prices rise and fall from day to day according to the law of demand and supply. Price lists giving the quotations are sent out weekly and sometimes daily by agents as the prices of materials rise or fall. The following are quotations of different grades of cotton, wool, and shoddy, quoted from a market list:

Quantity	Price
8103 lb. white yarn shoddy (best all wool)	$0.485
3164 lb. white knit stock (best all wool)	.365
2896 lb. pure indigo blue	.315
1110 lb. fine dark merino wool shoddy	.225
410 lb. fine light merino woolen rags	.115
718 lb. cloakings (cotton warp)	.02
872 lb. wool bat rags	.035
96 lb. 2/20's worsted (Bradford) yarn	.725
408 lb. 2/40's Australian yarn	1.35
593 lb. 1/50's delaine yarn	1.20
987 lb. 16 cut merino yarn (50% wool and 50% shoddy)	.285
697 lb. carpet yarn 60 yd. double reel wool filling	.235

Find the total cost of the above quantities and grades of textiles.

Alligation

It is customary in mills to mix different fibers at different prices in order to make a product of some intermediate quality or price. The process of finding the value of the product is called *alligation.*

RULE. — To find the average cost per pound of a mixture, when the proportion of the materials mixed and their prices are given, divide the total value of the materials mixed by the sum of the amounts put in, and the quotient will be the average price per pound.

A mill man desires to find the average value per pound for the following lot of wool:

218 lb. at 81 cts. per lb.	$218 \times .81 = \$\ 176.58$
413 lb. at 79 cts. per lb.	$413 \times .79 = 362.27$
284 lb. at 82 cts. per lb.	$284 \times .82 = 232.88$
264 lb. at 83 cts. per lb.	$264 \times .83 = 219.12$
18 lb. at 103 cts. per lb.	$18 \times 1.03 = 19.44$
	1197 lb. $= \$\,1074.29$

$1074.29 \div 1197$ lb. $= \$0.897$, price of mixture per pound.

EXAMPLES

Find the average value per pound of the following lots of wool:

1. 217 lb. at $0.82
   384 lb. at .86
   295 lb. at .89

2. 198 lb. at $0.78
   164 lb. at .81
   312 lb. at .74

3. 217 lb. at $0.94
   108 lb. at .78
   79 lb. at 1.12

4. 164 lb. at $0.94
   218 lb. at .81
   98 lb. at 1.08

5. 208 lb. at $0.81
   161 lb. at .78
   94 lb. at 1.12

6. 174 lb. at $0.79
   101 lb. at .91
   68 lb. at 1.08

7.

112 lb. at	$0.74
184 lb. at	.88
101 lb. at	1.02

8.

211 lb. at	$0.78
191 lb. at	.91
294 lb. at	1.11

9.

198 lb. at	$0.75
208 lb. at	.88
69 lb. at	1.03

10.

69 lb. at	$0.91
51 lb. at	.98

Alligation Alternate

The process of finding the quantities of different values required to produce a mixture of a given value is called *alligation alternate.*

A mill man often desires to find the amount of each kind of wool that must be mixed to produce a mixture of a definite amount.

EXAMPLE. — How much wool of each kind at respective values of 81, 85, and 96 cts. must be mixed to produce a mixture of 89 cts. per pound?

$$89 \left\{ \begin{array}{c} 81 \\ 85 \\ \\ 96 \end{array} \right\} \begin{array}{l} +8 \times 1 = 8 \\ +4 \times 1 = \underline{4} \\ \qquad\quad\;\; 12 \text{ gain} \\ -7 \times 1\tfrac{5}{7} = 12 \text{ loss} \end{array}$$

Place the price of the mixture on the left and the prices of the ingredients on the right of the brace. The differences are placed on the right of another brace as plus or minus as shown above. One pound of the first and 1 of the second will give a gain of 12 over the desired mixture; $1\frac{5}{7}$ lb. of the third will make up the difference of 12.

One pound of the first, one of the second, and $1\frac{5}{7}$ pounds of the third will give the desired mixture.

EXAMPLES

1. How much wool of each kind is required at the respective values of: 83, 87, and 94 to produce a mixture of 88 cts.?

2. How much wool of each kind is required at the respective values of: 78, 83, and 88 to produce a mixture of 86 cts.?

3. How much wool of each kind is required at the respective values of: 74, 84, and 91 to produce a mixture of 87 cts.?

4. How much wool of each kind is required at the respective values of: 79, 86, and 94 to produce a mixture of 88 cts.?

5. How much wool of each kind is required at the respective values of: 78, 88, and 93 to produce a mixture of 91 cts.?

EXAMPLE. — A manufacturer has 432 lb. of wool of a value 94 cts. on hand which he desires to use in producing a lot worth 82 cts. per lb. He has a large lot of a cheaper grade of wool (2587 lb. worth 75 cts.). How much of this cheaper grade must be used to produce a mixture worth 82 cts. per pound?

$$82\left\{\begin{matrix}94\\ \\ 75\end{matrix}\right\}\quad\begin{matrix}-12\times 432=5184\text{ loss}\\ \\ +7\times 740\tfrac{4}{7}=5184\text{ gain}\end{matrix}$$

He should mix 432 lb. of 94 cts. and $740\frac{4}{7}$ lb. of 75 cts. wool in order to produce a mixture of 82 cts.

Proof. — 94×432 lb. $=\$406.08$
$75\times 740\frac{4}{7}$ lb. $=\ \ 555.42\frac{6}{7}$
$1172\frac{4}{7}$ lb. $=\$961.50\frac{6}{7}$
1 lb. $=$ 82 cts.

EXAMPLES

1. With merino wool at $ 1.38 per lb. and Australian wool at $ 1.25 and $ 1.31 per lb., what proportion of each should be used to produce a mixture costing $ 1.32 per lb.?

2. How will four kinds of cotton fiber worth 37 cents, 42 cents, $43\frac{1}{2}$ cents, and 82 cents per pound be mixed to give off an intermediate grade of cotton to be valued at 50 cents per pound?

3. When wool is valued at $\$1.33\frac{1}{3}$ per lb. and cotton at 87 cents, how much of each must be used to produce a grade of mixture at $ 1.02 per lb.?

4. Find the average cost per lb. of a mixture made up as follows: $\frac{1}{3}$ at $ 1.23 per lb., $\frac{1}{9}$ at 87 cents per lb., and the remainder at 97 cents per lb.

5. What is the cost per lb. of the following mixture: $\frac{1}{4}$ Egyptian at \$ 1.02 per lb., $\frac{1}{2}$ upland at 39 cents per lb., and $\frac{1}{4}$ Indian cotton at 28 cents per lb.?

6. In a certain piece of cloth two woolen yarns of different grades and a silk thread are used; $\frac{1}{16}$ of the yarn is silk and equal amounts of the two grades of wool are used. The silk cost \$ 2.25 per lb. and the wool used weighed 750 lb. and was valued at \$ 1.23 and \$ 1.49 respectively. How much silk and wool was used? What was the cost of the yarn?

7. How many bales of cotton, the average weight being 490 lb. to a bale, would be required to supply a picker room having 12 finisher pickers, each machine producing 11,250 pounds of finished laps per week, the loss during the process being $3\frac{1}{2}\%$?

8. How many bales of cotton, the average weight being 495 lb. to a bale, would be required to supply a picker room having 9 finisher pickers, each machine producing 10,895 pounds of finished laps per week, the loss during the process being $3\frac{3}{4}\%$?

9. How many bales of cotton, the average weight being 500 lb. to a bale, would be required to supply a picker room having 20 finisher pickers, each machine producing 11,250 pounds of finished laps per week, the loss during the process being 4 %?

10. How many bales of cotton would be required for an order of 85,000 pounds of yarn, the average weight per bale being 485 lb.? The loss in the picker room is $2\frac{3}{4}\%$, the loss at the cards is $3\frac{3}{4}\%$, and the loss is 3 % (including invisible loss) during the remaining processes.

APPENDIX

METRIC SYSTEM

THE metric system is used in nearly all the countries of Continental Europe and among scientific men as the standard system of weights and measures. It is based on the **meter** as the unit of length. The meter is supposed to be one ten-millionth part of the length of the meridian passing from the equator to the poles. It is equal to about 39.37 inches. The unit of weight is the **gram**[1] which is equal to about one thirtieth of an ounce. The unit of volume is the **liter**, which is a little larger than a quart.

Measures of Length

10 millimeters (mm.)	= 1 centimeter	cm.
10 centimeters	= 1 decimeter	dm.
10 decimeters	= 1 meter	m.
10 meters	= 1 dekameter	Dm.
10 dekameters	= 1 hektometer	Hm.
10 hektometers	= 1 kilometer	Km.

Measures of Surface (not Land)

100 square millimeters (mm.)	= 1 square centimeter	sq. cm.
100 square centimeters	= 1 square decimeter	sq. dm.
100 square decimeters	= 1 square meter	sq. m.

Measures of Volume

1000 cubic millimeters (mm.)	= 1 cu. centimeter	cu. cm.
1000 cubic centimeters	= 1 cubic decimeter	cu. dm.
1000 cubic decimeters	= 1 cubic meter	cu. m.

[1] The gram is the weight of one cubic centimeter of pure distilled water at a temperature of 39.2° F.; the kilogram is the weight of 1 liter of water; the metric ton is the weight of 1 cubic meter of water.

Measures of Capacity

10 milliliters (ml.)	= 1 centiliter	cl.
10 centiliters	= 1 deciliter	dl.
10 deciliters	= 1 liter[1]	l.
10 liters	= 1 dekaliter	Dl.
10 dekaliters	= 1 hektoliter	Hl.
10 hektoliters	= 1 kiloliter	Kl.

Measures of Weight

10 milligrams (mg.)	= 1 centigram	cg.
10 centigrams	= 1 decigram	dg.
10 decigrams	= 1 gram	g.
10 grams	= 1 dekagram	Dg.
10 dekagrams	= 1 hektogram	Hg.
10 hektograms	= 1 kilogram	Kg.
1000 kilograms	= 1 ton	T.

Metric Equivalent Measures

Measures of Length

1 meter	= 39.37 in. = 3.28083 ft. = 1.0936 yd.
1 centimeter	= .3937 inch
1 millimeter	= .03937 inch, or $\frac{1}{25}$ inch nearly
1 kilometer	= .62137 mile
1 foot	= .3048 meter
1 inch	= 2.54 centimeters = 25.4 millimeters

Measures of Surface

1 square meter	= 10.764 sq. ft. = 1.196 sq. yd.
1 square centimeter	= .155 sq. in.
1 square millimeter	= .00155 sq. in.
1 square yard	= .836 square meter
1 square foot	= .0929 square meter
1 square inch	= 6.452 square centimeters = 645.2 square millimeters

Measures of Volume and Capacity

1 cubic meter	= 35.314 cu. ft. = 1.308 cu. yd. = 264.2 gal.
1 cubic decimeter	= 61.023 cu. in. = .0353 cu. ft.
1 cubic centimeter	= .061 cu. in.

[1] The liter is equal to the volume occupied by 1 cubic decimeter.

1 liter	=	1 cubic decimeter = 61.023 cu. in. = .0353 cu. ft. = 1.0567 quarts (U. S.) = .2642 gallon (U. S.) = 2.202 lb. of water at 62° F.
1 cubic yard	=	.7645 cubic meter
1 cubic foot	=	.02832 cubic meter = 28.317 cubic decimeters = 28.317 liters
1 cubic inch	=	16.387 cubic centimeters
1 gallon (British)	=	4.543 liters
1 gallon (U. S)	=	3.785 liters

Measures of Weight

1 gram	=	15.432 grains
1 kilogram	=	2.2045 pounds
1 metric ton	=	.9842 ton of 2240 lb. = 19.68 cwt. = 2204.6 lb.
1 grain	=	.0648 gram
1 ounce avoirdupois	=	28.35 grams
1 pound	=	.4536 kilogram
1 ton of 2240 lb.	=	1.016 metric tons = 1016 kilograms

Miscellaneous

1 kilogram per meter = .6720 pound per foot
1 gram per square millimeter = 1.422 pounds per square inch
1 kilogram per square meter = .2084 pound per square foot
1 kilogram per cubic meter = .0624 pound per cubic foot
1 degree centigrade = 1.8 degrees Fahrenheit
1 pound per foot = 1.488 kilograms per meter
1 pound per square foot = 4.882 kilograms per square meter
1 pound per cubic foot = 16.02 kilograms per cubic meter
1 degree Fahrenheit = .5556 degree centigrade
1 Calorie (French Thermal Unit) = 3.968 B. T. U. (British Thermal Unit)
1 horse power = 33,000 foot pounds per minute = 746 watts
1 watt (Unit of Electrical Power) = .00134 horse power = 44.24 foot pounds per minute
1 kilowatt = 1000 watts = 1.34 horse power = 44,240 foot pounds per minute

Table of Metric Conversion

To change meters to feet	multiply by	3.28083
feet to meters	multiply by	.3048
square feet to square meters . . .	multiply by	.0929
square meters to square feet . . .	multiply by	10.764

To change square centimeters to square inches .	multiply by	.155
square inches to square centimeters .	multiply by	6.452
inches to centimeters	multiply by	2.54
centimeters to inches	multiply by	.3937
grams to grains	multiply by	15.43
grains to grams	multiply by	.0648
grams to ounces	multiply by	.0353
ounces to grams	multiply by	28.35
pounds to kilograms	multiply by	.4536
kilograms to pounds	multiply by	2.2045
liters to quarts	multiply by	1.0567
liters to gallons	multiply by	.2642
gallons to liters	multiply by	3.78543
liters to cubic inches	multiply by	61.023
cubic inches to cubic centimeters . .	multiply by	16.387
cubic centimeters to cubic inches . .	multiply by	.061
cubic feet to cubic decimeters or liters	multiply by	28.317
kilowatts to horse power	multiply by	1.34
calories to British Thermal Units . .	multiply by	3.968

EXAMPLES

1. Change 8 m. to centimeters; to kilometers.

2. Reduce 4 Km., 6 m., and 2 m. to centimeters.

3. How many square meters of carpet will cover a floor which is 25.5 feet long and 24 feet wide?

4. (*a*) Change 6.5 centimeters into inches.
   (*b*) Change 48.3 square centimeters into square inches.

5. A cellar 18 m. × 37 m. × 2 m. is to be excavated; what will it cost at 13 cents per cubic meter to do the work?

6. How many liters of capacity has a tank containing 5.2 cu. m.?

7. What is the weight in grams of 31 cc. of water?

8. Give the approximate value of 36 millimeters in inches.

9. Change 84.9 square meters into square feet.

10. Change 23.6 liters to cubic inches.

11. If a tank is 7.6 m. by 1.4 m. by 2.2 m., how many kilograms of water will it hold?

12. A blue print of a table top measures 3.8 m. by 78.6 cm., what are its length and width in inches?

13. A package weighs 28 kg. What is its weight in pounds?

14. A box weighs 82.5 g. How many ounces does it weigh?

15. (*a*) Change 14.6 kilograms to pounds.
(*b*) Change 6.8 kilowatts to horse power.
(*c*) Change 20.4 liters to quarts.
(*d*) Change 83.2 liters to gallons.
(*e*) Change 10.25 inches to centimeters.

16. A cubic foot of water weighs about 62.5 lb. Find the weight in kilograms.

17. (*a*) Change 30.5 sq. in. to sq. cm.
(*b*) Change 84.9 sq. ft. to sq. m.
(*c*) Change 8.5 gal. to liters.
(*d*) Change 164.3 grains to grams.
(*e*) Change 212.8 oz. to grams.

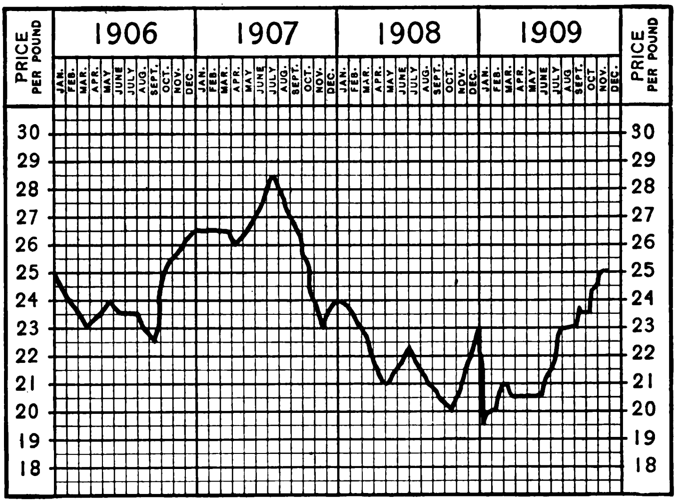
18. The dimensions of a meal chest on a foreign blue print are 2.6 m., .48 m., and 59.3 cm. How many liters of meal will it hold?

19. Change 6.1 pounds into kilograms.

20. A rectangular watering trough is 166 cm. long, 58 cm. wide, and 422 mm. deep. If it is full of water, what are its contents in liters?

GRAPHS

A sheet of paper, ruled with horizontal and vertical lines that are equally distant from each other, is called a sheet of cross-section, or coördinate, paper. Every tenth line is very distinct so that it is easy for one to measure off the horizontal and vertical distances without the aid of a ruler. Ruled or



Graph showing the Variation in Price of Cotton Yarn for a Series of Years

coördinate paper is used to record the rise and fall of the price of any commodity, or the rise and fall of the barometer or thermometer.

Trade papers and reports frequently make use of coördinate paper to show the results of the changes in the price of commodities. In this way one can see at a glance the changes

and condition of a certain commodity, and can compare these with the results of years or months ago. He also can see from the slope of the curve the rate of rise or fall in price.

If similar commodities are plotted on the same sheet, the effect of one on the other can be noted. Often experts are able to prophesy with some certainty the price of a commodity for a month in advance. The two quantities which must be employed in this comparison are time and value, or terms corresponding to them.

The lowest left-hand corner of the squared paper is generally used as an initial point, or origin, and is marked O, although any other corner may be used. The horizontal line from this corner, taken as a line of reference or axis, is called the *abscissa.* The vertical line from this corner is the other axis, and is called the *ordinate.*

Equal distances on the *abscissa* (horizontal line) represent definite units of time (hours, day's, months, years, etc.), while equal distances along the *ordinate* (vertical line) represent certain units of value (cost, degrees of heat, etc.).

By plotting, or placing points which correspond to a certain value on each axis and connecting these points, a line is obtained that shows at every point the relationship of the line to the axis.

EXAMPLES

1. Show the rise and fall of temperature in a day from 8 A.M. to 8 P.M., taking readings every hour.

2. Show the rise and fall of temperature at noon every day for a week.

3. Obtain stock quotation sheets and plot the rise and fall of cotton for a week.

4. Show the rise and fall of the price of potatoes for two months.

5. Show a curve giving the amount of coal used each day for a week.

FORMULAS

Most technical books and magazines contain many formulas. The reason for this is evident when we remember that rules are often long and their true meaning not comprehended until they have been reread several times. The attempt to abbreviate the length and emphasize the meaning results in the formula, in which whole clauses of the written rule are expressed by one letter, that letter being understood to have throughout the discussion the same meaning with which it started.

To illustrate: One of the fundamental laws of electricity is that the quantity of electricity flowing through a circuit (flow of electricity) is equal to the quotient (expressed in amperes) obtained by dividing the electric motive force (pressure, or expressed in volts, voltage) of the current by the resistance (expressed in ohms).

One unfamiliar with electricity is obliged to read this rule over several times before the relations between the different parts are clear. To show how the rule may be abbreviated,

Let $I =$ quantity of electricity through a wire (amperes)

$E =$ pressure of the current (volts)

$R =$ resistance of the current (ohms)

Then $I = E \div R = \frac{E}{R}$

It is customary to allow the first letter of the quantity to represent it in the formula, but in this case I is used because the letter C is used in another formula with which this might be confused.

Translating Rules into Formulas

The area of a trapezoid is equal to the sum of the two parallel sides multiplied by one half the perpendicular distance between them.

We may abbreviate this rule by letting

A = area of trapezoid
L = length of longest parallel side
M = length of shortest parallel side
N = length of perpendicular distance between them

Then $A = (L + M) \times \frac{N}{2}$, or

$$A = (L + M)\frac{N}{2}*$$

The area of a circle is equal to the square of the radius multiplied by 3.1416. When a number is used in the formula it is called a constant, and is sometimes represented by a letter. In this case 3.1416 is represented by the Greek letter π (pi).

Let A = area of circle
R = radius of circle

Then $A = \pi \times R^2$, or (the multiplication sign is usually left out between letters)

$$A = \pi R^2$$

Thus we see that a formula is a short and simple way of stating a rule. Any formula may be written or expressed in words and is then called a rule. The knowledge of formulas and of their use is necessary for nearly every one engaged in the higher forms of mechanical or technical work.

* When two or more quantities are to be multiplied or divided or otherwise operated upon by the same quantity, they are often grouped together by means of parentheses ( ) or braces { }, or brackets [ ]. Any number or letter placed before or after one of these parentheses, with no other sign between, is to multiply all that is grouped within the parentheses. In the trapezoid case above, $\frac{N}{2}$ is to multiply the sum of L and M, hence the parentheses. To prevent confusion, different kinds of parentheses may be used for different combinations in the same problem.
For instance,

$$V = \frac{1}{3}\pi H\left[\frac{3}{2}(r^2 + r'^2) + \frac{H^2}{2}\right] \text{ which equals}$$
$$V = \tfrac{1}{2}\pi H(r^2 + r'^2) + \tfrac{1}{6}\pi H^3$$

EXAMPLES

Abbreviate the following rules into formulas:

1. One electrical horse power is equal to 746 watts.

2. One kilowatt is equal to 1000 volts.

3. The number of watts consumed in a given electrical circuit, such as a lamp, is obtained by multiplying the volts by the amperes.

4. The volts equal the watts divided by the amperes.

5. Amperes equal the watts divided by the volts.

6. To find the pressure in pounds per square inch of a column of water, multiply the height of the column in feet by .434.

7. To find the diameter of a pump cylinder required to move a given quantity of water per minute (100 ft. of piston being the standard of speed), divide the number of gallons by 4, then extract the square root, and the product will be the diameter in inches of the pump cylinder.

8. To find quantity of water elevated in one minute (running at 100 feet of piston per minute), square the diameter of the water cylinder in inches and multiply by 4.

9. To find the horse power necessary to elevate water to a given height, multiply the weight of the water elevated per minute in pounds by the height in feet, and divide the product by 33,000. (An allowance should be added for water friction, and a further allowance for loss in steam cylinder, say from 20 to 30 per cent.)

10. In a steam pump the effective pressure equals the area of the steam piston, multiplied by the steam pressure, minus the area of the water piston, multiplied by the pressure of water per square inch, which gives the resistance. (A margin must be made between the power and the resistance to move the pistons at the required speed — say from 20 to 40 per cent, according to speed and other conditions.)

11. To find the capacity of a cylinder in gallons. Multiplying the area in inches by the length of stroke in inches will give the total number of cubic inches; divide this amount by 231 (which is the cubical contents of a United States gallon in inches) and the product is the capacity in gallons.

12. To find the length of an arc of a circle: Multiply the diameter of the circle by the number of degrees in the arc and this product by .0087266.

13. To find the area of a sector of a circle: Multiply the number of degrees in the arc of the sector by the square of the radius and by .008727; or, multiply the arc of the sector by half its radius.

Translating Formulas into Rules

In order to understand a formula, it is necessary to be able to express it in simple language.

1. One of the simplest formulas is that for finding the area of a circle, $A = \pi R^2$

Here A stands for the area of a circle,
R for the radius of the circle.
π is a constant quantity and is the ratio of the circumference of a circle to its diameter. The exact value cannot be expressed in figures, but for ordinary purposes is called 3.1416 or $3\frac{1}{7}$.

Therefore, the formula reads, the area of a circle is equal to the square of the radius multiplied by 3.1416.

2. The formula for finding the area of a rectangle is

$$A = L \times W$$

Here A = area of a rectangle
L = length of rectangle
W = width of rectangle

The area of a rectangle, therefore, is found by multiplying the length by the width.

EXAMPLES

Express the facts of the following formulas as rules:

1. Electromotive force or voltage of electricity delivered by a current, when current and resistance are given:

$$E = RI$$

2. For the circumference of a circle, when the length of the radius is given:

$$C = 2\pi R \text{ or } \pi D$$

3. For the area of an equilateral triangle, when the length of one side is given:

$$A = \frac{a^2\sqrt{3}}{4}$$

4. For the volume of a circular pillar, when the radius and height are given:

$$V = \pi R^2 h$$

5. For the volume of a square pyramid, when the height and one side of the base are given:

$$V = \frac{a^2 h}{3}$$

6. For the volume of a sphere, when the diameter is given:

$$V = \frac{\pi D^3}{6}$$

7. For the diagonal of a rectangle, when the length and breadth are given:

$$D = \sqrt{L^2 + b^2}$$

8. For the average diameter of a tree, when the average girth is known:

$$D = \frac{G}{\pi}$$

9. For the diameter of a ball, when the volume of it is known.

$$D = \sqrt[3]{\frac{6v}{\pi}}$$

10. The diameter of a circle may be obtained from the area by the following formula:

$$D = 1.1283 \times \sqrt{A}$$

11. The number of miles in a given length, expressed in feet, may be obtained from the formula

$$M = .00019 \times F$$

12. The number of cubic feet in a given volume expressed in gallons may be obtained from the formula

$$C = .13367 \times G$$

13. Contractors express excavations in cubic yards; the number of bushels in a given excavation expressed in yards may be obtained from the formula

$$C = .0495 \times Y$$

14. The circumference of a circle may be obtained from the area by the formula

$$C = 3.5446 \times \sqrt{A}$$

15. The area of the surface of a cylinder may be expressed by the formula $A = (C \times L) + 2\,a$

When $C =$ circumference
$L =$ length
$a =$ area of one end

16. The surface of a sphere may be expressed by the formula

$$S = D^2 \times 3.141\dot{6}$$

17. The solidity of a sphere may be obtained from the formula

$$S = D^3 \times .5236$$

18. The side of an inscribed cube of a sphere may be obtained from the formula

$S = R \times 1.1547$, where $S =$ length of side,
$R =$ radius of sphere.

19. The solidity or contents of a pyramid may be expressed by the formula

$$S = A \times \frac{F}{3}, \qquad \text{where } A = \text{area of base,}$$
$$F = \text{height of pyramid.}$$

20. The length of an arc of a circle may be obtained from the formula

$$L = N \times .017453\,R, \qquad \text{where } L = \text{length of arc,}$$
$$N = \text{number of degrees,}$$
$$R = \text{radius of circle.}$$

21. The horse power of a single leather belt may be determined by the formula

$$HP = \frac{DRW}{2520}, \qquad \text{where } D = \text{diameter of pulley in inches,}$$
$$W = \text{width of belt in inches,}$$
$$R = \text{revolutions per minute,}$$
$$HP = \text{horse power transmitted.}$$

22. The formula for finding the weight of an iron ball may be calculated by the following:

$$W = D^3 \times 0.1377$$

23. The formula for finding the diameter of an iron ball when the weight is given is

$$D = 1.936\sqrt[3]{W}, \qquad \text{where } D = \text{diameter of the ball in inches,}$$
$$W = \text{weight of ball in pounds.}$$

24. The volume of a sphere when the circumference of a great circle is known may be determined by the formula

$$V = \frac{C^3}{6\,\pi^2}$$

25. The diameter of a circle the circumference of which is known may be found by the formula

$$D = \frac{C}{\pi}$$

26. The area of a circle the circumference of which is known may be found by the formula

$$A = \frac{C^2}{4\pi}$$

Coefficients and Similar Terms

When a quantity may be separated into two factors, one of these is called the *coefficient* of the other; but by the coefficient of a term is generally meant its numerical factor.

Thus, $4b$ is a quantity composed of two factors 4 and b; 4 is a coefficient of b.

Similar terms are those that have as factors the same letters with the same exponents.

Thus, in the expression, $6a$, $4b$, $2a$, $5ab$, $5a$, $2b$. $6a$, $2a$, $5a$ are similar terms; $4b$, $2b$ are similar terms; $5ab$ and $6a$ are not similar terms because they do not have the same letters as factors. $3ab$, $5ab$, $1ab$, $8ab$ are similar terms. They may be united or added by simply adding the letters to the numerical sum, $17ab$.

In the following, $8b$, $5b$, $3ab$, $4a$, ab, and $2a$, $8b$ and $5b$ are similar terms; $3ab$ and ab are similar terms; $4a$ and $2a$ are similar terms; $8b$, $3ab$, and $4a$ are dissimilar terms.

In addition the numerical coefficients are algebraically added; in subtraction the numerical coefficients are algebraically subtracted; in multiplication the numerical coefficients are algebraically multiplied; in division the numerial coefficients are algebraically divided.

EXAMPLES

State the similar terms in the following expressions:

1. $5x$, $8ax$, $3x$, $2ax$.
2. $8abc$, $7c$, $2ab$, $3c$, $8ab$, $9abc$.
3. $2pq$, $5p$, $8q$, $2p$, $3q$, $5pq$.
4. $4y$, $5yz$, $2y$, $15z$, $5z$, $2yz$.
5. $18mn$, $6m$, $5n$, $4mn$, $2m$.
6. $15abc$, $2abc$, $4abc$, $2ab$, $3ab$.
7. $8x$, $6x$, $13xy$, $5x$, $7y$.
8. $7y$, $2y$, $2xy$, $3y$, $2xy$.
9. 2π, $5\pi r^2$, $\frac{\pi}{2}$, πr^2, $2\pi r$.

Equations

A statement that two quantities are equal may be expressed mathematically by placing one quantity on the left and the other on the right of the equality sign (=). The statement in this form is called an equation.

The quantity on the left hand of the equation is called the left-hand member and the quantity on the right hand of the equation is called the right-hand member.

An equation may be considered as a balance. If a balance is in equilibrium, we may add or subtract or multiply or divide the weight on each side of the balance by the same weight and the equilibrium will still exist. So in an equation we may perform the following operations on each member without changing the value of the equation:

We may add an equal quantity or equal quantities to each member of the equation.

We may subtract an equal quantity or equal quantities from each member of the equation.

We may multiply each member of the equation by the same or equal quantities.

We may divide each member of the equation by the same or equal quantities.

We may extract the square root of each member of the equation.

We may raise each member of the equation to the same power.

The expression, $A = \pi R^2$ is an equation. Why?

If we desire to obtain the value of R instead of A we may do so by the process of transformation according to the above rules. To obtain the value of R means that a series of operations must be performed on the equation so that R will be left on one side of the equation.

(1) $A = \pi R^2$

(2) $\frac{A}{\pi} = R^2$ (Dividing equation (1) by the coefficient of R^2.)

(3) $\sqrt{\frac{A}{\pi}} = R$ (Extracting the square root of each side of the equation.)

Methods of Representing Operations

Multiplication

The multiplication sign ($\times$) is used in most cases. It should not be used in operations where the letter (x) is also to be employed.

Another method is as follows:

$$2 \cdot 3 \quad a \cdot 6 \quad 2a \cdot 3b \quad 4x \cdot 5a$$

This method is very convenient, especially where a number of small terms are employed. Keep the dot above the line, otherwise it is a decimal point.

Where parentheses, etc., are used, multiplication signs may be cmitted. For instance, $(a + b) \times (a - b)$ and $(a + b)(a - b)$ are identical; also, $2 \cdot (x - y)$ and $2(x - y)$.

The multiplication sign is very often omitted in order to simplify work. To illustrate, $2a$ means 2 times a; $5xyz$ means $5 \cdot x \cdot y \cdot z$; $x(a - b)$ means x times $(a - b)$, etc.

A number written to the right of, and above, another (x^4) is a sign indicating the special kind of multiplication known as involution.

In multiplication we add exponents of similar terms.

Thus,
$$x^2 \cdot x^3 = x^{2+3} = x^5$$
$$abc \cdot ab \cdot a^2b = a^4b^3c$$

The multiplication of dissimilar terms may be indicated.

Thus,
$$a \cdot b \cdot c \cdot x \cdot y \cdot z = abcxyz.$$

Division

The division sign ($\div$) is used in most cases. In many cases, however, it is best to employ a horizontal line to indicate division. To illustrate, $\frac{a + b}{x - y}$ means the same as $(a + b) \div (x - y)$ in simpler form. The division sign is *never* omitted.

A root or radical sign ($\sqrt{3}$, $\sqrt[4]{x^2}$) is a sign indicating the special form of division known as evolution.

In division, we subtract exponents of similar terms.

Thus,
$$x^3 \div x^2 = \frac{x^3}{x^2} = x^{3-2} = x$$
$$a^4b^2c^3 \div a^2bc^2 = \frac{a^4b^2c^3}{a^2bc^2} = a^2bc.$$

The division of dissimilar terms may be indicated.

Thus,
$$(abc) \div xyz = \frac{abc}{xyz}.$$

Substituting and Transposing

A formula is usually written in the form of an equation. The left-hand member contains only one quantity, which is the quantity that we desire to find. The right-hand member contains the letters representing the quantity and numbers whose values we are given either directly or indirectly.

To find the value of the formula we must (1) substitute for every letter in the right-hand member its exact numerical value, (2) carry out the various operations indicated, remembering to perform all the operations of multiplication and division before those of addition and subtraction, (3) if there are any parentheses, these should be removed, one pair at a time, inner parentheses first. A minus sign before a parenthesis means that when the parenthesis is removed, all the signs of the terms included in the parenthesis must be changed.

Find the value of the expression

$$3a + b(2a - b + 18), \text{ where } a = 5,\ b = 3.$$

Substitute the value of each letter. Then perform all addition or subtraction in the parentheses.

$$3 \times 5 + 3(10 - 3 + 18)$$
$$15 + 3(28 - 3)$$
$$15 + 3(25)$$
$$15 + 75 = 90$$

EXAMPLES

Find the value of the following expressions:

1. $2A \times (2 + 3A) \times 8$, when $A = 10$.

2. $8a \times (6 - 2a) \times 7$, when $a = 7$.

3. $8b + 3c + 2a(a + b + c) - 8$, when $a = 9$; $b = 11$; $c = 13$.

4. $8(x + y)$, when $x = 9$; $y = 11$.

5. $13(x - y)$, when $x = 27$; $y = 9$.

6. $24y + 8z(2 + y) - 3y$, when $y = 8$; $z = 11$.

7. $Q(6M + 3N) + 2O$, when $M = 4$, $N = 5$, $Q = 6$, $O = 8$.

8. Find the value of X in the formula $X = \frac{3(MN + P)}{P - M}$, when $M = 11$, $N = 9$, $P = 28$.

9. $x = \frac{8(n + m)}{P - Q}$, when $n = 5$, $m = 6$, $P = 8$, $Q = 7$.

10. Find the value of T in the equation

$$T = \frac{8(x + y) + 7(x + y)}{(x + y)(x - y)}, \text{ when } x = 7,\ y = 6.$$

11. $3a + 4(b - 2a + 3c) - c$, when $a = 4$, $b = 6$, $c = 2$.

12. $5p - 8q(p + r - S) - q$, when $p = 5$, $q = 7$, $r = 9$, $S = 11$.

13. $S^2 + t^2 - p^2 - 3(S + t + p)$, when $p = 5$, $S = 8$, $t = 9$.

14. $a^2 - b^3 + c^2$, when $a = 9$, $b = 6$, $c = 4$.

15. $(a + b)(a + b - c)$, when $a = 2$, $b = 3$, $c = 4$.

16. $(a^2 - b^2)(a^2 + b^2)$, when $a = 8$, $b = 4$.

17. $(c^3 + d^3)(c^3 - d^3)$, $c = 9$, $d = 5$.

18. $\sqrt{a^2 + 2ab + b^2}$, when $a = 7$, $b = 8$.

19. $\sqrt[3]{c^3 - 61}$, when $c = 5$.

PROBLEMS

Solve the following problems by first writing the formula from the rule on page 300, and then substituting for the answer.

1. How many electrical horse power in 4389 watts?

2. How many kilowatts in 2389 watts?

3. (*a*) Give the number of watts in a circuit of 110 volts and 25 amperes.

(*b*) How many electrical horse power?

4. What is the voltage of a circuit if the horse power is 2740 watts and the quantity of electricity delivered is 25 amperes?

5. What is the resistance of a circuit if the voltage is 110 and the quantity of electricity is 25 amperes?

6. What is the pressure per square inch of water 87 feet high?

7. What is the capacity of a cylinder with a base of 16 square inches and 6 inches high? (Capacity in gallons is equal to cubical contents obtained by multiplying base by the height and dividing by 231 cubic inches.)

8. What is the length of a 30° arc of a circle with 16″ diameter?

9. What is the area of a sector with an arc of 40° and a diameter of 18″?

10. What is the weight of the rim of a flywheel of a 25 H. P. engine?

11. What is the area of a cylinder of a 50 H. P. engine with the piston making 120 ft. per minute?

12. What is the H. P. of a $2\frac{15}{16}''$ shaft making 180 R. P. M.?

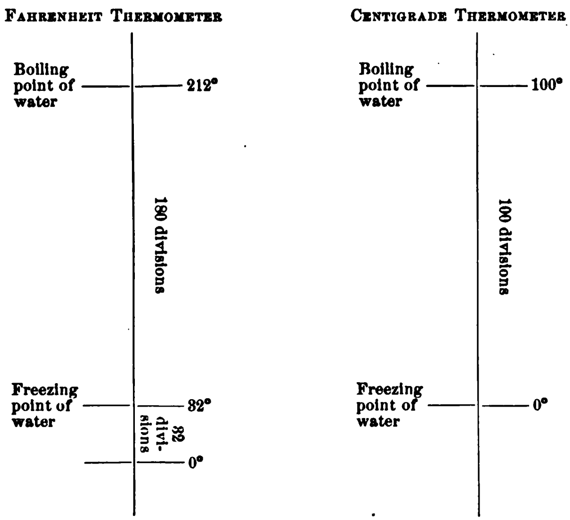
13. What is the capacity of a pail 14″ (diameter of top), 11″ (diameter of bottom), and 16″ in height?

14. What is the area of an ellipse with the greatest length 16″ and the greatest breadth 10″?

Interpretation of Negative Quantities

The quantity or number -12 has no meaning to us according to our knowledge of simple arithmetic, but in a great many problems in practical work the minus sign before a number assists us in understanding the different solutions.

To illustrate:



On the Centigrade scale the freezing point of water is marked 0°. Below the freezing point of water on the Centigrade scale all readings are expressed as minus readings.

$-30°$ C means thirty degrees below the freezing point. In other words, all readings, in the direction below zero are expressed as $-$, and all readings above zero are called $+$. Terms are quantities connected by a plus or minus sign. Those preceded by a plus sign (when no sign precedes a quantity plus is understood) are called positive quantities, while those connected by a minus sign are called negative quantities.

Let us try some problems involving negative quantities. Find the corresponding reading on the Fahrenheit scale corresponding to $-18°$ C.

$$F = \tfrac{9}{5}\,C + 32°$$
$$F = \tfrac{9}{5}(-18°) + 32°$$

Notice that a minus quantity is placed in parenthesis when it is to be multiplied by another quantity.

$$F = -\tfrac{162}{5}° + 32° = -32\tfrac{2}{5}° + 32°\,;\ F = -\tfrac{2}{5}°.$$

The value $-\frac{2}{5}°$ is explained by saying it is $\frac{2}{5}$ of a degree below zero point on Fahrenheit scale.

Let us consider another problem. Find the reading on the Centigrade scale corresponding to $-40°$ F.

Substituting in the formula, we have

$$C = \tfrac{5}{9}(-40° - 32°) = \tfrac{5}{9}(-72) = -40°.$$

Since subtracting a negative number is equivalent to adding a positive number of the same value, and subtracting a positive number is equivalent to adding a negative number of the same value, the rule for subtracting may be expressed as follows: Change the sign of the subtrahend and proceed as in addition.

For example, 40 minus -28 equals 40 plus 28, or 68.
40 minus $+28$ equals 40 plus -28, or 12.
-40 minus $+32$ equals -40 plus $-32 = -72$.

(Notice that a positive quantity multiplied by a negative quantity or a negative quantity multiplied by a positive quantity always gives a negative product. Two positive quantities multiplied together will give a positive product, and two negative quantities multiplied together will give a positive product.) To illustrate:

$$5 \text{ times } 5 = 5 \times 5 = 25$$
$$5 \text{ times } -5 = 5 \times (-5) = -25$$
$$(-5) \text{ times } (-5) = +25$$

In adding positive and negative quantities, first add all the positive quantities and then add all the negative quantities

together. Subtract the smaller from the larger and prefix the same sign before the remainder as is before the larger number.

For example:

$$2a,\ 5a,\ -6a,\ 8a,\ -2a$$
$$2a + 5a + 8a = 15a;\ -6a - 2a = -8a$$
$$15a - 8a = 7a$$

EXAMPLES

Add the following terms:

1. $3x,\ -x,\ 7x,\ 4x,\ -2x.$
2. $6y,\ 2y,\ 9y,\ -7y.$
3. $9ab,\ 2ab,\ 6ab,\ -4ab,\ 7ab,\ -5ab.$

Multiplication of Algebraic Expressions

Each term of an algebraic expression is composed of one or more factors, as, for example, $2ab$ contains the factors *2*, a, and b. The factors of a term have, either expressed or understood, a small letter or number in the upper right-hand corner, which states how many times the quantity is to be used as a factor. For instance, ab^2. The factor a has the exponent 1 understood and the factor b has the exponent 2 expressed, meaning that a is to be used once and b twice as a factor. ab^2 means, then, $a \times b \times b$. The rule of algebraic multiplication by terms is as follows: Add the exponents of all like letters in the terms multiplied and use the result as exponent of that letter in the product. Multiplication of unlike letters may be expressed by placing the letters side by side in the product.

For example:

$$2ab \times 3b^2 = 6ab^3$$
$$4a \times 3b = 12ab$$

Algebraic or literal expressions of more than one term are multiplied in the following way: begin with the first term to the left in the multiplier and multiply every term in the multiplicand, placing the partial products underneath the line. Then

repeat the same operation, using the second term in the multiplier. Place similar products of the same factors and degree (same exponents) in same column. Add the partial products.

Thus, $a + b$ multiplied by $a - b$.

$$\begin{array}{l} a + b \\ \underline{a - b } \\ a^2 + ab - b^2 \\ \underline{ - ab } \\ a^2 - b^2 \end{array}$$

Notice the product of the sum and difference of the quantities is equal to the difference of their squares.

EXAMPLES

1. Multiply $a + b$ by $a + b$.

State what the square of the sum of the quantities equals.

2. Multiply $x - y$ by $x - y$.

State what the square of the difference of the quantities equals.

3. Multiply $(p + q)(p - q)$.
4. Multiply $(p + q)(p + q)$.
5. Multiply $(r + s)(r - s)$.
6. Multiply $(a + b)(a + b)$.
7. Multiply $(x - y)(x - y)$.
8. $(x + y)^2 = ?$
9. $(x - y)^2 = ?$
10. $(x + y)(x - y) = ?$

LOGARITHMS

THE **logarithm** of a number to the base 10 is defined as the power to which 10 must be raised in order to equal the number.

Thus the logarithm (or log, as it is more generally written) of 10 is 1, for the first power of 10 is 10. The log of 100 is 2, for the second power of 10 is 100. Hence the log of a number between 10 and 100 is a number between 1 and 2, and is, therefore, 1 plus a decimal. The whole number 1 is called the *characteristic*, and the decimal part is called the *mantissa*. In like manner, the log of a number between 100 and 1000 is 2 plus a decimal, for $10^2 = 100$ and $10^3 = 1000$. The log of a number between 1 and 10 is 0 plus a decimal.

The Characteristic. — The *characteristic* of the log of a number between 1 and 10 is 0; that is, the characteristic of the log of a number that has one digit is 0. The characteristic of the log of a number between 10 and 100 is 1. The characteristic of the log of a number between 100 and 1000 is 2. In each case, the characteristic is seen to be one less than the number of digits at the left of the decimal point. Hence the rule:

RULE I. — *The characteristic of the logarithm of a number is one less than the number of digits (or the number of figures) in the numbers that are at the left of the decimal point.*

If, however, there are no figures at the left of the decimal point; that is, if the number is less than 1, we must use the following rule:

RULE II. — *The characteristic of a number less than one is (minus) one more than the number of zeros at the right of the decimal point before the first significant figure.*

Thus the characteristic of the log of .41 is -1; of 0.0302 is -2; of 0.0032 is -3, etc. These are usually written $9-10$; $8-10$; $7-10$.

The Mantissa. — Consider the three numbers 7.21, 72.1, and 721. $721. = 10 \times 72.1 = 10^2 \times 7.21$. Therefore, $\log 721 = \log 10 + \log 72.1 = \log 10^2 + \log 7.21$. (In order to multiply the numbers, we may add the logarithms of these numbers.) Therefore, $\log 721 = 1 + \log 72.1 = 2 + \log 7.21$, ($\log 10 = 1$; $\log 10^2 = 2$). Therefore, we see that the logarithms of numbers made up of the same sequence of

digits, but differing in the position of the decimal point, differ only in the characteristic, the *mantissæ* remaining the same.

For this reason, in making tables of logarithms, only the mantissæ are given, the characteristics being added according to the rules above. The tables we are to use are made for three significant digits in the number and four digits in the mantissa of the log.

To Find the Logarithm of a Number

A. When the number has three significant figures.

The first two figures are found in the column headed No. (pages 318–319), and the third figure in the top row. The mantissa is stated in the column and row so determined.

To illustrate, find the log of 62.8. In the 62 row and in the column under 8 is the mantissa, 7980. By Rule I we find the characteristic to be 1; therefore, the log of 62.8 is 1.7980.

Find the log of .00709. The significant figures are 709; in the 70 row and in the column under 9, we find the mantissa 8506. By Rule II the characteristic is -3. Therefore the log of .00709 is -3.8506. To prevent confusion in the operations of arithmetic -3 is usually written $\bar{3}$, so the log is stated to be $\bar{3}.8506$, or $7.8506 - 10$.

EXAMPLES

Find log of 117; of 1280; of 16.5; of 2.09; of .721; of .0121.

B. When the number has four or more significant figures.

Ex. 1.— Find the log of 7987. The tables give only the mantissa of numbers of three figures, so the mantissa of 7987 cannot be found in the table; 7987, however, lies between 7980 and 7990, hence its log is between the log of 7980 and the log of 7990. We find the mantissa 7980 to be 9020, and the mantissa of 7990 to be 9025. Now the difference between these mantissæ is 5, while the difference between the two numbers, 7980 and 7990, is 10. But the difference between 7980 and 7987 is $\frac{7}{10}$ of the difference between 7980 and 7990, and the mantissa of 7987 will be $\frac{7}{10}$ of the difference between the mantissa of 7980 and the mantissa of 7990. Therefore it will equal 9020 (the mantissa of 7980) plus $\frac{7}{10}$ of 5 (the difference between the mantissæ of 7980 and 7990): $5 \times .7 = 3.5$, hence the mantissa of 7987 is 9024. In such reckoning all decimal parts less than .5 are counted as 0, and all decimal parts greater than .5 are counted as 1; .5 is counted as either 1 or 0. Log 7987 is .39024.

Ex. 2. — Find the log of 12.554.

First find the mantissa. In looking for the mantissa the decimal point need not be considered.

Solution. — 12554 lies between 12500 and 12600.

Mantissa of 12600 is 1004.

Mantissa of 12500 is 0969.

35 is the difference between the mantissæ.

$12600 - 12500 = 100 =$ difference between numbers.

$12554 - 12500 = 54 =$ difference between original numbers.

The multiplier is $\frac{54}{100} = .54$.

$35 \times .54 = 18.90$ or 19.

$0969 + 19 = 0988$.

log 12.564 is 1.0988. *Ans.*

EXAMPLES

Find log of 17.89; of 2172; of 652.12; of 4213; of 3342000.

C. To find the number corresponding to a given logarithm.

Ex. 1. — What number has the log 1.6085?

The mantissa 6085 is found in the 40 row and in the column under 6. The number corresponding to mantissa 6085 is therefore 406. The characteristic 1 states that there are two digits to the left of the decimal point. The number is therefore 40.6.

Ex. 2. — What number has the log 5.8716?

The mantissa 8716 is in the 74 row and in the column under 4. The characteristic 5 states that there are six digits to left of decimal point. The number is therefore 744000.

Ex. 3. — What number has the log 2.6538?

The mantissa 6538 is not found in the tables, but lies between 6532 and 6542, hence the number (not considering the decimal point) lies between 450 and 451. The difference between the mantissæ of 450 and 451 is 10; the difference between the mantissa of 450 and of the number to be found is 6. The difference between 450 and 451 is 1. $1 \times .6 = .6$. The number corresponding to mantissa 6538 is 4506, hence the number corresponding to log 2.6538 is 450.6.

EXAMPLES

Find number corresponding to log 1.5481; to log 0.6681; to log $\bar{1}.9559$; to log $\bar{2}.9324$.

No.	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
No.	0	1	2	3	4	5	6	7	8	9

No.	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996
No.	0	1	2	3	4	5	6	7	8	9

TRIGONOMETRY

THERE are many problems in the shop that involve a knowledge of angles and the relation between the parts of triangles — angles and sides. Such problems include laying out angles, depth of screw threads, diagonal distance across bolts, etc.

Trigonometry is the subject which deals with the properties and measurement of angles and sides of triangles, and is spoken of in the machine shop as simply "trig."

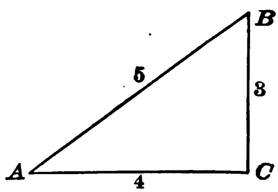
Trigonometric Functions (Ratios). — Since the sum of all the angles of a triangle equals 180°, and since a right triangle is composed of a right angle and two acute angles, it follows that, if we know one acute angle, we may obtain the other by subtracting it from 90°. The angle found by subtracting a definite angle from 90° is called the *complement* of the given angle.

The complement of 45° is 45°, for $90° - 45° = 45°$.

The complement of 30° is 60°, for $90° - 30° = 60°$.

The *supplement* of an angle is the difference between it and 180°. The supplement of 60° is 120°, for $180° - 60° = 120°$.

Examine a right angled triangle very carefully. Notice the position of the two acute angles and of the right angle. The longest side of the triangle, called the *hypotenuse*, is opposite the right angle. The sides of the triangle called the *legs*, are the two smallest sides. These sides are perpendicular to each other, and the longer side is opposite the greater angle.



RIGHT ANGLED TRIANGLE

Draw a right angle with sides respectively 3 and 4 inches long.

Then draw the hypotenuse and measure the length of it, which will be 5 inches.

Divide the length of the side opposite $\angle A$ by the hypotenuse, $3 \div 5 = .6$.

Divide the length of the side adjacent to $\angle A$ by the hypotenuse, $4 \div 5 = .8$.

Divide the length of the side opposite $\angle A$ by the adjacent side, $3 \div 4 = .75$.

These values are called ratios. The ratio of the length of the side opposite $\angle A$ to the length of the hypotenuse is called the *sine* of $\angle A$.

The ratio of the adjacent $\angle A$ to the length of hypotenuse is called the *cosine* of $\angle A$.

The ratio of the length of side opposite $\angle A$ to the length of the adjacent leg is called the *tangent* of $\angle A$.

The ratio of the length of the adjacent side of $\angle A$ to the opposite side is called the *cotangent.*

The ratio of the length of the hypotenuse to the adjacent side of $\angle A$ is called the *secant.*

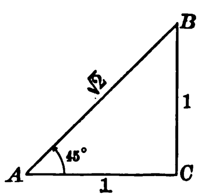
The ratio of the length of the hypotenuse to the opposite side of $\angle A$ is called the *cosecant.*

These six ratios or constants represent a definite relation between the sides and angles of right triangles.

Since all triangles may be divided into right angled triangles, by dropping a perpendicular from one of the vertices of the triangle, we might say these relations apply to all triangles. The same relations apply to all figures, since any figure may be divided into triangles, and then into right angled triangles.

These definite relations between the sides and angles are called the functions of the angles, and are really constants representing the fixed proportions between the sides and angles of a triangle. The exact values of these functions, or the proportion of the various parts of the triangles, have been figured out for every degree that will be used in daily practice. They are given in the tables for sine, cosine, and tangent at the end of this chapter.

By means of a protractor construct an angle of 45°, $\angle CAB$. At a point on AC 1″ from A, erect a perpendicular, CB. Connecting parts A and B by the line AB, we have a rt. Δ with $BC = AC$, because $\angle A =$ 45° and $\angle B = 45°$.



Measure accurately the length of all the sides of the triangle.

Divide the length of side opposite $\angle A$ by the hypotenuse, $\frac{BC}{AB} = .7$.

Divide the length of side adjacent to $\angle A$ by the hypotenuse, $\frac{AC}{AB} = .7$.

Divide the length of side opposite $\angle A$ by the side adjacent to $\angle A$, $\frac{BC}{AC} = 1.$

Divide the length of the adjacent side by the side opposite $\angle A$, $\frac{AC}{BC} = 1.$

Divide the length of the hypotenuse by the side adjacent to $\angle A$, $\frac{AB}{AC} = 1.4.$

Divide the length of the hypotenuse by the side opposite $\angle A$, $\frac{AB}{BC} = 1.4.$

If the work is carried out accurately, the above values should be obtained.

Find the value of sine, cosine, and tangent when angles of 25°, 30°, 60°, 75°, are constructed.

The reciprocal of the tangent of an angle ($\angle A$) is called the *co-tangent.*

The reciprocal of the sine of an angle ($\angle A$) is called the *cosecant.*

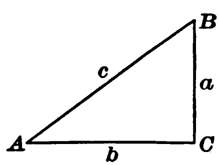
The reciprocal of the cosine of an angle ($\angle A$) is called the *secant.*

If the angles of the triangles are always marked with capital letters, and the length of the sides opposite the angles are marked with small letters as shown below, it will be possible to abbreviate functions of the angle in terms of the sides.

Trigonometrical Formulas, etc.

Geometrical Solution of Right Angled Triangles.

$$c = \sqrt{a^2 + b^2}$$
$$b = \sqrt{c^2 - a^2}$$
$$a = \sqrt{c^2 - b^2}$$



$$\sin A = \frac{a}{c} = \frac{\text{opposite side}}{\text{hypotenuse}} \qquad \cos A = \frac{b}{c} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

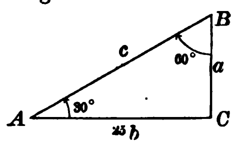
$$\tan A = \frac{a}{b} = \frac{\text{opposite side}}{\text{adjacent side}} \qquad \cot A = \frac{b}{a} = \frac{\text{adjacent side}}{\text{opposite side}}$$

$$\sec A = \frac{c}{b} = \frac{\text{hypotenuse}}{\text{adjacent side}} \qquad \text{cosec } A = \frac{c}{a} = \frac{\text{hypotenuse}}{\text{opposite side}}$$

Practical Value of a Trigonometric Ratio

Since each function of an angle may be expressed as the ratio of two sides, it follows that if we know the size of an angle and the length of one side of a right triangle, we may determine the other sides by substituting in the equation.

Ex. If one angle of a right angled triangle is 30°, and the adjacent side is 25 inches, what is the length of the other leg?



$$\angle B = 90° - 30° = 60°$$

Since we desire the other leg we should use

(Formula involving two legs)

$$\text{Tangent} \angle A = \frac{a}{25} \qquad \tan 30° = \frac{a}{25}$$

Looking up tables for tangent A, we find it to be .57735; substituting, we have

$$.577 = \frac{a}{25} \qquad a = 14.4$$

EXAMPLES

1. The simplest application of trigonometry in the shop is the problem for finding the depth of a V screw thread. Since the angle at the depth of the screw thread is 60° and the sides are equal, an equilateral triangle is formed. Forming right angled triangles by dropping a perpendicular from the angle of the thread, we divide the angle of 60° into two angles of 30° each. If we consider the pitch 1 inch, then the sides are each an inch and the perpendicular divides the base into two parts of ½ inch each.

The perpendicular, or depth of the thread, is the cosine of the angle of 30°. Looking in the table for 30°, then across until we come to the column headed cosine, we find .8. This gives us the depth directly.

2. Find the side of the thread if the depth is 1 inch.

3. What is the depth of a V thread with 8 threads to the inch?

4. What is the distance across the corners of a square bar $3\frac{1}{2}''$ on each side?

5. What size circular stock will be required to mill a square nut $1\frac{1}{16}''$ on a side?

6. What is the distance across the corners of a bar $2\frac{1}{2}''$ by $\frac{3}{4}''$? What angle does the diagonal make with the base?

Natural Trigonometric Functions

The tables on pages 326–329 give the numerical values of the trigonometric functions of angles between 0° and 90° with intervals of 10 minutes. For angles from 0° to and including 44°, read from the top of the table downwards; for angles from 45° to and including 89°, read from bottom of table upwards. The degrees and minutes are found in the column marked A or Angle, and the value of the function in the column marked by the name of the function. For instance, the value of the sine of 15° 40′ is found, in the column marked sine and in the row 40 under 15, to be .2700. The value of the cotangent of 63° 10′ is found (reading from the bottom of the table upwards, since the angle is between 45° and 89°), in the column marked cotangent (at bottom of table) and in the row marked 10 above 63, to be .5059. The values of the sine, tangent, and secant increase as the angle increases, while the values of the cosine, cotangent, and cosecant decrease as the angle increases, so that for the former functions the correction must be subtracted from the value given in the table.

I. To find the value of a function of an angle which is not given in the table.

Ex. 1. Find the value of sine 35° 21′.

Angle 35 is given in table, but 21′ is not. 21′, however, lies between 20′ and 30′, hence the value of sine of 35° 21′ will lie between the value of sine of 35° 20′ and the value of sine of 35° 30′. The value of sine of 35° 20′ is given in table as .5783, and the value of sine of 35° 30′ is .5807; the difference between these two values is .0024. The tabular difference in the angles (that is, the angular difference between 35° 20′ and 35° 30′) is 10′, while the angular difference between the smaller angle given in the table (35° 20′) and the angle of which we are trying to find the value (35° 21′) is 1′. The correction which we will have to add to .5783, the value of sine of 35° 20′, is $\frac{1}{10} \times .0024 = .00024$. Hence the value of sine of 35° 21′ is $.5783 + .00024 = .57854$.

Ex. 2. Find the value of cotangent 82° 41′.

82° 41′ lies between 82° 40′ and 82° 50′
cotan 82° 40′ = .12869
cotan 82° 50′ = .12574
Difference = .00295

The tabular difference between angles 82° 40′ and 82° 50′ is 10′. The difference between the smaller angle and given angle (82° 41′ − 82° 40′) is 1′. The correction is therefore $\frac{1}{10} \times .00295$ or .000295, but since the last figure is 5 we called it .00300. This correction must be subtracted from the value of cotan 82° 40′ since, as above, the value of the cotan decreases as the angle increases.

cotan 82° 41′ = .12869 − .003 = .12569

EXAMPLES

Find value of tan 45° 19′; cosine 32° 8′; cotan 78° 51′; sine 62° 37′.

II. To find the value of an angle, when the value of a function is given.

Ex. 1. Find the angle whose cotan is .6873.

In the column marked cotan look for the numbers .6873. Since it is found in the column marked cotan at the bottom of the table, the degrees and minutes will be found at the right-hand side of the table, reading from the bottom upwards. It is in the row marked 30 above 55. Hence, angle 55° 30′ has the cotan .6873.

Ex. 2. Find the angle whose cosine is .9387.

This number is found in the column marked cosine at the top of the table, therefore the degrees and minutes will be found on the left of the table reading from top downwards. It is in the row marked 10 under 20. Hence, the angle 20° 10′ has the cosine .9387.

EXAMPLES

Find the angle that has the sine .7642; cotan 1.5607; cosine .4746; tangent 1.5108.

A.	Sin.	Cos.		A.	Sin.	Cos.		A.	Sin.	Cos.	
0°	.000000	1.0000	**90°**	30′	.1305	.9914	30′	**15°**	.2588	.9659	**75°**
10′	.002909	1.0000	50′	40′	.1334	.9911	20′	10′	.2616	.9652	50′
20′	.005818	1.0000	40′	50′	.1363	.9907	10′	20′	.2644	.9644	40′
30′	.008727	1.0000	30′	**8°**	.1392	.9903	**82°**	30′	.2672	.9636	30′
40′	.011635	.9999	20′	10′	.1421	.9899	50′	40′	.2700	.9628	20′
50′	.014544	.9999	10′	20′	.1449	.9894	40′	50′	.2728	.9621	10′
1°	.017452	.9998	**89°**	30′	.1478	.9890	30′	**16°**	.2756	.9613	**74°**
10′	.02036	.9998	50′	40′	.1507	.9886	20′	10′	.2784	.9605	50′
20′	.02327	.9997	40′	50′	.1536	.9881	10′	20′	.2812	.9596	40′
30′	.02618	.9997	30′	**9°**	.1564	.9877	**81°**	30′	.2840	.9588	30′
40′	.02908	.9996	20′	10′	.1593	.9872	50′	40′	.2868	.9580	20′
50′	.03199	.9995	10′	20′	.1622	.9868	40′	50′	.2896	.9572	10′
2°	.03490	.9994	**88°**	30′	.1650	.9863	30′	**17°**	.2924	.9563	**73°**
10′	.03781	.9993	50′	40′	.1679	.9858	20′	10′	.2952	.9555	50′
20′	.04071	.9992	40′	50′	.1708	.9853	10′	20′	.2979	.9546	40′
30′	.04362	.9990	30′	**10°**	.1736	.9848	**80°**	30′	.3007	.9537	30′
40′	.04653	.9989	20′	10′	.1765	.9843	50′	40′	.3035	.9528	20′
50′	.04943	.9988	10′	20′	.1794	.9838	40′	50′	.3062	.9520	10′
3°	.05234	.9986	**87°**	30′	.1822	.9833	30′	**18°**	.3090	.9511	**72°**
10′	.05524	.9985	50′	40′	.1851	.9827	20′	10′	.3118	.9502	50′
20′	.05814	.9983	40′	50′	.1880	.9822	10′	20′	.3145	.9492	40′
30′	.06105	.9981	30′	**11°**	.1908	.9816	**79°**	30′	.3173	.9483	30′
40′	.06395	.9980	20′	10′	.1937	.9811	50′	40′	.3201	.9474	20′
50′	.06685	.9978	10′	20′	.1965	.9805	40′	50′	.3228	.9465	10′
4°	.06976	.9976	**86°**	30′	.1994	.9799	30′	**19°**	.3256	.9455	**71°**
10′	.07266	.9974	50′	40′	.2022	.9793	20′	10′	.3283	.9446	50′
20′	.07556	.9971	40′	50′	.2051	.9787	10′	20′	.3311	.9436	40′
30′	.07846	.9969	30′	**12°**	.2079	.9781	**78°**	30′	.3338	.9426	30′
40′	.08136	.9967	20′	10′	.2108	.9775	50′	40′	.3365	.9417	20′
50′	.08426	.9964	10′	20′	.2136	.9769	40′	50′	.3393	.9407	10′
5°	.08716	.9962	**85°**	30′	.2164	.9763	30′	**20°**	.3420	.9397	**70°**
10′	.09005	.9959	50′	40′	.2193	.9757	20′	10′	.3448	.9387	50′
20′	.09295	.9957	40′	50′	.2221	.9750	10′	20′	.3475	.9377	40′
30′	.09585	.9954	30′	**13°**	.2250	.9744	**77°**	30′	.3502	.9367	30′
40′	.09874	.9951	20′	10′	.2278	.9737	50′	40′	.3529	.9356	20′
50′	.10164	.9948	10′	20′	.2306	.9730	40′	50′	.3557	.9346	10′
6°	.10453	.9945	**84°**	30′	.2334	.9724	30′	**21°**	.3584	.9336	**69°**
10′	.10742	.9942	50′	40′	.2363	.9717	20′	10′	.3611	.9325	50′
20′	.11031	.9939	40′	50′	.2391	.9710	10′	20′	.3638	.9315	40′
30′	.11320	.9936	30′	**14°**	.2419	.9703	**76°**	30′	.3665	.9304	30′
40′	.11609	.9932	20′	10′	.2447	.9696	50′	40′	.3692	.9293	20′
50′	.11898	.9929	10′	20′	.2476	.9689	40′	50′	.3719	9283	10′
7°	.12187	.9925	**83°**	30′	.2504	.9681	30′	**22°**	.3746	.9272	**68°**
10′	.12476	.9922	50′	40′	.2532	.9674	20′	10′	.3773	.9261	50′
20′	.12764	.9918	40′	50′	.2560	.9667	10′	20′	.3800	.9250	40′
30′	.13053	.9914	30′	**15°**	.2588	.9659	**75°**	30′	.3827	.9239	30′
	Cos.	Sin.	A.		Cos.	Sin.	A.		Cos.	Sin.	A.

A.	Sin.	Cos.		A.	Sin.	Cos.		A.	Sin.	Cos.	
30′	.3827	.9239	30′	30°	.5000	.8660	60°	30′	.6088	.7934	30′
40′	.3854	.9228	20′	10′	.5025	.8646	50′	40′	.6111	.7916	20′
50′	.3881	.9216	10′	20′	.5050	.8631	40′	50′	.6134	.7898	10′
23°	.3907	.9205	67°	30′	.5075	.8616	30′	38°	.6157	.7880	52°
10′	.3934	.9194	50′	40′	.5100	.8601	20′	10′	.6180	.7862	50′
20′	.3961	.9182	40′	50′	.5125	.8587	10′	20′	.6202	.7844	40′
30′	.3987	.9171	30′	31°	.5150	.8572	59°	30′	.6225	.7826	30′
40′	.4014	.9159	20′	10′	.5175	.8557	50′	40′	.6248	.7808	20′
50′	.4041	.9147	10′	20′	.5200	.8542	40′	50′	.6271	.7790	10′
24°	.4067	.9135	66°	30′	.5225	.8526	30′	39°	.6293	.7771	51°
10′	.4094	.9124	50′	40′	.5250	.8511	20′	10′	.6316	.7753	50′
20′	.4120	.9112	40′	50′	.5275	.8496	10′	20′	.6338	.7735	40′
30′	.4147	.9100	30′	32°	.5299	.8480	58°	30′	.6361	.7716	30′
40′	.4173	.9088	20′	10′	.5324	.8465	50′	40′	.6383	.7698	20′
50′	.4200	.9075	10′	20′	.5348	.8450	40′	50′	.6406	.7679	10′
25°	.4226	.9063	65°	30′	.5373	.8434	30′	40°	.6428	.7660	50°
10′	.4253	.9051	50′	40′	.5398	.8418	20′	10′	.6450	.7642	50′
20′	.4279	.9038	40′	50′	.5422	.8403	10′	20′	.6472	.7623	40′
30′	.4305	.9026	30′	33°	.5446	.8387	57°	30′	.6494	.7604	30′
40′	.4331	.9013	20′	10′	.5471	.8371	50′	40′	.6517	.7585	20′
50′	.4358	.9001	10′	20′	.5495	.8355	40′	50′	.6539	.7566	10′
26°	.4384	.8988	64°	30′	.5519	.8339	30′	41°	.6561	.7547	49°
10′	.4410	.8975	50′	40′	.5544	.8323	20′	10′	.6583	.7528	50′
20′	.4436	.8962	40′	50′	.5568	.8307	10′	20′	.6604	.7509	40′
30′	.4462	.8949	30′	34°	.5592	.8290	56°	30′	.6626	.7490	30′
40′	.4488	.8936	20′	10′	.5616	.8274	50′	40′	.6648	.7470	20′
50′	.4514	.8923	10′	20′	.5640	.8258	40′	50′	.6670	.7451	10′
27°	.4540	.8910	63°	30′	.5664	.8241	30′	42°	.6691	.7431	48°
10′	.4566	.8897	50′	40′	.5688	.8225	20′	10′	.6713	.7412	50′
20′	.4592	.8884	40′	50′	.5712	.8208	10′	20′	.6734	.7392	40′
30′	.4617	.8870	30′	35°	.5736	.8192	55°	30′	.6756	.7373	30′
40′	.4643	.8857	20′	10′	.5760	.8175	50′	40′	.6777	.7353	20′
50′	.4669	.8843	10′	20′	.5783	.8158	40′	50′	.6799	.7333	10′
28°	.4695	.8829	62°	30′	.5807	.8141	30′	43°	.6820	.7314	47°
10′	.4720	.8816	50′	40′	.5831	.8124	20′	10′	.6841	.7294	50′
20′	.4746	.8802	40′	50′	.5854	.8107	10′	20′	.6862	.7274	40′
30′	.4772	.8788	30′	36°	.5878	.8090	54°	30′	.6884	.7254	30′
40′	.4797	.8774	20′	10′	.5901	.8073	50′	40′	.6905	.7234	20′
50′	.4823	.8760	10′	20′	.5925	.8056	40′	50′	.6926	.7214	10′
29°	.4848	.8746	61°	30′	.5948	.8039	30′	44°	.6947	.7193	46°
10′	.4874	.8732	50′	40′	.5972	.8021	20′	10′	.6967	.7173	50′
20′	.4899	.8718	40′	50′	.5995	.8004	10′	20′	.6988	.7153	40′
30′	.4924	.8704	30′	37°	.6018	.7986	53°	30′	.7009	.7133	30′
40′	.4950	.8689	20′	10′	.6041	.7969	50′	40′	.7030	.7112	20′
50′	.4975	.8675	10′	20′	.6065	.7951	40′	50′	.7050	.7092	10′
30°	.5000	.8660	60°	30′	.6088	.7934	30′	45°	.7071	.7071	45°
	Cos.	Sin.	A.		Cos.	Sin.	A.		Cos.	Sin.	A.

A.	Tan.	Cot.		A.	Tan.	Cot.		A.	Tan.	Cot.	
0°	.000000	∞	90°	30'	.1317	7.5958	30'	15°	.2679	3.7321	75°
10'	.002909	343.7737	50'	40'	.1346	7.4287	20'	10'	.2711	3.6891	50'
20'	.005818	171.8854	40'	50'	.1376	7.2687	10'	20'	.2742	3.6470	40'
30'	.008727	114.5887	30'	8°	.1405	7.1154	82°	30'	.2773	3.6059	30'
40'	.011636	85.9398	20'	10'	.1435	6.9682	50'	40'	.2805	3.5656	20'
50'	.014545	68.7501	10'	20'	.1465	6.8269	40'	50'	.2836	3.5261	10'
1°	.017455	57.2900	89°	30'	.1495	6.6912	30'	16°	.2867	3.4874	74°
10'	.02036	49.1039	50'	40'	.1524	6.5606	20'	10'	.2899	3.4495	50'
20'	.02328	42.9641	40'	50'	.1554	6.4348	10'	20'	.2931	3.4124	40'
30'	.02619	38.1885	30'	9°	.1584	6.3138	81°	30'	.2962	3.3759	30'
40'	.02910	34.3678	20'	10'	.1614	6.1970	50'	40'	.2994	3.3402	20'
50'	.03201	31.2416	10'	20'	.1644	6.0844	40'	50'	.3026	3.3052	10'
2°	.03492	28.6363	88°	30'	.1673	5.9758	30'	17°	.3057	3.2709	73°
10'	.03783	26.4316	50'	40'	.1703	5.8708	20'	10'	.3089	3.2371	50'
20'	.04075	24.5418	40'	50'	.1733	5.7694	10'	20'	.3121	3.2041	40'
30'	.04366	22.9038	30'	10°	.1763	5.6713	80°	30'	.3153	3.1716	30'
40'	.04658	21.4704	20'	10'	.1793	5.5764	50'	40'	.3185	3.1397	20'
50'	.04949	20.2056	10'	20'	.1823	5.4845	40'	50'	.3217	3.1084	10'
3°	.05241	19.0811	87°	30'	.1853	5.3955	30'	18°	.3249	3.0777	72°
10'	.05533	18.0750	50'	40'	.1883	5.3093	20'	10'	.3281	3.0475	50'
20'	.05824	17.1693	40'	50'	.1914	5.2257	10'	20'	.3314	3.0178	40'
30'	.06116	16.3499	30'	11°	.1944	5.1446	79°	30'	.3346	2.9887	30'
40'	.06408	15.6048	20'	10'	.1974	5.0658	50'	40'	.3378	2.9600	20'
50'	.06700	14.9244	10'	20'	.2004	4.9894	40'	50'	.3411	2.9319	10'
4°	.06993	14.3007	86°	30'	.2035	4.9152	30'	19°	.3443	2.9042	71°
10'	.07285	13.7267	50'	40'	.2065	4.8430	20'	10'	.3476	2.8770	50'
20'	.07578	13.1969	40'	50'	.2095	4.7729	10'	20'	.3508	2.8502	40'
30'	.07870	12.7062	30'	12°	.2126	4.7046	78°	30'	.3541	2.8239	30'
40'	.08163	12.2505	20'	10'	.2156	4.6382	50'	40'	.3574	2.7980	20'
50'	.08456	11.8262	10'	20'	.2186	4.5736	40'	50'	.3607	2.7725	10'
5°	.08749	11.4301	85°	30'	.2217	4.5107	30'	20°	.3640	2.7475	70°
10'	.09042	11.0594	50'	40'	.2247	4.4494	20'	10'	.3673	2.7228	50'
20'	.09335	10.7119	40'	50'	.2278	4.3897	10'	20'	.3706	2.6985	40'
30'	.09629	10.3854	30'	13°	.2309	4.3315	77°	30'	.3739	2.6746	30'
40'	.09923	10.0780	20'	10'	.2339	4.2747	50'	40'	.3772	2.6511	20'
50'	.10216	9.7882	10'	20'	.2370	4.2193	40'	50'	.3805	2.6279	10'
6°	.10510	9.5144	84°	30'	.2401	4.1653	30'	21°	.3839	2.6051	69°
10'	.10805	9.2553	50'	40'	.2432	4.1126	20'	10'	.3872	2.5826	50'
20'	.11099	9.0098	40'	50'	.2462	4.0611	10'	20'	.3906	2.5605	40'
30'	.11394	8.7769	30'	14°	.2493	4.0108	76°	30'	.3939	2.5386	30'
40'	.11688	8.5555	20'	10'	.2524	3.9617	50'	40'	.3973	2.5172	20'
50'	.11983	8.3450	10'	20'	.2555	3.9136	40'	50'	.4006	2.4960	10'
7°	.12278	8.1443	83°	30'	.2586	3.8667	30'	22°	.4040	2.4751	68°
10'	.12574	7.9530	50'	40'	.2617	3.8208	20'	10'	.4074	2.4545	50'
20'	.12869	7.7704	40'	50'	.2648	3.7760	10'	20'	.4108	2.4342	40'
30'	.13165	7.5958	30'	15°	.2679	3.7321	75°	30'	.4142	2.4142	30'
	Cot.	Tan.	A.		Cot.	Tan.	A.		Cot.	Tan.	A.

A.	Tan.	Cot.		A.	Tan.	Cot.		A.	Tan.	Cot.	
30′	.4142	2.4142	30′	**30°**	.5774	1.7321	**60°**	30′	.7673	1.3032	30′
40′	.4176	2.3945	20′	10′	.5812	1.7205	50′	40′	.7720	1.2954	20′
50′	.4210	2.3750	10′	20′	.5851	1.7090	40′	50′	.7766	1.2876	10′
23°	.4245	2.3559	**67°**	30′	.5890	1.6977	30′	**38°**	.7813	1.2799	**52°**
10′	.4279	2.3369	50′	40′	.5930	1.6864	20′	10′	.7860	1.2723	50′
20′	.4314	2.3183	40′	50′	.5969	1.6753	10′	20′	.7907	1.2647	40′
30′	.4348	2.2998	30′	**31°**	.6009	1.6643	**59°**	30′	.7954	1.2572	30′
40′	.4383	2.2817	20′	10′	.6048	1.6534	50′	40′	.8002	1.2497	20′
50′	.4417	2.2637	10′	20′	.6088	1.6426	40′	50′	.8050	1.2423	10′
24°	.4452	2.2460	**66°**	30′	.6128	1.6319	30′	**39°**	.8098	1.2349	**51°**
10′	.4487	2.2286	50′	40′	.6168	1.6212	20′	10′	.8146	1.2276	50′
20′	.4522	2.2113	40′	50′	.6208	1.6107	10′	20′	.8195	1.2203	40′
30′	.4557	2.1943	30′	**32°**	.6249	1.6003	**58°**	30′	.8243	1.2131	30′
40′	.4592	2.1775	20′	10′	.6289	1.5900	50′	40′	.8292	1.2059	20′
50′	.4628	2.1609	10′	20′	.6330	1.5798	40′	50′	.8342	1.1988	10′
25°	.4663	2.1445	**65°**	30′	.6371	1.5697	30′	**40°**	.8391	1.1918	**50°**
10′	.4699	2.1283	50′	40′	.6412	1.5597	20′	10′	.8441	1.1847	50′
20′	.4734	2.1123	40′	50′	.6453	1.5497	10′	20′	.8491	1.1778	40′
30′	.4770	2.0965	30′	**33°**	.6494	1.5399	**57°**	30′	.8541	1.1708	30′
40′	.4806	2.0809	20′	10′	.6536	1.5301	50′	40′	.8591	1.1640	20′
50′	.4841	2.0655	10′	20′	.6577	1.5204	40′	50′	.8642	1.1571	10′
26°	.4877	2.0503	**64°**	30′	.6619	1.5108	30′	**41°**	.8693	1.1504	**49°**
10′	.4913	2.0353	50′	40′	.6661	1.5013	20′	10′	.8744	1.1436	50′
20′	.4950	2.0204	40′	50′	.6703	1.4919	10′	20′	.8796	1.1369	40′
30′	.4986	2.0057	30′	**34°**	.6745	1.4826	**56°**	30′	.8847	1.1303	30′
40′	.5022	1.9912	20′	10′	.6787	1.4733	50′	40′	.8899	1.1237	20′
50′	.5059	1.9768	10′	20′	.6830	1.4641	40′	50′	.8952	1.1171	10′
27°	.5095	1.9626	**63°**	30′	.6873	1.4550	30′	**42°**	.9004	1.1106	**48°**
10′	.5132	1.9486	50′	40′	.6916	1.4460	20′	10′	.9057	1.1041	50′
20′	.5169	1.9347	40′	50′	.6959	1.4370	10′	20′	.9110	1.0977	40′
30′	.5206	1.9210	30′	**35°**	.7002	1.4281	**55°**	30′	.9163	1.0913	30′
40′	.5243	1.9074	20′	10′	.7046	1.4193	50′	40′	.9217	1.0850	20′
50′	.5280	1.8940	10′	20′	.7089	1.4106	40′	50′	.9271	1.0786	10′
28°	.5317	1.8807	**62°**	30′	.7133	1.4019	30′	**43°**	.9325	1.0724	**47°**
10′	.5354	1.8676	50′	40′	.7177	1.3934	20′	10′	.9380	1.0661	50′
20′	.5392	1.8546	40′	50′	.7221	1.3848	10′	20′	.9435	1.0599	40′
30′	.5430	1.8418	30′	**36°**	.7265	1.3764	**54°**	30′	.9490	1.0538	30′
40′	.5467	1.8291	20′	10′	.7310	1.3680	50′	40′	.9545	1.0477	20′
50′	.5505	1.8165	10′	20′	.7355	1.3597	40′	50′	.9601	1.0416	10′
29°	.5543	1.8040	**61°**	30′	.7400	1.3514	30′	**44°**	.9657	1.0355	**46°**
10′	.5581	1.7917	50′	40′	.7445	1.3432	20′	10′	.9713	1.0295	50′
20′	.5619	1.7796	40′	50′	.7490	1.3351	10′	20′	.9770	1.0235	40′
30′	.5658	1.7675	30′	**37°**	.7536	1.3270	**53°**	30′	.9827	1.0176	30′
40′	.5696	1.7556	20′	10′	.7581	1.3190	50′	40′	.9884	1.0117	20′
50′	.5735	1.7437	10′	20′	.7627	1.3111	40′	50′	.9942	1.0058	10′
30°	.5774	1.7321	**60°**	30′	.7673	1.3032	30′	**45°**	1.0000	1.0000	**45°**
	Cot.	Tan.	A.		Cot.	Tan.	A.		Cot.	Tan.	A.

TABLE OF FORMULAS

Circumference of the circle See page 62.

$$C = \pi D$$

Area of the circle See page 63.

$$A = C \times \frac{R}{2}$$

$$A = \pi 2R \times \frac{R}{2} = \pi R^2$$

$$A = \tfrac{1}{2} D \times \tfrac{1}{2} C$$

$$A = .7854\, D^2$$

Area of a ring See page 64.

$$B = .7854\,(D^2 - d^2)$$

Area of a triangle See page 68.

$$A = \tfrac{1}{2}\ \mathit{Base} \times \mathit{Altitude}$$

Area of a rectangle See page 69.

$$A = ba$$

Area of a trapezoid See page 69.

$$A = (b + c) \times \tfrac{1}{2} a$$

Area of a polygon See page 71.

$$A = \tfrac{1}{2} aP$$

Area of an ellipse

$$A = \pi \frac{d_1 d_2}{4}$$

Circumference of an ellipse See page 71.

$$C = \frac{d_1 + d_2}{2} \pi$$

Contents of a cylinder See page 72.

$$S = \pi R^2 H$$

Volume of a pyramid

$$V = \tfrac{1}{3} ba$$

Volume of a frustum of a pyramid

$$V = \tfrac{1}{3} h\,(b + b^1 + \sqrt{bb^1})$$

Surface of a regular pyramid See page 73.

$$S = P \times \tfrac{1}{2}\, sh$$

Volume of a cone See page 73

$$V = \tfrac{1}{3}\, \pi R^2 H$$
$$V = .2618\, D^2 H$$

Lateral surface of a cone See page 73.

$$S = \tfrac{1}{2}\, sh \times P$$

Volume of a frustum of a cone See page 74.

$$V = \tfrac{1}{3}\, H(B + B^1 + \sqrt{BB^1})$$

Volume of a sphere

$$V = \frac{4\,\pi R^3}{3}$$

Surface of a sphere See page 74.

$$S = 4\,\pi R^2$$

Lateral surface of a frustum of a cone See page 74.

$$S = \tfrac{1}{2}\, sh \times (P + P_1)$$

Volume of a barrel See page 75.

$$V = (D^2 \times 2) + d^2 \times L \times .2618$$

Diameter of blank for square bolt See page 127.

$$B = 1.414\, A$$

Diameter of blank for hexagonal bolt See page 127.

$$B = 1.155\, X$$

Pitch of a screw with V-shaped thread See page 140.

$$P = \frac{1}{No.\ of\ threads\ per\ inch}$$

Depth for V-shaped thread See page 140.

$$D = P \times .8660$$

Size of tap drill for V-shaped thread See page 141.

$$S = T - \frac{1.733}{N}$$

Size of tap drill for U. S. Standard Thread See page 142.

$$S = T - \frac{1.3}{N}$$

Depth of thread of U. S. Standard See page 142.

$$D = P \times .6495$$

Flat

$$F = \frac{P}{8}$$

Acme Standard Thread See page 143.

$$d = D - \frac{1.3732}{N}$$

Square Thread See page 144.

$$d = D - \frac{1}{N}$$

Belting See page 147.

$$L = 0.1309\, N(D + d)$$

Pulleys See page 151.

$$F = \frac{\pi DR}{12}$$

$$D = \frac{12\,F}{\pi R} \qquad R = \frac{12\,F}{\pi D}$$

Surface speed of pulleys See page 152.

$$D = \frac{D'N'}{N} \qquad N = \frac{D'N'}{D}$$

Thickness of tooth of gearing See page 162.

$$T = \frac{1.57}{Diametral\ Pitch}$$

Circular pitch See page 162.

$$CP = \frac{3.1416}{Diametral\ Pitch}$$

Diametral pitch See page 162.

$$DP = \frac{3.1416}{Circular\ Pitch}$$

Dimensions of gears by diametrical pitch See page 165.

$$P=\frac{N+2}{D}\qquad P=\frac{N}{D'}\qquad t=\frac{1.57}{P}$$

$$D'=\frac{D\times N}{N+2}\qquad D'=\frac{N}{P}\qquad D''=\frac{2}{P}$$

$$N=PD'\qquad N=PD-2\qquad f=\frac{t}{10}$$

$$D=\frac{N+2}{P}\qquad D=D'+\frac{2}{P}.$$

$$D''+f=\text{whole depth of tooth}$$

$$P=\frac{\pi}{P'}\qquad P'=\frac{\pi}{P}$$

Distance between centers of two gears See page 168.

$$a=\frac{D'+d'}{2}$$

$$a=\frac{b}{2P}$$

Volume of rectangular tank See page 173.

$$L\times B\times H\times 7.48=V$$

Volume of cylindrical tank See page 174.

$$C=d^2h\times .0034 \text{ or } .0034\,d^2h$$

Weight in lb. of lead pipe See page 175.

$$W=(D^2-d^2)\times 3.8697\times l$$

Cubical contents of a foot of pipe See page 176.

$$C=D^2\times .7854\times 12\div 231$$

$$C=D^2\times .0408$$

Capacity of a pipe of any length and any diameter See page 176.

$$C=D^2\times .0408\times L$$

$$C=D^2\times .7854\times L\div 231$$

Pressure of water per sq. in. See page 183.

$$P=h\times 0.434 \text{ lb. per sq. in.}$$

Head of water in feet See page 183.

$$h = \frac{p}{0.434} = \frac{1}{0.434} \times p = 2.31\,p$$

Thickness of pipe

$$T = \frac{h \times .s}{750}$$

Velocity of water See page 186.

$$V = \sqrt{\frac{H \times 2500}{l \times \dfrac{13.9}{d}}}$$

Head to produce a given velocity See page 187.

$$h = \frac{V^2 \times L \times \dfrac{13.9}{d}}{2500}$$

Twaddell scale into specific gravity See page 194.

$$\frac{(5 \times N) + 1000}{1000} = SG$$

To change specific gravity into Twaddell scale See page 195.

$$\frac{(SG \times 1000) - 1000}{5} = \text{Degrees Twaddell}$$

To convert Centigrade to Fahrenheit See page 196.

$$F = \frac{9\,C}{5} + 32°$$

To convert Fahrenheit to Centigrade See page 208.

$$C = \frac{5}{9}(F - 32°)$$

Thickness of boiler plate See page 208.

$$T = \frac{P \times R \times F.S.}{T.S. \times \%}$$

Diameter of boiler See page 210.

$$D = \frac{T \times T.S. \times \%}{P.F.} \times 2$$

Size of safety valve See page 216.

$$D = \sqrt{\frac{W \times 70}{P} \times 11 \div .7854}$$

$$D = \sqrt{\frac{22.5\,G}{(P + 8.62) \times .7854}}$$

Horse power of an engine See page 225.

$$H.\,P. = \frac{A \times P \times V}{33{,}000}$$

$$H.\,P.\ (\text{approx.}) = \frac{D^2}{2.5}$$

Diameter of cylinder See page 225.

$$D = \sqrt{\frac{5500 \times H.\,P.}{.7854 \times V}}$$

Diameter of supply pipe See page 225.

$$D = \sqrt{\frac{H.\,P.}{6}}$$

Electric current See page 232.

$$I = E \div R \text{ or } I = \frac{E}{R}$$

Power in watts See page 242.

$$P = \frac{E^2}{R}$$

Resistance of wire in ohms See page 244.

$$R = \frac{KL}{d^2}$$

Size of wire See page 246.

$$CM = \frac{21.6 \times DI}{e}$$

Resistance of cables See page 250.

$$R = \frac{10{,}000}{c.\,m.} \text{ in ohms per 1000 ft.}$$

Weight of cables See page 250.

$$W = .00305 \times c.\,m. \text{ in lb. per 1000 ft.}$$

Table of Decimal Equivalents of the Fraction of an Inch

By 8ths, 16ths, 32ds, and 64ths

8ths	32ds	64ths	64ths Continued
$\frac{1}{8}$ = .125	$\frac{1}{32}$ = .03125	$\frac{1}{64}$ = .015625	$\frac{33}{64}$ = .515625
$\frac{1}{4}$ = .250	$\frac{3}{32}$ = .09375	$\frac{3}{64}$ = .046875	$\frac{35}{64}$ = .546875
$\frac{3}{8}$ = .375	$\frac{5}{32}$ = .15625	$\frac{5}{64}$ = .078125	$\frac{37}{64}$ = .578125
$\frac{1}{2}$ = .500	$\frac{7}{32}$ = .21875	$\frac{7}{64}$ = .109375	$\frac{39}{64}$ = .609375
$\frac{5}{8}$ = .625	$\frac{9}{32}$ = .28125	$\frac{9}{64}$ = .140625	$\frac{41}{64}$ = .640625
$\frac{3}{4}$ = .750	$\frac{11}{32}$ = .34375	$\frac{11}{64}$ = .171875	$\frac{43}{64}$ = .671875
$\frac{7}{8}$ = .875	$\frac{13}{32}$ = .40625	$\frac{13}{64}$ = .203125	$\frac{45}{64}$ = .703125
16ths	$\frac{15}{32}$ = .46875	$\frac{15}{64}$ = .234375	$\frac{47}{64}$ = .734375
$\frac{1}{16}$ = .0625	$\frac{17}{32}$ = .53125	$\frac{17}{64}$ = .265625	$\frac{49}{64}$ = .765625
$\frac{3}{16}$ = .1875	$\frac{19}{32}$ = .59375	$\frac{19}{64}$ = .296875	$\frac{51}{64}$ = .796875
$\frac{5}{16}$ = .3125	$\frac{21}{32}$ = .65625	$\frac{21}{64}$ = .328125	$\frac{53}{64}$ = .828125
$\frac{7}{16}$ = .4375	$\frac{23}{32}$ = .71875	$\frac{23}{64}$ = .359375	$\frac{55}{64}$ = .859375
$\frac{9}{16}$ = .5625	$\frac{25}{32}$ = .78125	$\frac{25}{64}$ = .390625	$\frac{57}{64}$ = .890625
$\frac{11}{16}$ = .6875	$\frac{27}{32}$ = .84375	$\frac{27}{64}$ = .421875	$\frac{59}{64}$ = .921875
$\frac{13}{16}$ = .8125	$\frac{29}{32}$ = .90625	$\frac{29}{64}$ = .453125	$\frac{61}{64}$ = .953125
$\frac{15}{16}$ = .9375	$\frac{31}{32}$ = .96875	$\frac{31}{64}$ = .484375	$\frac{63}{64}$ = .984375

By 64ths; from $\frac{1}{64}$ to 1 inch

$\frac{1}{64}$ = .015625	$\frac{17}{64}$ = .265625	$\frac{33}{64}$ = .515625	$\frac{49}{64}$ = .765625
$\frac{1}{32}$ = .031250	$\frac{9}{32}$ = .281250	$\frac{17}{32}$ = .531250	$\frac{25}{32}$ = .781260
$\frac{3}{64}$ = .046875	$\frac{19}{64}$ = .296875	$\frac{35}{64}$ = .546875	$\frac{51}{64}$ = .796875
$\frac{1}{16}$ = .062500	$\frac{5}{16}$ = .312500	$\frac{9}{16}$ = .562500	$\frac{13}{16}$ = .812500
$\frac{5}{64}$ = .078125	$\frac{21}{64}$ = .328125	$\frac{37}{64}$ = .578125	$\frac{53}{64}$ = .828125
$\frac{3}{32}$ = .093750	$\frac{11}{32}$ = .343750	$\frac{19}{32}$ = .593750	$\frac{27}{32}$ = .843750
$\frac{7}{64}$ = .109375	$\frac{23}{64}$ = .359375	$\frac{39}{64}$ = .609375	$\frac{55}{64}$ = .859375
$\frac{1}{8}$ = .125000	$\frac{3}{8}$ = .375000	$\frac{5}{8}$ = .625000	$\frac{7}{8}$ = .875000
$\frac{9}{64}$ = .140625	$\frac{25}{64}$ = .390625	$\frac{41}{64}$ = .640625	$\frac{57}{64}$ = .890625
$\frac{5}{32}$ = .156250	$\frac{13}{32}$ = .406250	$\frac{21}{32}$ = .656250	$\frac{29}{32}$ = .906250
$\frac{11}{64}$ = .171875	$\frac{27}{64}$ = .421875	$\frac{43}{64}$ = .671875	$\frac{59}{64}$ = .921875
$\frac{3}{16}$ = .187500	$\frac{7}{16}$ = .437500	$\frac{11}{16}$ = .687500	$\frac{15}{16}$ = .937500
$\frac{13}{64}$ = .203125	$\frac{29}{64}$ = .453125	$\frac{45}{64}$ = .703125	$\frac{61}{64}$ = .953125
$\frac{7}{32}$ = .218750	$\frac{15}{32}$ = .468750	$\frac{23}{32}$ = .718750	$\frac{31}{32}$ = .968750
$\frac{15}{64}$ = .234375	$\frac{31}{64}$ = .484375	$\frac{47}{64}$ = .734375	$\frac{63}{64}$ = .984375
$\frac{1}{4}$ = .250000	$\frac{1}{2}$ = .500000	$\frac{3}{4}$ = .750000	1 = 1.000000

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